*See back cover for an English translation of this cover*

2

**91262M**

912625

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

**QUALIFY FOR THE FUTURE WORLD**
**KIA NOHO TAKATŪ KI TŌ ĀMUA AO!**

Tohua tēnei pouaka mēnā KĀORE koe i tuhi kōrero ki tēnei pukapuka

# Te Pāngarau me te Tauanga, Kaupae 2, 2022

## 91262M Te whakamahi tikanga tuanaki i te wā e whakaoti rapanga ana

Ngā whiwhinga: E rima

| Paetae | Kaiaka | Kairangi |
|---|---|---|
| Te whakamahi tikanga tuanaki i te wā e whakaoti rapanga ana. | Te whakamahi tikanga tuanaki, mā roto i te whakaaro whai pānga, i te wā e whakaoti rapanga ana. | Te whakamahi tikanga tuanaki, mā roto i te whakaaro waitara e whānui ana, i te wā e whakaoti rapanga ana. |

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

**Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.**

Whakaaturia ngā whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te Pepa Ture Tātai L2–MATHMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapa o ngā whārangi 2 – 27, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (). Ka poroa pea taua wāhanga ka mākahia ana te pukapuka.

**HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU**.

## TE TŪMAHI TUATAHI

(a) Ka tuhia tētahi pānga $f$ e pēnei ana $f(x) = 2x^4 + 4x^3 - 20x^2 - 5$.

Whakamahia te tuanaki ki te whiriwhiri i te rōnaki o te kauwhata o te pānga i te pūwāhi o $x = 3$.

(b) Mō te pānga $f$:

$$f'(x) = 4 - 6x + 2x^2$$

Ka whakawhiti te kauwhata o $f(x)$ mā te pūwāhi (3,4).

Whiriwhiria te whārite o te pānga $f$.

## QUESTION ONE

(a) A function $f$ is given by $f(x) = 2x^4 + 4x^3 - 20x^2 - 5$.

Use calculus to find the gradient of the graph of the function at the point where $x = 3$.

(b) For the function $f$:

$$f'(x) = 4 - 6x + 2x^2$$

The graph of $f(x)$ passes through the point (3,4).

Find the equation of the function $f$.

(c) Whakamahia te tuanaki hei whiriwhiri i ngā uara o x e heke nei te kauwhata o te pānga $f(x) = \frac{2x^3}{3} + \frac{3x^2}{2} - 20x - 3$ .

(d) Kei te pātapa ki te kōpiko $f(x) = \text{p}x - \text{q}x^2$ kei te pūwāhi (2,–10) he –6 te rōnaki.

Whiriwhiria ngā uara o ngā pūmau p me q.

(c) Use calculus to find the values of $x$ for which the graph of the function $f(x)=\frac{2x^3}{3}+\frac{3x^2}{2}-20x-3$ is decreasing.

(d) The tangent to the curve $f(x) = \text{p}x - \text{q}x^2$ at the point (2,–10) has a gradient of –6.

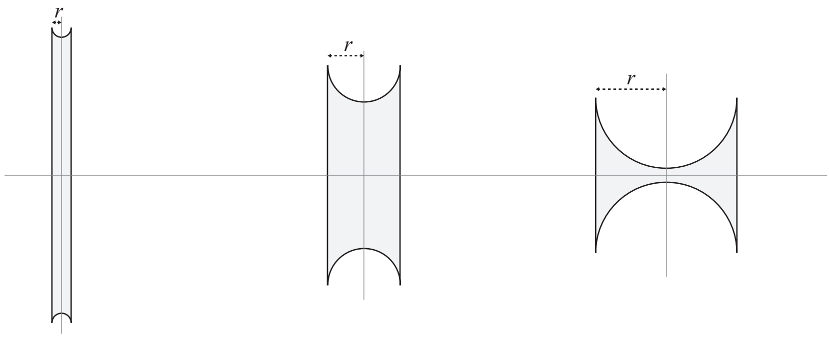
Find the values of the constants p and q.

(e) Kei te hoahoatia e tētahi pakihi hou tētahi tohu ko tētahi auahatanga o te āhua ‘H’ tōna pūtake, e kitea ana i ngā hoahoa.

Ka hoahoatia te tohu ki ngā porowhita haurua e rua, me ngā rārangi torotika e rua. E hiahia ana te kaiwhakahaere o te pakihi hou kia whai wāhi te ingoa o te pakihi ki roto i te āhua, nā reira kua tonoa te kaihoahoa e ia kia eke te nui o te wāhi o roto i te āhua ki tōna mōrahi katoa ka taea.

E mea ana te kaiwhakahaere ki te hanga i tētahi kape o te tohu, ā, nā te kōpiri o ngā rauemi, me kaua e neke atu i te 80cm te paenga katoa o te āhua.

E toru ngā tauira kei te kitea i konei:




(i) Whiriwhiria te wāhi mōrahi katoa.

(ii) Whakamahia ngā tikanga tuanaki hei whakaatu atu, he mōrahi tēnei.

Te paenga o tētahi porowhita: $C = 2\pi r$

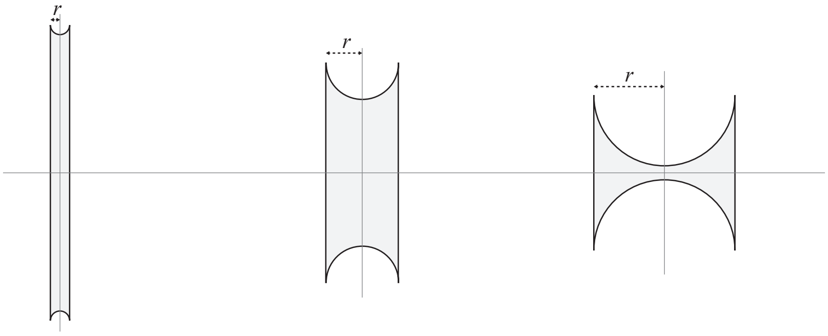
Te horahanga o tētahi porowhita : $A = \pi r^2$

(e) A new business is designing a logo based on a stylised 'H' shape, as seen in the diagrams.

