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91261M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

SUPERVISOR'S USE ONLY

Tohua tēnei pouaka mēnā KĀORE koe i tuhi kōrero ki tēnei pukapuka

Te Pāngarau me te Tauanga, Kaupae 2, 2022

91261M Te whakamahi tikanga taurangi i te wā e whakaoti rapanga ana

Ngā whiwhinga: E whā

Paetae	Kaiaka	Kairangi
Te whakamahi tikanga taurangi i te wā e whakaoti rapanga ana.	Te whakamahi tikanga taurangi i te wā e whakaoti rapanga ana, mā roto i te whakaaro whai pānga.	Te whakamahi tikanga taurangi i te wā e whakaoti rapanga ana, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Whakaaturia ngā whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te Pepa Ture Tātai L2–MATHMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapa o ngā whārangi 2–31, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (). Ka poroa pea taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whakaotihia $\frac{2x-3}{x+4}-3=0$.

(b) (i) Whakatauwehea katoatia $6x^3y-15x^2\sqrt{y}$.

(ii) Whakarūnātia katoatia $\frac{6x^2-x-12}{3x^2-5x-12}$.

QUESTION ONE

(a) Solve $\frac{2x-3}{x+4}-3=0$.

(b) (i) Factorise completely $6x^3y-15x^2\sqrt{y}$.

(ii) Simplify fully $\frac{6x^2-x-12}{3x^2-5x-12}$.

(c) I ētahi wā ka pēnei te takotoranga o te wātaka, arā, ka tukua tētahi nama i te 1 ki te 365 ki ia rā. Anei te tīmatatanga o te wātaka o tētahi tau:

M	T	W	T	P	H	H
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

Ka tuhi a Jo i tētahi tapawhā 4-ki-4 hei āta tirotiro i tētahi whakapae kua rangona e ia:

"*he ōrite i ngā wā katoa ngā tapeke o ngā kokonga tauaro e hauroki ana, ahakoa ki hea tuhia ai tō tapawhā* ". Arā, e mea ana, ki te tāpiri koe i ngā tau i ngā kokonga karaka, e ōrite ana ki tō tāpiri i ngā tau i ngā kokonga kahurangi.

Ka whakaaro ake a Jo mehemea ka tika tonu te whakapae ahakoa ki hea tīmatahia ai e ia te tapawhā, ā, ka tīmata ia i tētahi tūhura mā te whakamahi i te taurangi:

A			

(i) Whakamahia te taurangi hei hāpono i te kōrero, ahakoa ki hea tuhia ai te tapawhā 4-ki-4 i te wātaka, **me ōrite te tapeke o ngā kokonga karaka ki te tapeke o ngā kokonga kahurangi**.

(ii) Ka whakaaroaro ake a Jo ki ngā **whakareatanga** o ngā kokonga tauaro e hauroki ana: ka ōrite rānei?

Whakamahia te taurangi hei hāpono i te kōrero **kāore** e āhei ana te tuhia o tētahi tapawhā 4-ki-4 e ōrite ana te whakareatanga o ngā kokonga tauaro e hauroki ana.

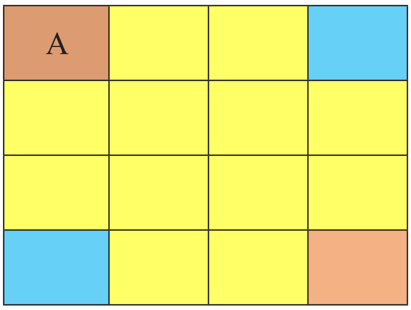
(c) A calendar can be presented in the following way, where each day is given a number from 1 to 365. This is the beginning of a year's calendar:

M	**T**	**W**	**TH**	**F**	**SA**	**SU**
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

Jo draws a 4-by-4 square on the calendar to check a claim that she heard:

"*the sums of the diagonally opposite corners are always the same, no matter where you make your square*". In other words, when you add the numbers in the orange corners, it is the same as when you add the numbers in the blue corners.

Jo wonders if the claim will still be true no matter where she starts the square, so she begins an investigation using algebra:



(i) Use algebra to prove that, no matter where the 4-by-4 square is drawn on the calendar, **the sum of the orange corners must be the same as the sum of the blue corners**.

(ii) Jo wonders about the **products** of the diagonally opposite corners: could they be the same?

Use algebra to prove that it is **not** possible to draw any 4-by-4 square for which the products of the diagonally opposite corners are the same.

(d) Ka tika rānei i ngā wā katoa, me ōrite te **tapeke** o ngā kokonga karaka ki te tapeke o ngā kokonga kahurangi, ahakoa te rahi, te āhua rānei o te tapawhā ka tuhia e Jo?

Whakamahia te taurangi hei tautoko i tō whakautu mā te whai whakaaro ki tētahi tapawhā m-ki-n ka tuhia ki te wātaka i raro nei (arā, ko m me n ngā tauoti e nui ake ana i te 1, ā, $m \neq n$).

Ka hiahia pea koe ki te tā i tētahi hoahoa ki te wātaka, ki tōna taha rānei, hei āwhina i te whakamāramatanga o ō whakaaro.

M	**T**	**W**	**T**	**P**	**H**	**H**
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
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60	61	62	63	64	65	66
67	68	69	70	71	72	73

(d) Will it always be true that the **sum** of the orange corners must be the same as the sum of the blue corners, regardless of the size or shape of the rectangle Jo draws?

Use algebra to support your answer by considering an m-by-n rectangle drawn on the calendar below (where m and n are whole numbers greater than 1, and $m \neq n$).

