See back cover for an English translation of this cover

3

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91586M

915865

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tohua tēnei pouaka mēnā KĀORE koe i tuhituhi i roto i tēnei pukapuka

Te Pāngarau me te Tauanga (Tauanga), Kaupae 3, 2021

91586M Te whakamahi tuari tūponotanga hei whakaoti rapanga

Ngā whiwhinga: Whā

Paetae	Kaiaka	Kairangi
Te whakamahi tuari tūponotanga hei whakaoti rapanga.	Te whakamahi tuari tūponotanga mā te whakaaro tūhonohono hei whakaoti rapanga.	Te whakamahi tuari tūponotanga mā te whakaaro waitara hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–STATMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki roto i tētahi wāhi kauruku whakahāngai (). Ka tapahia pea tēnei wāhi ina mākahia te pukapuka.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TŪMAHI TUATAHI

E matapaetia ana i Memorial Avenue i Ōtautahi he 0.45 te tūponotanga me tū koe (ka tūpono atu ki tētahi rama pākā, whero rānei) i ngā rama ārahi waka e 6 kei te rori. Ko te awhe o ngā tawhiti i waenga i tēnā rama ārahi waka me tēnā rama ārahi waka mai i te 0.6 km te mea poto rawa ki te 1.2 km ki te mea roa rawa.

(a) Me kī ko te maha o ngā wā me tū he kaitaraiwa i ngā rama ārahi waka e 6 ina taraiwahia e ia te roanga o Memorial Avenue ka taea te whakatauira mā tētahi tuaritanga huarua.

(i) Parahautia te whakamahinga o te tuaritanga huarua hei whakatauira i te maha o ngā wā me tū he kaitaraiwa i te wā e taraiwa ana ia i te katoa o te roa o Memorial Avenue.

(ii) Tātaitia he whakatau tata mō te tūponotanga kāore e mate te kaitaraiwa ki te tū i tētahi o ngā rama ārahi waka e 6, me tū RĀNEI ia ki ngā rama ārahi waka katoa e 6 ina taraiwa ana i te katoa o te roa o Memorial Avenue.

QUESTION ONE

It is estimated that on Memorial Avenue in Christchurch there is probability of 0.45 of being required to stop (encountering an amber or red light) at each of the 6 sets of traffic lights on the road. Distances between each set of traffic lights range from 0.6 km at the shortest to 1.2 km at the longest.

(a) Suppose that the number of times a driver is required to stop at the 6 sets of traffic lights when they drive the complete length of Memorial Avenue can be modelled by a binomial distribution.

(i) Justify the use of the binomial distribution to model the number of times a driver is required to stop when they drive the complete length of Memorial Avenue.

(ii) Calculate an estimate for the probability that the driver is not required to stop at any of the 6 sets of lights OR is required to stop at all of the 6 sets of lights when driving the complete length of Memorial Avenue.

(b) Mō tētahi pūtahitanga whai rama ārahi waka i Memorial Avenue, nā ngā panoni haumaru e marohitia ana ka piki te tūponotanga me tū koe mā te 0.45 ki te 0.6. Me kī he 'whakarapa' he kaitaraiwa mēnā ka whakaritea ia kia tū ki ngā rama ārahi waka neke atu i te haurua o ngā mea e 6 ina taraiwa ia i te katoa o te roa o Memorial Avenue.

(i) Tātaitia tētahi whakatau tata mō te tūponotanga ka 'whakarapa' he kaitaraiwa ina taraiwa ia i te katoa o te roa o Memorial Avenue i runga i ngā panoni haumaru e marohitia ana.

(ii) Ka taraiwa tētahi tangata i te katoa o te roa o Memorial Avenue ki te mahi me te hoki ki te kāinga i te wiki mahi (mai i te Mane ki te Paraire).

Tūhuratia te whakapae kāore e pā ki tēnei tangata te tūponotanga nui ake ka 'whakarapa' ia i ia haerenga ōna i Memorial Avenue i tana wiki mahi, mēnā ka whakaurua mai ngā panoni haumaru e marohitia ana.

(b) For any given intersection with traffic lights on Memorial Avenue, proposed safety changes will increase the probability of being required to stop from 0.45 to 0.6. A driver can be considered 'unlucky' if they are required to stop at more than half of the 6 sets of traffic lights when they drive the complete length of Memorial Avenue.

(i) Calculate an estimate for the probability that a driver will be 'unlucky' when driving the complete length of Memorial Avenue with the proposed safety changes.