The logo is to be designed with two semi-circles and two straight lines. The owner of the new business wants to include space for the company's name inside the shape, so he has asked the designer to maximise the area inside the shape.

The owner plans to build a replica of the logo, and due to material constraints, the total perimeter of the shape must not be greater than 80 cm.

Three examples are shown here:




(i) Find the maximum area.

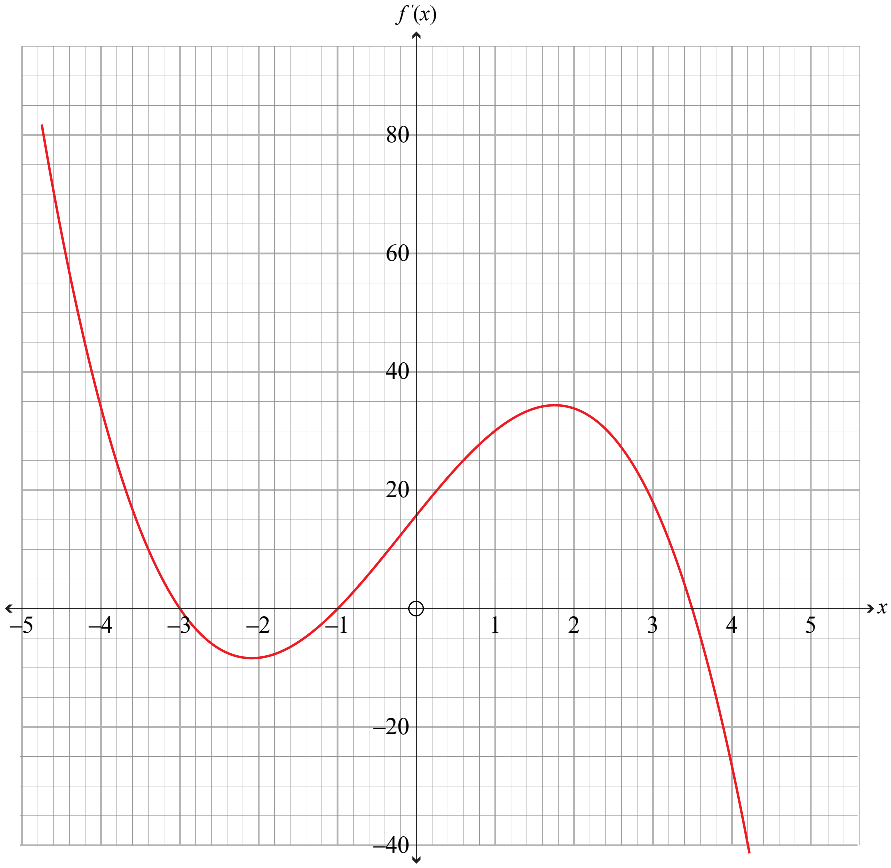
(ii) Use calculus methods to show that this is a maximum.

Circumference of a circle: $C = 2\pi r$

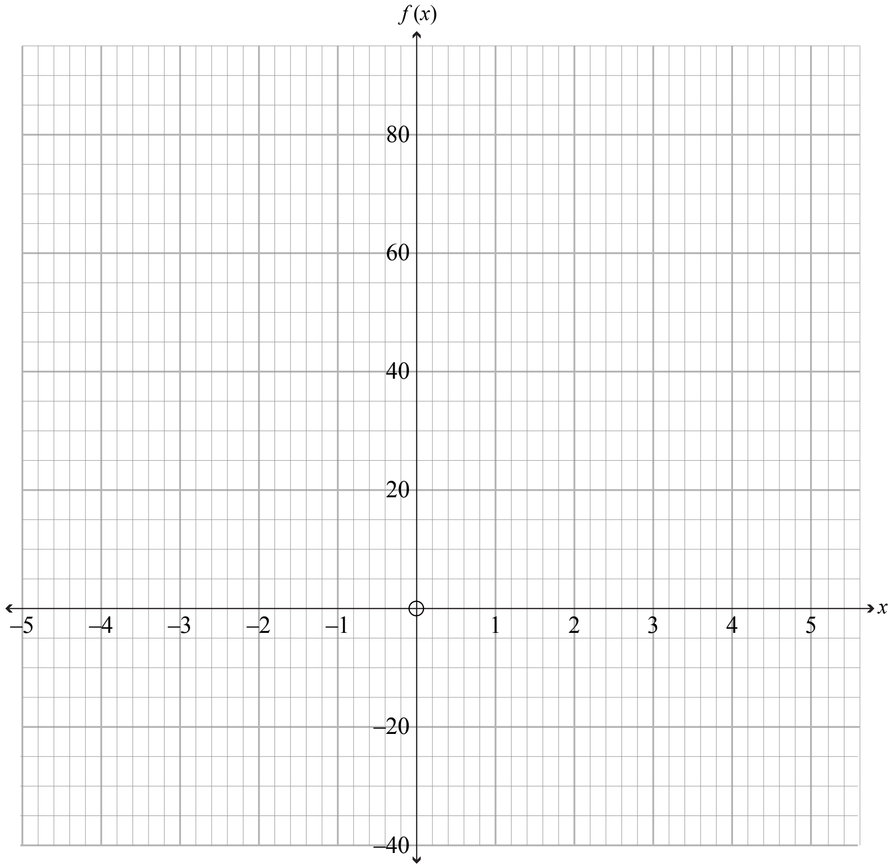
Area of a circle : $A = \pi r^2$

**TE TŪMAHI TUARUA**

(a) Kei te whakaahuatia te kauwhata rōnaki o tētahi pānga $y = f(x)$ i ngā tuaka i raro nei.