You may wish to draw a diagram on the calendar, or beside it, to help explain your reasoning.

M	T	W	TH	F	SA	SU
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

TE TŪMAHI TUARUA

(a) Kei tētahi whārite pūrua, ax^2 + bx + c = 0, ēnei otinga $\frac{1}{3}$ me $\frac{-2}{7}$.

Whiriwhiria ngā uara o ngā tau tōpū a, b, me c.

a = ______ b = ______ c = ______

(b) (i) He aha te tātaritanga o te whārite $2x^2 - 12x + 7 = 0$?

(ii) Whakapaetia y = $2x^2 - 12x$ + k, arā, e pūmau ana a k.

He aha te uara o k e kotahi anake ai te otinga o te whārite $y = 0$?

QUESTION TWO

(a) A quadratic equation, $ax^2 + bx + c = 0$, has solutions of $\frac{1}{3}$ and $\frac{-2}{7}$.

Find the values of the integers a, b, and c.

a = ______ b = ______ c = ______

(b) (i) What is the discriminant of the equation $2x^2 - 12x + 7 = 0$?

(ii) Suppose $y = 2x^2 - 12x + k$, where k is a constant.

For what value of k will the equation $y = 0$ have exactly one solution?

(c) Whakaotihia $\sqrt{2x+3}=3x$.

(d) Whakapaetia $Q(x) = fx^2 + gx + h$, arā, ko f, ko g, me h e pūmau ana. Ko te "whārite huripoki" o $Q(x)$ e tautuhia ana hei $Q^*(x) = hx^2 + gx + f$, arā, kua whakaraupapa whakamurihia ngā tau whakarea.

(i) Whiriwhiria ngā otinga o te whārite $Q(x) = Q^*(x)$.

(c) Solve $\sqrt{2x+3} = 3x$.

(d) Suppose $Q(x) = fx^2 + gx + h$, where f, g, and h are constants. The "reciprocal polynomial" of $Q(x)$ is defined as $Q^*(x) = hx^2 + gx + f$, where the coefficients are in the reverse order.

(i) Find the solutions of the equation $Q(x) = Q^*(x)$.

(ii) Whakapaetia e 2 ngā pūtake rerekē o $Q(x) = 0$, arā, ko A me B.

He whakareatanga nō A me B ngā pūtake o $Q^*(x) = 0$, arā, ko kA me kB ngā pūtake mō ētahi tau pūmau, mō k.

Whiriwhiria tētahi rerenga mō k e hāngai ana ki te f, ki te g, me te h hoki/rānei.

(ii) Suppose that $Q(x) = 0$ has 2 different roots, A and B.

The roots of $Q^*(x) = 0$ are multiples of A and of B, i.e. the roots are kA and kB for some constant k.

Find an expression for k in terms of f, g, and / or h.

TE TŪMAHI TUATORU

(a) (i) Whakarūnātia katoatia $\sqrt{49y^{36}}$.

(ii) Whakaotihia te whārite $2^x = 2022$.

(b) Whakarūnātia katoatia te rerenga e whai ake nei, tuhia tō tuhinga hei pūkōaro māmā.

$$\log(3a) + 2\log\left(\frac{a}{6}\right)$$

QUESTION THREE

(a) (i) Simplify fully $\sqrt{49y^{36}}$.

(ii) Solve the equation $2^x = 2022$.

(b) Simplify the following expression fully, writing your answer as a single logarithm.

$$\log(3a) + 2\log\left(\frac{a}{6}\right)$$

(c) Whakaarohia te whārite $\log_2(x - \mathrm{a}) - \log_2(x + \mathrm{a}) = \mathrm{c}$, arā, e pūmau ana te a me te c.

(i) Whakaaturia ka noho ana te x hei tumu mō tēnei whārite, ka $x = \mathrm{a}\dfrac{1+2^{\mathrm{c}}}{1-2^{\mathrm{c}}}$.

Me whakamahi koe i ngā kīanga pāngarau e tika ana i tō whakamāramatanga.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(c) Consider the equation $\log_2(x - \mathrm{a}) - \log_2(x + \mathrm{a}) = \mathrm{c}$, where a and c are constants.

(i) Show that when x is made the subject of this equation, $x = \mathrm{a}\dfrac{1+2^{\mathrm{c}}}{1-2^{\mathrm{c}}}$.

Ensure that you use correct mathematical statements in your reasoning.

Question Three continues on the following page.

(ii) Ko ētahi uara anake o te a me te c e tika ana hei otinga mō te whārite $\log_2(x - \mathrm{a}) - \log_2(x + \mathrm{a}) = \mathrm{c}$.

Mā te āta whakamārama i ō whakaaro, whakaahuatia mā ēhea uara o te a me te c e taea ai te whārite te whakaoti.

Tērā pea ka whai hua tō maumahara ake, ka noho ana te x hei tumu i tēnei whārite, ka $x = \mathrm{a}\dfrac{1+2^{\mathrm{c}}}{1-2^{\mathrm{c}}}$

(ii) The equation $\log_2(x - a) - \log_2(x + a) = c$, is only possible to solve for some values of a and for some values of c.

Explaining your reasoning clearly, describe which values of a and of c will make the equation possible to solve.

You may find it useful to recall that, when x is made the subject of this equation, $x = a\dfrac{1+2^c}{1-2^c}$.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 2 Mathematics and Statistics 2022

91261M Apply algebraic methods in solving problems

Credits: Four

91261M

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 31 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.