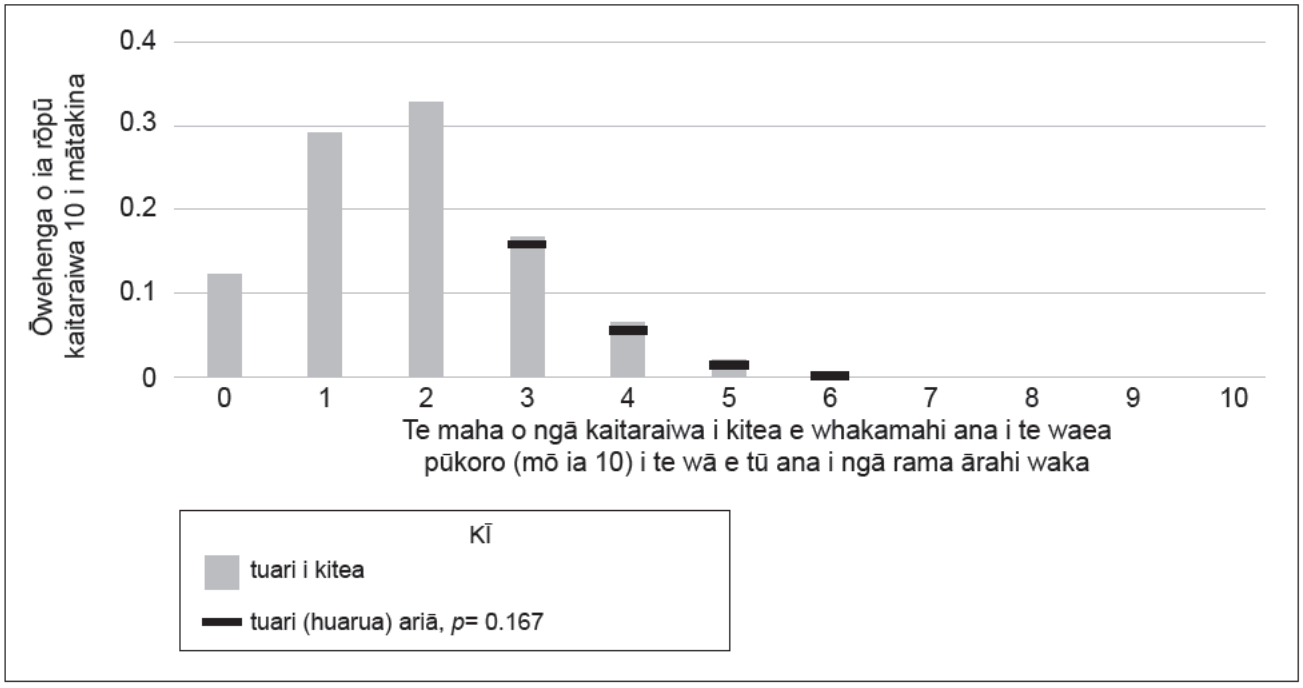
(ii) A person drives the entire length of Memorial Avenue to work and back home over a working week (Monday to Friday inclusive).

Investigate the claim that this person will not experience an increased likelihood of being 'unlucky' on every trip they take along Memorial Avenue during their working week, if the proposed safety changes are put in place.

(c) I pātai tētahi rangahau o Aotearoa o nā tata nei ki ngā kaitaraiwa kia kirimuna tā rātau whakautu mō ā rātau tikanga waea pūkoro ina taraiwa ana. 16.7% o ngā tāngata i whāki mai he whakamahi rātau i tā rātau waea i te wā e tū ana ki tētahi rama ārahi waka. I te hiahia tētahi kairangahau o Waitaha ki te kite mēnā he ōrite te whanonga o ngā kaitaraiwa i Memorial Avenue ki ngā kaitaraiwa i roto i tēnei rangahau.

I te Mahuru 2021, he mea whakaemi ngā raraunga mai i ngā rōpū kaitaraiwa 10 i tū ki ngā rama ārahi waka i Memorial Avenue. I tuhia te maha o ngā kaitaraiwa (mai i te 10) i te whakamahi i ā rātau waea pūkoro i te wā e tū ana i ngā rama ārahi waka. E whakaaturia ana ngā otinga ki te kauwhata i raro (te tuari whakamātau e whakaaturia ana ki te kiwikiwi). Ka whakatauritea ēnei otinga ki te tauira tuari huarua (te tuari ariā e whakaaturia ana ki te pango).