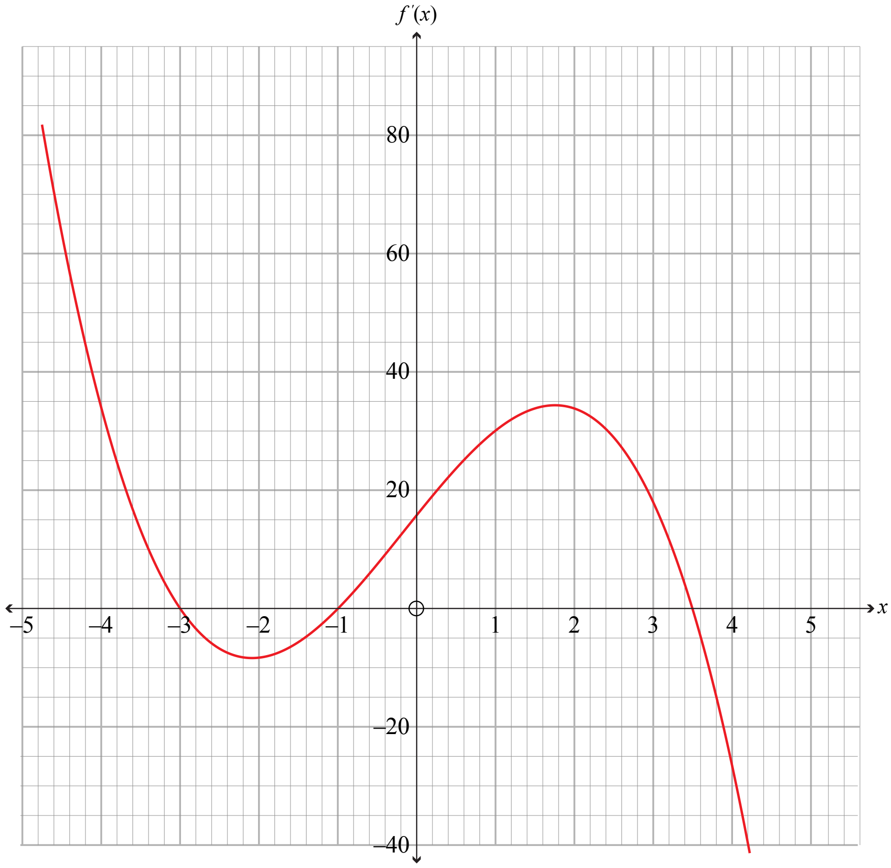

Huahuatia tētahi kauwhata, e taea ana, o te pānga taketake $y = f(x)$ i ngā tuaka i raro nei.



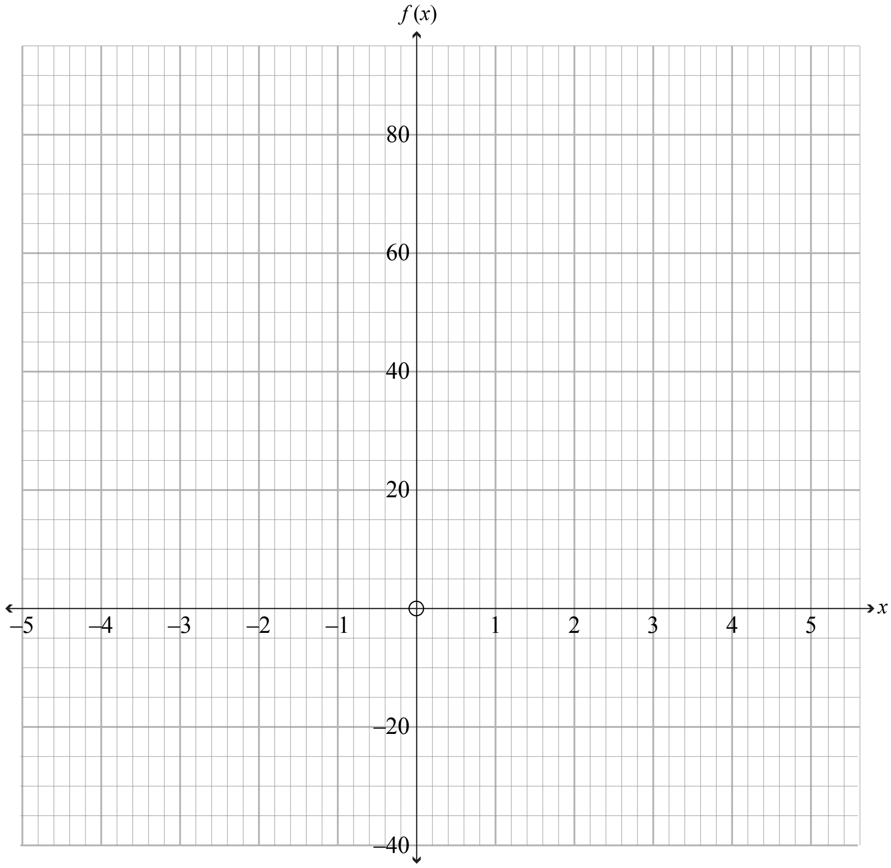

*Ki te hiahia koe ki te tā anō i tō kauwhata, whakamahia te tukutuku kei te whārangi 22.*

**QUESTION TWO**

(a) The gradient graph of a function $y = f(x)$ is shown on the axes below.




Sketch a possible graph of the original function $y = f(x)$ on the axes below.




*If you need to redraw your graph, use the grid on page 23.*

(b) Kei te tākarotia tētahi kēmu kirikiti. Ka patu tētahi kaitākaro i te pōro, ā, ko tōna teitei, $h$, i runga i te papa tākaro, ā-mita nei, ka whakatauiratia e te pānga:

$$h = 22.5t - 4.9t^2 + 1$$

arā, ko $t$ te wā, ā-hēkona, mai i te patunga o te pōro.

Whiriwhiria te mōrahi o te teitei i runga ake i te papa ka ekea e te pōro mā te whakamahi i ngā tikanga tuanaki.

(c) (i) I te wā o te kēmu kirikiti, ka tae mai, ka wehe hoki ngā kaimātakitaki i te wāhi tākaro. Ka whakatauiratia te tokomaha o ngā kaimātakitaki e te whārite e whai ake nei:

$$P = 100t^2 - 2t^4 + 750 \quad (0 \le t \le 7.5)$$

arā, ko $P$ te tokomaha o ngā kaimātakitaki i te wāhi, ā, ko $t$ te wā, ā-hāora, mai i te tīmatanga o te kēmu.