(i) E whakaatu ana te kauwhata i raro i te tuaritanga i kitea (ngā pae kiwikiwi) me tētahi tuari huarua o te $p = 0.167$, te tuari o te tauira ariā (e whakaaturia ana ki te pango).



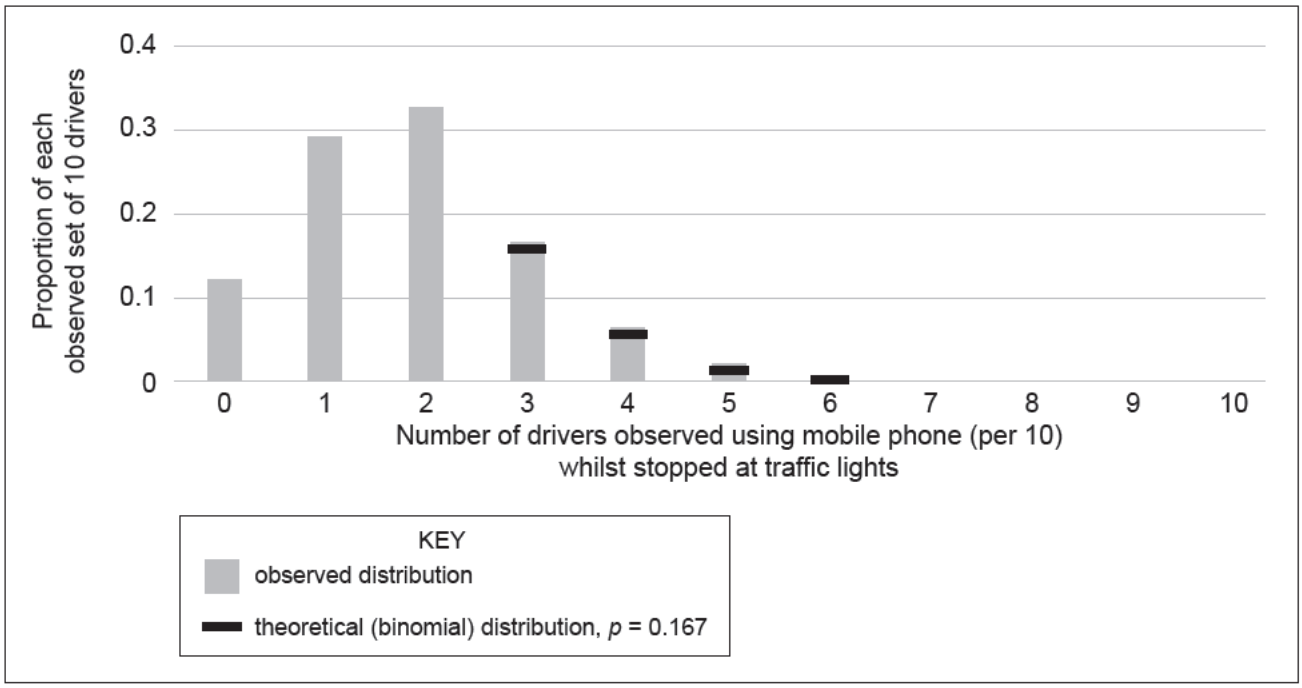
Whakaotihia te kauwhata mā te whakaatu i ngā uara e toe ana mō te tauira tuari huarua ariā.

Ki te hiahia koe ki te tuhi anō i tō urupare, whakamahia te kauwhata i te whārangi 24.

(c) A recent New Zealand study asked drivers to respond anonymously about their mobile phone habits whilst driving. 16.7% of people admitted to using their mobile phone while stopped at a traffic light. A Canterbury researcher was interested to see if the behaviour of drivers on Memorial Avenue was similar to the drivers in this New Zealand study.

In September 2021, data was collected from groups of 10 drivers stopped at traffic lights on Memorial Avenue. The number of drivers (out of 10) who were using their mobile phones whilst stopped at traffic lights was recorded. The results are shown in the graph below (the experimental distribution shown in grey). These results are compared to the binomial distribution model (the theoretical distribution shown in black).

(i) The graph below shows the observed distribution (grey bars) and a binomial distribution with $p = 0.167$, the theoretical model distribution (shown in black).



Complete the graph by showing the remaining values for the theoretical binomial distribution model.

If you need to redraw your response, use the graph on page 25.