Whakaaturia ko -528 tāngata i te hāora te pāpātanga panoni o ngā kaimātakitaki ka pēnei ana $t = 6$.

(ii) He aha te tikanga o taua uara mō te pāpātanga panoni?

(b) A cricket match is being played. A cricketer hits the ball, and its height, $h$, above the grass cricket pitch in metres, can be modelled by the function:

$$h = 22.5t - 4.9t^2 + 1$$

where $t$ is the time in seconds since the ball was hit.

Using calculus methods, find the maximum height above the ground that the ball reaches.

(c) (i) During the cricket match, spectators both come and leave the venue. The number of spectators can be modelled by the following equation:

$$P = 100t^2 - 2t^4 + 750 \quad (0 \leq t \leq 7.5)$$

where $P$ represents the number of spectators in attendance, and $t$ represents the time in hours since the start of the match.

Show that the rate of change of spectators is –528 people per hour when $t = 6$.

(ii) What is the meaning of this value for the rate of change?

(iii) I tētahi atu kēmu, ka whakatauiratia te tokomaha o ngā kaimātakitaki e te whārite $P = kt^2 - 2t^4 + 750$, arā, ko $k$ e pūmau ana.

Ko te pikinga tere katoa o te tokomaha o ngā kaimātakitaki hei te wā ka pēnei, $t = 4$.

Whiriwhiria te uara o $k$.

Āta whakamāramatia ō whakaaro, mā roto i ngā kīanga pāngarau e tika ana.

(iii) At another match, the number of spectators is modelled by the equation $P = kt^2 - 2t^4 + 750$, where $k$ is a constant.

The number of spectators is growing fastest when $t = 4$.

Find the value of $k$.

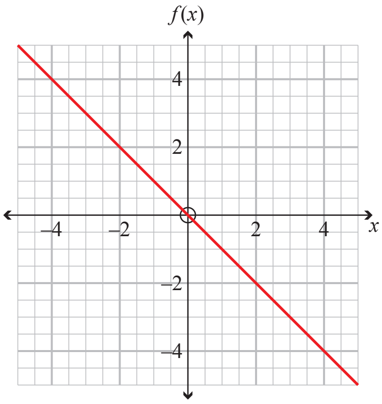
Explain your reasoning clearly, using correct mathematical statements.

## TE TŪMAHI TUATORU

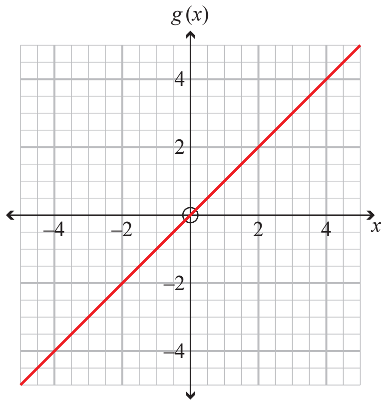
(a) Whiriwhiria te whārite o te pātapa o te kōpiko $y = 2x(x - 3)$, mēnā ka pēnei, $x = 1$.

(b) Kei te whakaatu ngā hoahoa i raro nei i ngā kauwhata mō ngā pānga o $y = f(x)$, $y = g(x)$, me $y = h(x)$.

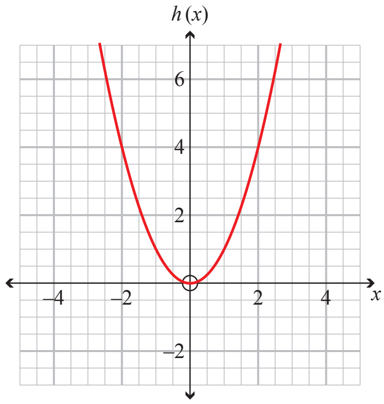
$y = f(x)$




$y = g(x)$

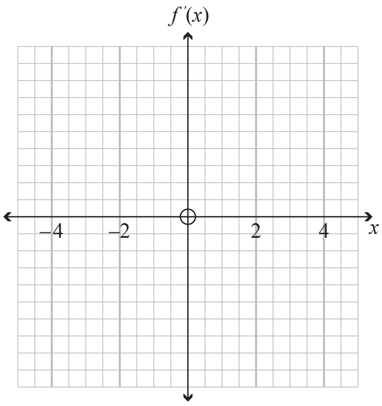



$y = h(x)$

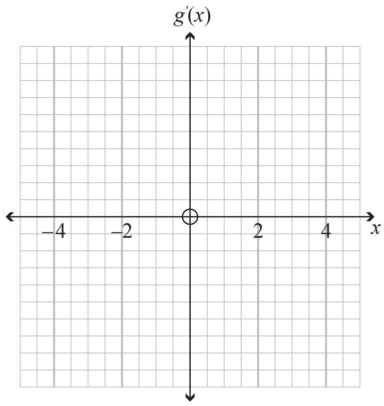



(i) I ngā tuaka i raro nei, huahuatia ngā kauwhata o ngā pānga rōnaki e hāngai ana ki tēnā, ki tēnā.

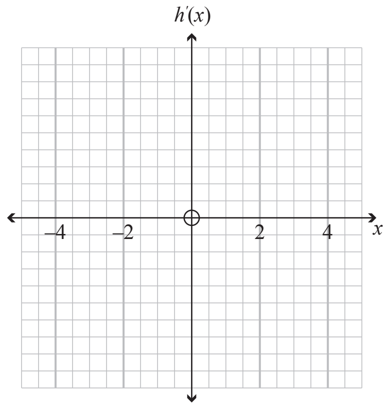
$y = f'(x)$




$y = g'(x)$




$y = h'(x)$



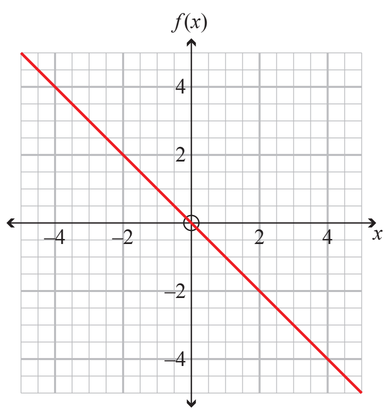

*Ki te hiahia koe ki te tā anō i ō kauwhata, whakamahia ngā tukutuku kei te whārangi 22*

## QUESTION THREE

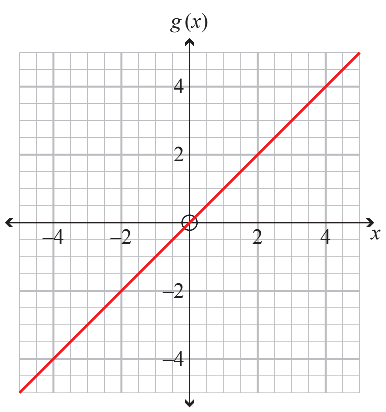
(a) Find the equation of the tangent of the curve $y = 2x(x - 3)$, where $x = 1$.