(ii) Matapakitia ngā āhuatanga E RUA o ngā tuari i kitea me ngā tuari ariā e parahau ana i te tōtika o tēnei tauira tuari huarua mō te maha o ngā kaitaraiwa (mai i te 10) i kitea e whakamahi ana i ā rātau waea pūkoro.

(ii) Discuss TWO features of the observed and theoretical distributions that justify the suitability of this binomial distribution model for the number of drivers (out of 10) observed using their mobile phone.

TŪMAHI TUARUA

He whakaraerae nui ngā rākau paina e tipu haere noa ki te taiao tūturu o Aotearoa. Ko te rākau paina e tipu haere noa he rākau tērā kua tipu mai i tētahi kākano kāore i āta whakatipuria, ā, ka tipu ki tētahi wāhi kāore i te hiahiatia. E ai ki ngā rangahau i runga o Rangitoto, ko te tau toharite o ngā paina e tipu haere noa ana he 1.8 mō ia heketea.

He mea urutau mai i: https://www.google.com/maps

(a) (i) Whakamahia tētahi tuari tūponotanga tōtika hei tātai i tētahi whakatau tata mō te tūponotanga neke atu i te 4 ngā rākau paina e tipu haere noa ana i tētahi heketea i Rangitoto.

(ii) E ai ki ngā kaitauwhiro[1] taiao he angitu ina eke te tūponotanga o te kite i tētahi paina kotahi e tipu haere noa ana i tētahi heketea he kotahi hautoru.

Tātaihia te tawhā tauira i puta ai tēnei otinga.

[1] kaiwhāomoomo

QUESTION TWO

Wilding pine trees are a serious threat to the natural landscapes of New Zealand. A wilding pine is one that has grown from a seed that was not deliberately planted, and grows in a place where it is not wanted. Research has shown that on Rangitoto Island, the average number of wilding pines per hectare is 1.8.

Adapted from: https://www.google.com/maps

(a) (i) Use a suitable probability distribution to calculate an estimate for the probability that there are more than 4 wilding pines in any hectare on Rangitoto Island.

(ii) Conservationists consider it a success when the probability of finding at least one wilding pine in a hectare is one-third.

Calculate the model parameter that provides this result.

(iii) He 2311 heketea te horahanga o Rangitoto. Ko te 100 heketea he 1 kiromita pūrua te horahanga.

Homai kia RUA ngā pūtake kāore pea e tika ki te whakamahi i tētahi tauira Poisson mō te maha o ngā rākau paina e tipu haere noa ana i Rangitoto i tētahi kiromita pūrua.

Pūtake Tuatahi: _____

Pūtake Tuarua: _____

(b) I roto i ngā tau maha e ngana ana ki te whakakore atu i ngā rākau paina e tipu haere noa ana i Aotearoa, e mōhiotia ana i ngā wāhi mārakerake kei reira te tūponotanga ka hoki mai anō. Mō Te Ika-a-Māui, e ai ki ngā raraunga ko te āhua nei ka pā mai tēnei i waenganui i te 3 me te 18 marama i muri i te whakawāteatanga o te whenua. Mō Te Waipounamu, te āhua nei kei waenga i te 3 me te 27 marama, ā, ko te tino wā pea i te 12 marama i muri i te whakawāteatanga o te whenua.

Mēnā ka tipu haere noa ana ēnei paina i roto i te 8 marama o te whakawāteatanga o tētahi wāhi, he nui te utu ki te whakawātea anō.

(i) Ki te tukutuku i raro, tātuhia ngā tauira tuari tūponotanga e rua mō te wā ka tipu haere noa anō ngā paina mō Te Ika-a-Māui me Te Waipounamu.

Me mārama te tautohu ko tēhea a Te Ika-a-Māui, ko tēhea a Te Waipounamu.

Ki te hiahia koe ki te tuhi anō i ō tauira, whakamahia te tukutuku i te whārangi 24.

(iii) Rangitoto has an area of 2311 hectares. 100 hectares make up an area of 1 square kilometre.

Provide TWO reasons why it may not be appropriate to use a Poisson model for the number of wilding pines on Rangitoto Island in any square kilometre.

Reason One: _____

Reason Two: _____

(b) Over many years of trying to eliminate wilding pines from New Zealand, it is known that in cleared areas there is a probability of them returning. For the North Island, data suggests this is likely to occur anywhere between 3 and 18 months after the land was cleared. For the South Island, it is likely to be anywhere between 3 and 27 months, with the most likely time at 12 months after the land was cleared.