(b) The diagrams below show the graphs for the functions of $y = f(x)$, $y = g(x)$, and $y = h(x)$.

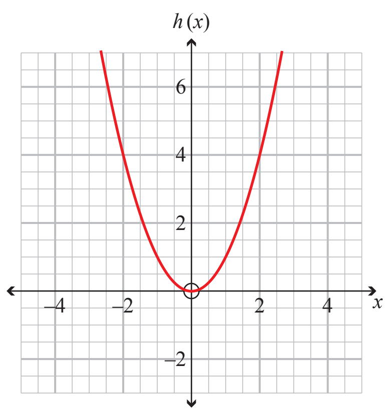
$y = f(x)$




$y = g(x)$

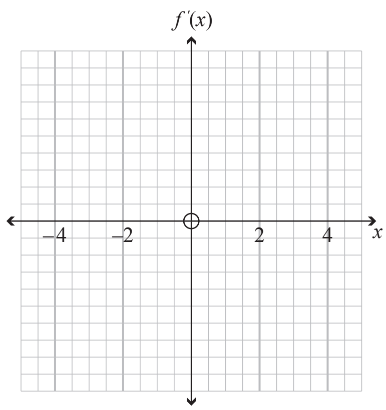



$y = h(x)$

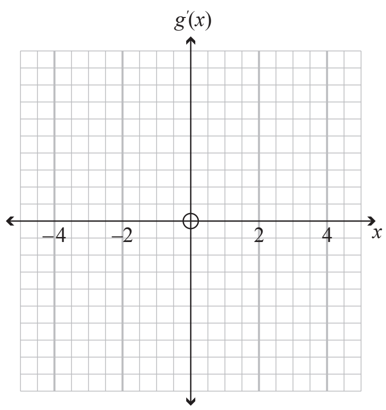



(i) On the axes below, sketch the graphs of their respective gradient functions.

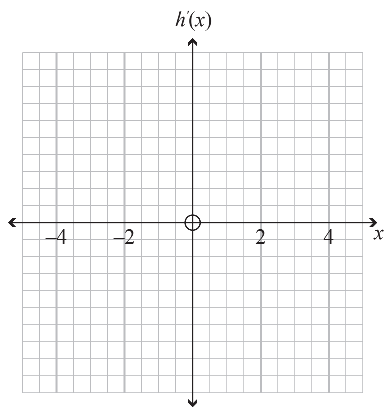
$y = f'(x)$




$y = g'(x)$

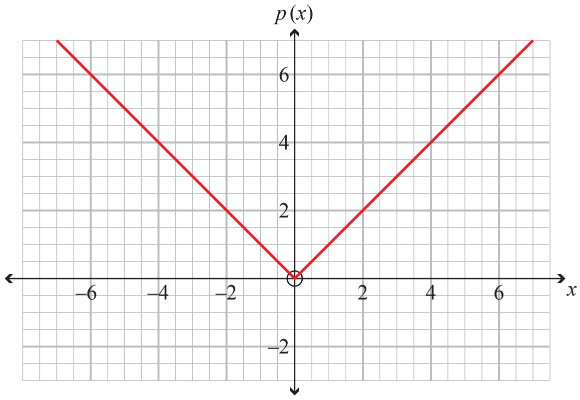



$y = h'(x)$

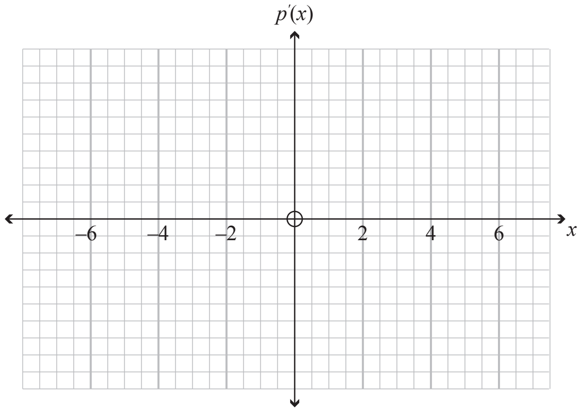



*If you need to redraw any of your graphs, use the grids on page 23.*

(ii) Kei te whakaatu te hoahoa i raro nei i te kauwhata o te pānga $y = p(x)$.




I ngā tuaka i raro nei, huahuatia te kauwhata o te pānga rōnaki $y = p'(x)$.

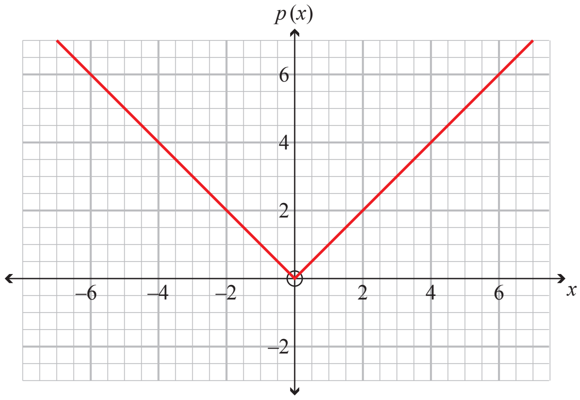



*Ki te hiahia koe ki te tā anō i ō kauwhata, whakamahia te tukutuku kei te whārangi 24.*

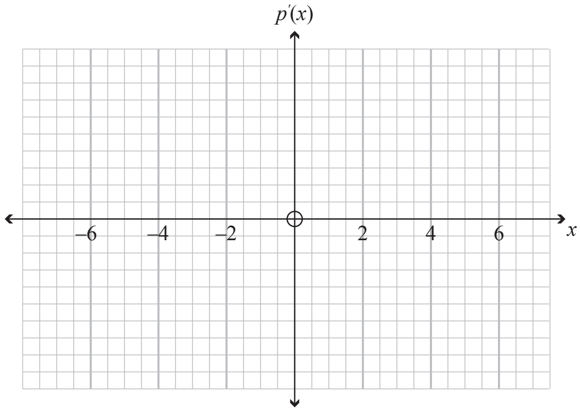
(iii) Kei te tika rānei te kōrero, ko te rōnaki o te pānga $p(x)$ ka kore i te wā e pēnei ana, $x = 0$?

Parahautia tō whakautu mā te whakamahi i ngā kauwhata i ngā whārangi 14 me 16, mā te whakaaro pāngarau hoki/rānei.

(ii) The diagram below shows the graph of the function $y = p(x)$.




On the axes below sketch the graph of the gradient function $y = p'(x)$.

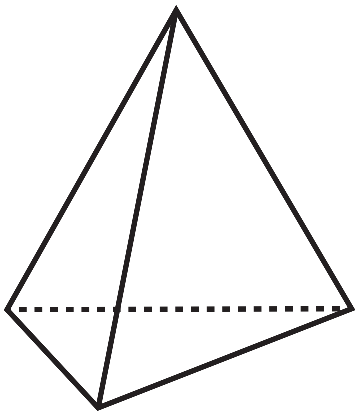
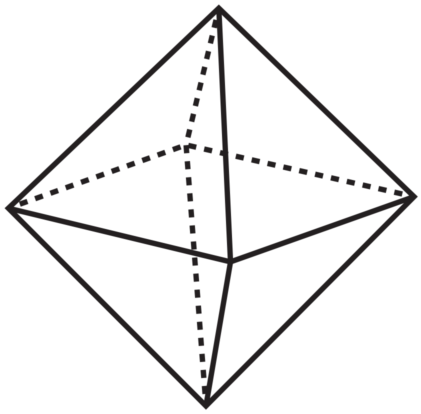



*If you need to redraw any of your graphs, use the grid on page 25.*

(iii) Is it true to say that the gradient of the function $p(x)$ is zero when $x = 0$?

Justify your response using the graphs on pages 14 and 16, and / or mathematical reasoning.

(c) Kei te tūhuratia te mahi toi āhuahanga ahu-3 e tētahi karaehe tuanaki. Ka whakatau ngā tauira ki te waihanga i ngā tauira waea o tētahi matawhā me tētahi matawaru, pēnei i ngā mea i raro nei.

| He matawhā | He matawaru |
|---|---|
|  |  |
| Te horahanga mata $=\sqrt{3}a^2$<br>arā, ko $a$ e tohu ana i te roa<br>o ia tapa o te matawhā | Te horahanga mata $=2\sqrt{3}b^2$<br>arā, ko $b$ e tohu ana i te roa<br>o ia tapa o te matawaru |

Ka hanga rātou i ia tapa ki tētahi wāhanga waea, ka tūhono ai i ia wāhanga, kātahi ka uhi i ia mata ki te pepa hei hanga rātana pēnei i ngā mea i raro nei.

Te mātāpuna: https://timesofindia.indiatimes.com/city/ahmedabad/20-unique-diy-mathematical-lamps-of-paper-by-ccl-iitgn/

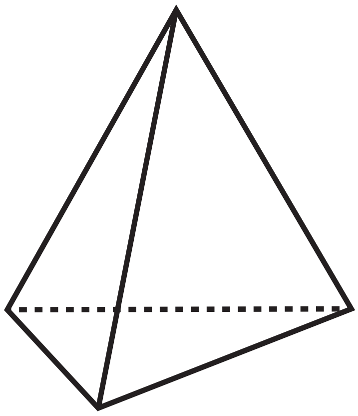
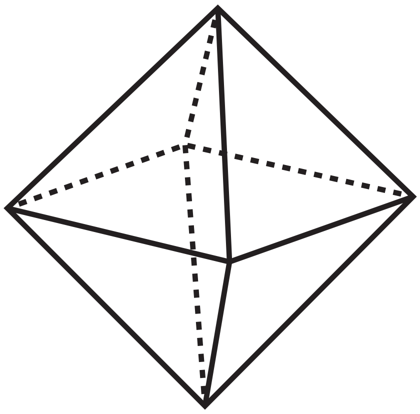
Kei te hiahia rātou ki te hanga i ia tauira kia mōkito katoa te tapeke o te horahanga mata o ngā āhua e rua i tōna otinga.

180 cm katoa te roa o te waea kei a rātou hei tapahi, hei whakamahi hoki mō ngā tauira e rua.

Whakatauria ngā roa o ngā tapa, $a$ me $b$ e tika ana, e mōkito katoa ai te tapeke o te horahanga mata o ngā āhua e rua hei te wā kua whakamahia te waea katoa.

Whakamahia te tuanaki hei parahau ake, he mōkito te horahanga mata.

(c) A calculus class is exploring 3-D geometrical art. The students decide that they want to create wireframe models of a tetrahedron and an octahedron, similar to those shown below.

| Tetrahedron | Octahedron |
|---|---|
|  |  |
| Surface area $= \sqrt{3}a^2$ where $a$ represents the length of each edge of the tetrahedron | Surface area $= 2\sqrt{3}b^2$ where $b$ represents the length of each edge of the octahedron |

They will make each edge with a piece of wire, and join each piece together, and then cover each surface with paper to create lanterns similar to those below.

Source: https://timesofindia.indiatimes.com/city/ahmedabad/20-unique-diy-mathematical-lamps-of-paper-by-ccl-iitgn/

They wish to create each model so that the total resulting surface area of the two shapes is a minimum.