If wilding pines re-establish themselves within 8 months of an area being cleared, it is expensive to clear the area again.

(i) On the grid below, sketch the two probability distribution models for the time for the wilding pines to return for the North Island and the South Island.

Clearly identify which is the North Island and which is the South Island.

If you need to redraw your models, use the grid on page 25.

(ii) Tātaitia he whakatau tata mō te tūponotanga o tētahi wāhi i tīpakohia matapōkeretia i Te Ika-a-Māui me tētahi wāhi i tīpakohia matapōkeretia i Te Waipounamu kua tipu haere noa ngā paina i mua i te 8 marama i muri i te whakawāteatanga.

(iii) Hei tātai i te tūponotanga i (b)(ii) kotahi te whakapae i te iti rawa me tuku.

Me whakamārama mai mēnā e tika ana tēnei whakapae (kāore rānei).

(ii) Calculate an estimate for the probability that a randomly selected area in the North Island and a randomly selected area in the South Island both see a re-emergence of wilding pines before 8 months after clearing.

(iii) Calculating the probability in (b)(ii) required at least one assumption to be made.

Explain whether this assumption is likely (or unlikely) to be valid.

TŪMAHI TUATORU

He tino hira ngā pūnaha whakahaere rorohiko rerekē e rua, Pūnaha A me Pūnaha W.

(a) E whakaatu ana te papatau i raro i ngā tuari tūponotanga o te taurangi matapōkere E, arā, te maha o ngā tāpiritanga (pērā i ngā tukuoro, ngā papapātuhi ahokore) he kaipupuri o te Pūnaha A me tētahi kaipupuri o Pūnaha W ka hokona mai mō tana rorohiko i te wā kei a ia.

e	0	1	2	3	4	5	6	7
Pūnaha A P($E = e$)	0.01	0.04	0.10	0.39	0.32	0.08	0.04	0.02
Pūnaha W P($E = e$)	0.23	0.12	0.09	0.04	0.05	0.18	0.14	0.15

Ko ngā uara e tūmanakohia ana mō te maha o ngā tāpiritanga mō Pūnaha A me Pūnaha W he tino ōrite, engari ko te ine mahora (SD) mō Pūnaha W he tino nui ake i te Pūnaha A (tirohia ngā uara i raro).

Pūnaha A: E(E) = 3.47 SD(E) = 1.2
Pūnaha W: E(E) = 3.41 SD(E) = 2.6

(i) Whakamahia te whakaaro whaitake o te tauanga hei whakamārama he aha ngā āhuatanga o ngā tuari e puta ai ēnei ōritetanga me ēnei rerekētanga i roto i ngā uara e tūmanakohia ana me ngā ine mahora mō te maha o ngā tāpiritanga kua hokona mai.

Te (ngā) pūtake mō ngā uara ōrite e tūmanakohia ana: _____

Te (ngā) pūtake mō ngā ine mahora rerekē: _____

QUESTION THREE

Two different computer operating systems, System A and System W, are very popular.

(a) The table below shows the probability distributions of the random variable E, the number of extras (such as speakers or wireless keyboards) an owner of System A and an owner of System W buys for their computer over the time they own it.

e	0	1	2	3	4	5	6	7
System A $P(E = e)$	0.01	0.04	0.10	0.39	0.32	0.08	0.04	0.02
System W $P(E = e)$	0.23	0.12	0.09	0.04	0.05	0.18	0.14	0.15

The expected values for the number of extras for System A and System W are very similar, but the standard deviation for System W is much larger than System A (see values below).

System A: E(E) = 3.47 SD(E) = 1.2
System W: E(E) = 3.41 SD(E) = 2.6

(i) Use statistical reasoning to explain what features of the distributions cause these similarities and differences in expected values and standard deviations for the number of extras purchased.

Reason(s) for similar expected values: _____

Reason(s) for different standard deviations: _____

(ii) Whakatauritea ngā tūponotanga o te hoko i ngā tāpiritanga e 5, neke atu rānei mō Pūnaha A me Pūnaha W.

(b) Mō ngā momo pūnaha e rua, ko te whakapono mō te wā e pā mai ai tētahi hapa pūnaha nui (mai i te hokonga) ka taea te whakatauira mā tētahi tuari māori me tētahi toharite o te 5.5 tau me tētahi ine mahora o te 0.9 tau.

(i) Whakatau tatatia te tūponotanga, ina he neke atu i te 6 tau te roa e mahi tika ana tētahi rorohiko mai i ngā pūnaha e rua tae noa ki te pānga mai o te hapa nui tuatahi, ka mahi tika mō neke atu i te 7½ tau.

(ii) Compare the probabilities of buying 5 or more extras for System A and System W.

(b) For both system types, it is believed that the time taken for a serious system error to occur (from purchase) can be modelled by a normal distribution with a mean of 5.5 years and a standard deviation of 0.9 years.

(i) Estimate the probability that, given a computer from either system lasts for more than 6 years until its first serious error, it will last more than 7½ years.

(ii) Ko tētahi atu pūnaha whakahaere, Pūnaha L, e mōhiotia ana ka pā mai tētahi hapa pūnaha nui i roto i te 2½ tau tuatahi i muri i te hokotanga, neke atu rānei i te 8½ tau i muri i te hokotanga, me tētahi tūponotanga tapeke neke atu i te 4%.

Parahautia mēnā ka taea ngā tawhā tuari māori mai i (b)(i) (toharite = 5.5 tau, ine mahora = 0.9 tau) te whakatauira te wā ki te hapa nui mō Pūnaha L.

Tautokona tō tuhinga ki te whakaaro whaitake o te tauanga, me ngā tātaitanga.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 22.

(ii) Another operating system, System L, is known to experience a serious system error within the first 2½ years after purchase or more than 8½ years after purchase, with a total probability of just over 4%.

Justify whether the normal distribution parameters from (b)(i) (mean = 5.5 years, standard deviation = 0.9 years) can be used to model the time to serious error for System L.

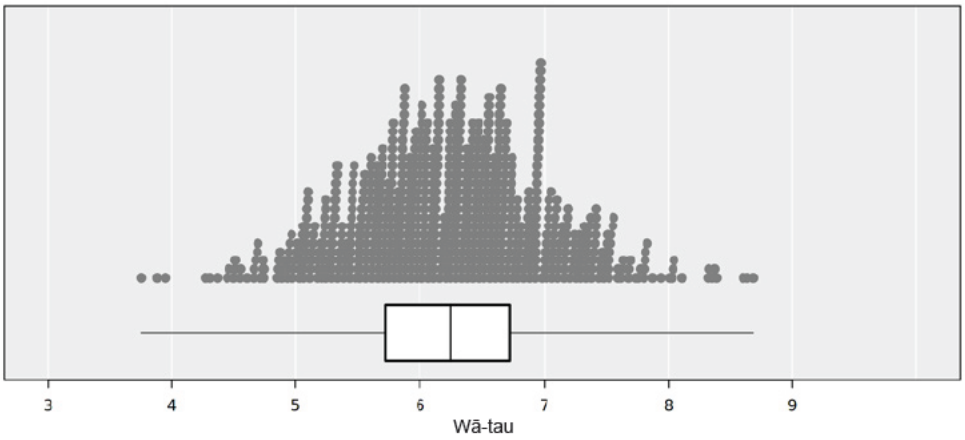
Support your answer with statistical reasoning, including calculations.

Question Three continues on page 23.

(c) I hangaia e tētahi atu kamupene he pūnaha whakahaere, e kīia ana ko Pūnaha C. E āhua ū ana ngā whakaaro o te kamupene he mea hanga e rātau a Pūnaha C kia roa ake te wā mō te pā mai o tētahi hapa pūnaha nui i te toharite tēnā i ētahi atu pūnaha whakahaere (Pūnaha A, W, me L). Mō Pūnaha C, ko te wā e pā mai ai he hapa pūnaha nui he 6.25 tau te toharite me te ine mahora o te 0.8 tau.

Ka tukuna ki te kamupene ngā taipitopito o tētahi tīpako o ngā rorohiko 800 e pā mai ai he hapa nui.

Ko te tuari me ngā tatauranga whakarāpopototanga mō te wā ki te hapa pūnaha nui i muri i te hokotanga mō tēnei tīpako ka whakaaturia ki te kauwhata i raro.