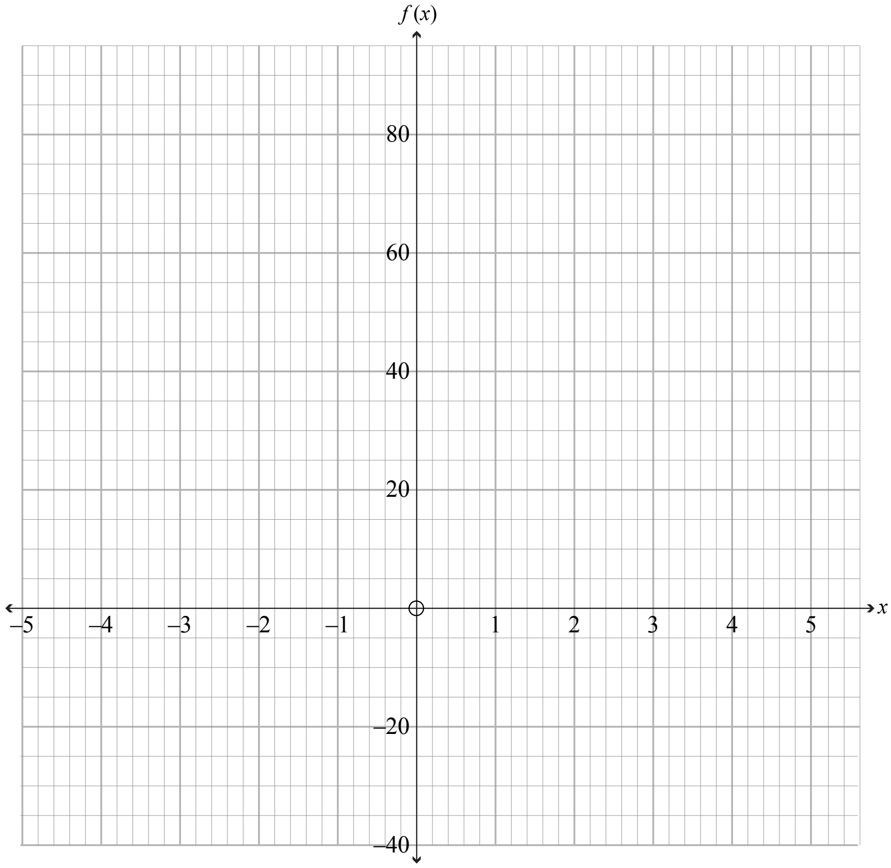
They have a total of 180 cm of wire to cut and use for both models.

Determine the lengths of the edges, $a$ and $b$, required to minimise the total surface area of the two shapes when all of the wire is used.

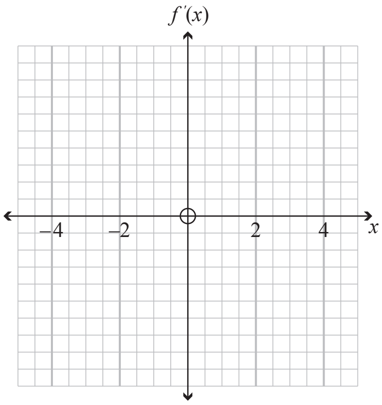
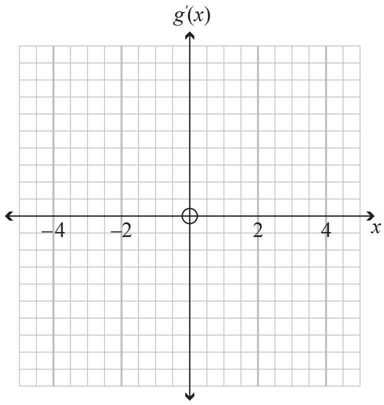
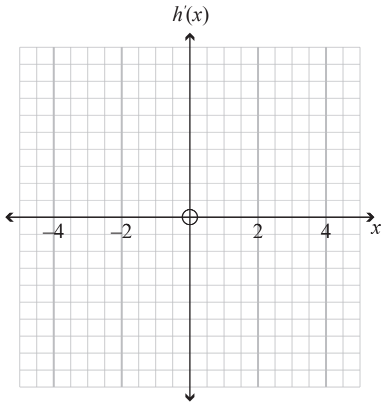
Use calculus to justify that the surface area is a minimum.

**NGĀ HOAHOA WĀTEA**

Ki te hiahia koe ki te tā anō i tō whakautu ki te Tūmahi Tuarua (a), whakamahia te tukutuku i raro nei. Kia mārama te whakamōhiotanga mai ko tēhea te whakautu e hiahia ana koe kia mākahia.