Mōkito	25%	Tau waenga	75%	Mōrahi	Tau toharite	Ine mahora (SD)	Rahi o te Tīpako
3.754	5.718	6.246	6.722	8.688	6.236	0.7755	800

Wā ki te hapa nui mō ngā rorohiko 800 e whakamahi ana i te Pūnaha C

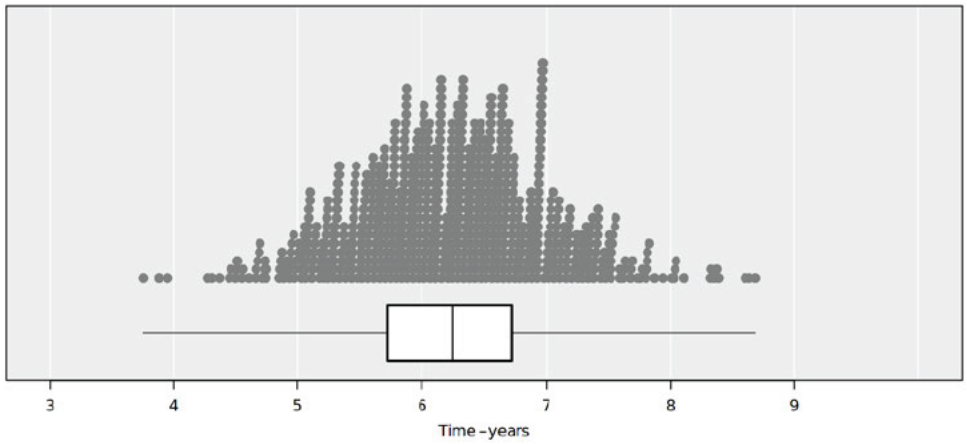
Parahautia te whakamahinga o tētahi tuari māori he 6.25 tau te toharite me te 0.8 tau te ine mahora hei whakatauira i te wā ki te hapa nui i muri mai i te hokotanga mō te Pūnaha C.

Whakamahia ngā tātai o te whakaaro whaitake o te tauanga hei parahau i tō tuhinga.

(c) Another company developed an operating system, named System C. The company is fairly sure that they have developed System C so that the time for a serious system error to occur is longer on average than the other operating systems (Systems A, W, and L). For System C, the time for a serious system error to occur is on average 6.25 years with standard deviation 0.8 years.

Details of a sample of 800 computers that develop a serious error are sent to the company.

The distribution and summary statistics for the time to serious system error after purchase for this sample are shown in the graph below.



Min	25%	Median	75%	Max	Mean	SD	Sample Size
3.754	5.718	6.246	6.722	8.688	6.236	0.7755	800

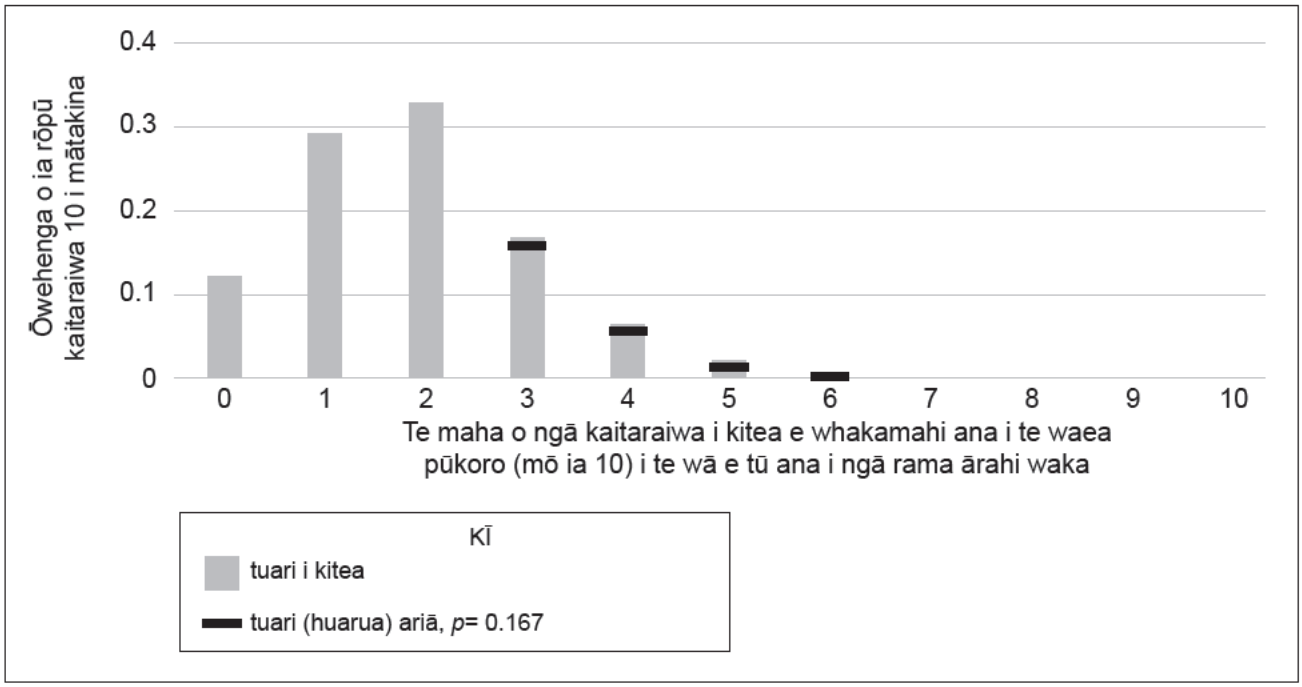
Time to serious error for 800 computers using System C

Justify the use of a normal distribution with mean 6.25 years and standard deviation 0.8 years to model the time to serious error after purchase for System C.

Use statistical reasoning calculations to justify your answer.

HE HOAHOA WĀTEA

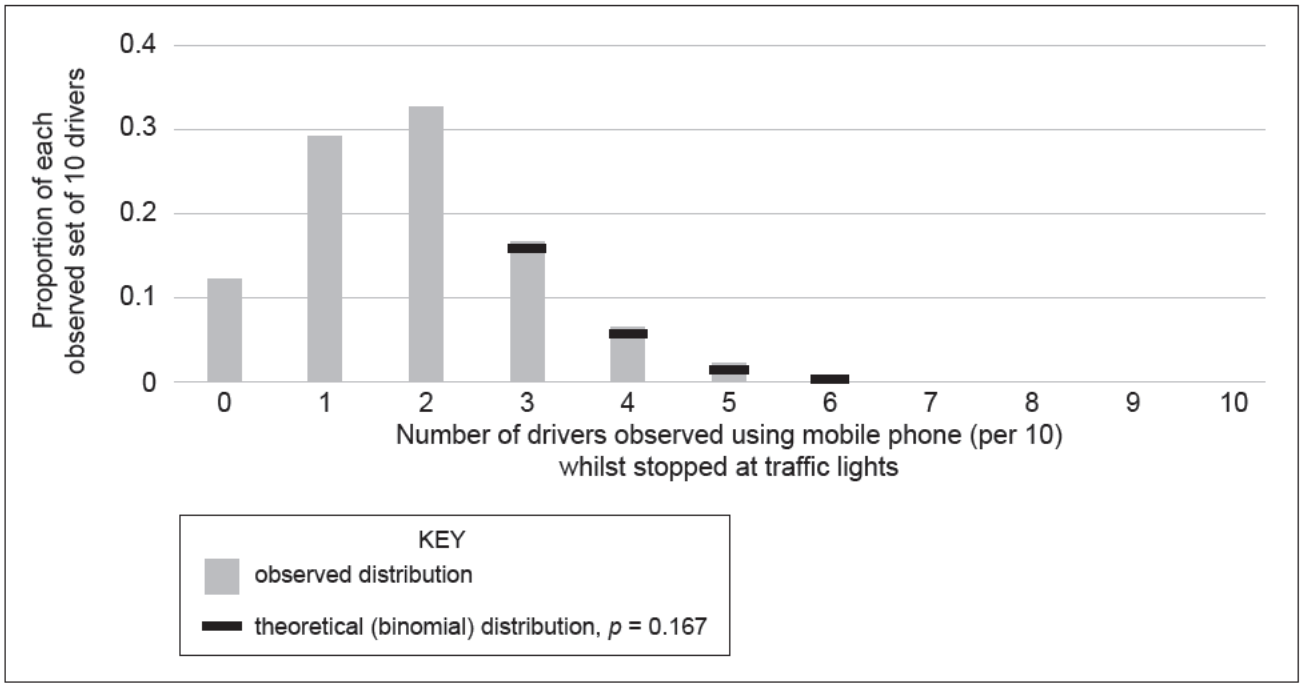
Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuatahi (c)(i), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.



Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuarua (b)(i), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

SPARE DIAGRAMS

If you need to redraw your response to Question One (c)(i), use the grid below. Make sure it is clear which answer you want marked.



If you need to redraw your response to Question Two (b)(i), use the grid below. Make sure it is clear which answer you want marked.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Mathematics and Statistics (Statistics) 2021

91586M Apply probability distributions in solving problems

Credits: Four

91586M

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.