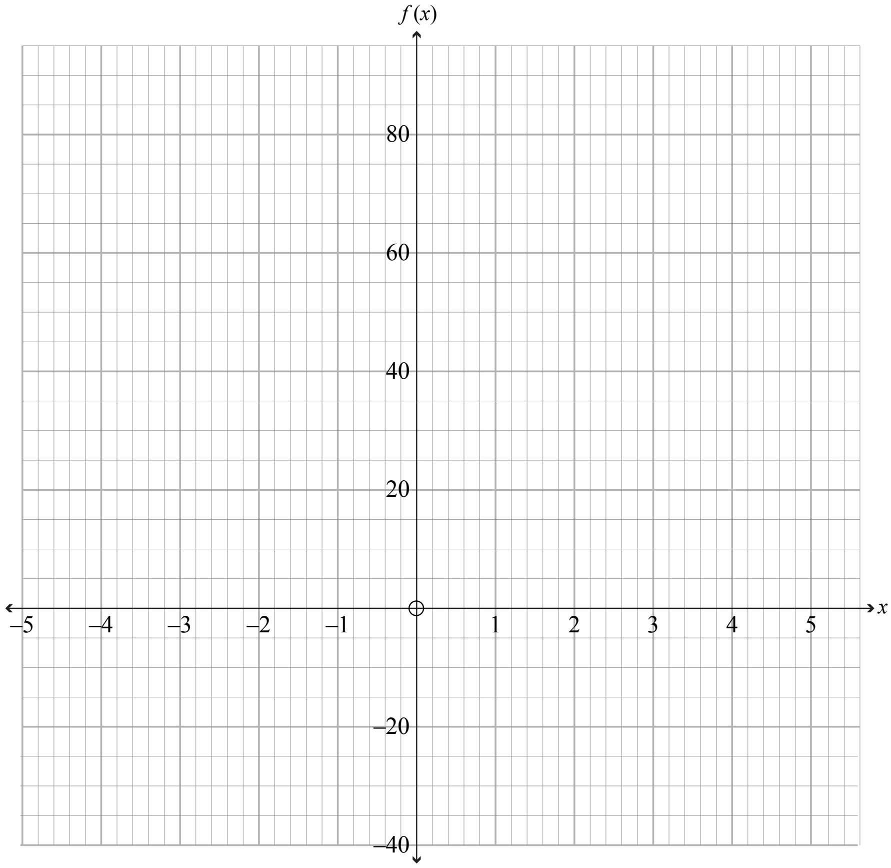



Ki te hiahia koe ki te tā anō i tētahi o ō whakautu ki te Tūmahi Tuatoru (b)(i), whakamahia ngā tukutuku i raro nei. Kia mārama te whakamōhiotanga mai ko ēhea ngā whakautu e hiahia ana koe kia mākahia.

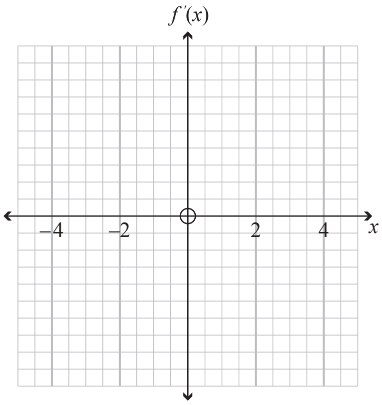
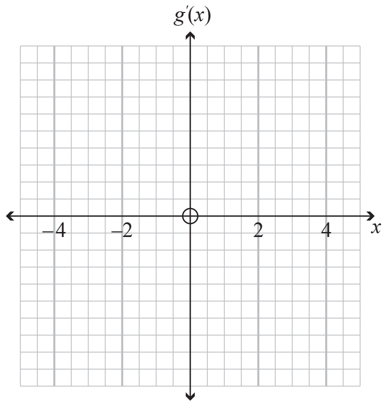
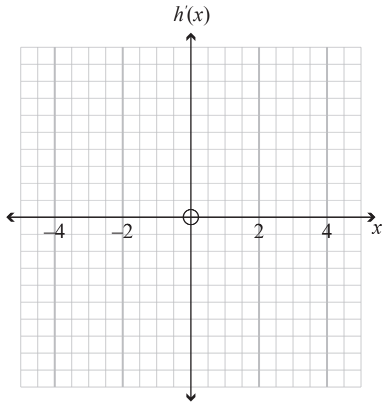
| $y = f'(x)$ | $y = g'(x)$ | $y = h'(x)$ |
|---|---|---|
|  |  |  |

## SPARE DIAGRAMS

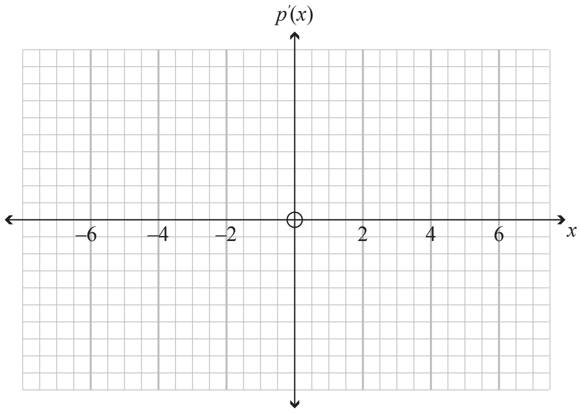
If you need to redraw your response to Question Two (a), use the grid below. Make sure it is clear which answer you want marked.



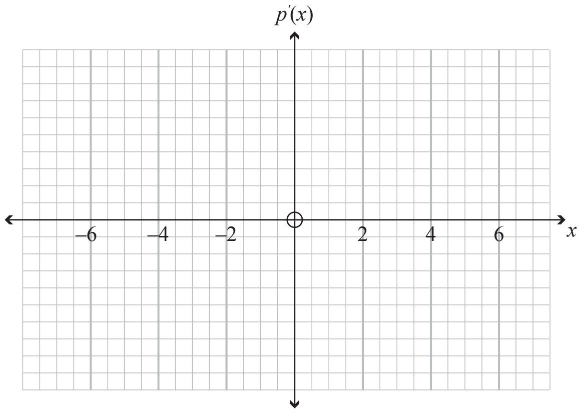

If you need to redraw any of your responses to Question Three (b)(i), use the grids below. Make sure it is clear which answers you want marked.

| $y = f'(x)$ | $y = g'(x)$ | $y = h'(x)$ |
|---|---|---|
|   |   |   |

Ki te hiahia koe ki te tā anō i tō whakautu ki te Tūmahi Tuatoru (b)(ii), whakamahia te tukutuku i raro nei. Kia mārama te whakamōhiotanga mai ko tēhea te tuhinga e hiahia ana koe kia mākahia.



If you need to redraw your response to Question Three (b)(ii), use the grid below. Make sure it is clear which answer you want marked.



**He whārangi anō ki te hiahiatia.**
**Tuhia te tau tūmahi mēnā e hāngai ana.**

TE TAU TŪMAHI

**Extra space if required.**
**Write the question number(s) if applicable.**

QUESTION NUMBER

*English translation of the wording on the front cover*

# Level 2 Mathematics and Statistics 2022

## 91262M Apply calculus methods in solving problems

Credits: Five

91262M

| Achievement | Achievement with Merit | Achievement with Excellence |
| --- | --- | --- |
| Apply calculus methods in solving problems. | Apply calculus methods, using relational thinking, in solving problems. | Apply calculus methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**