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QUALIFY FOR THE FUTURE WORLD
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Tick this box if you have NOT written in this booklet

Level 2 Mathematics and Statistics 2022

91267 Apply probability methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (//). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

WATER, WATER, EVERYWHERE …

QUESTION ONE

In New Zealand, the average person uses a mean of 227 litres of water per day, with a standard deviation of 16 litres. Assume that water consumption can be modelled by a normal distribution.

(a) Find the probability that a randomly chosen New Zealander uses between 227 and 247 litres in a day.

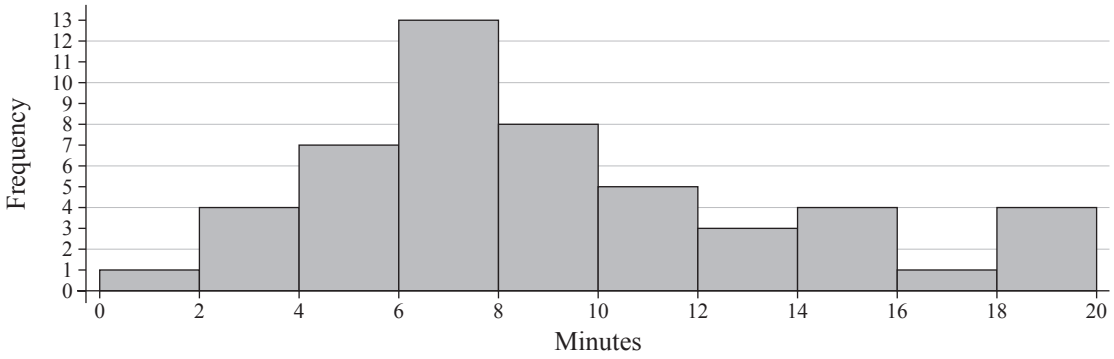
(b) (i) If two New Zealanders are randomly chosen, what is the probability that they both use less than 210 litres per day?

(ii) If the two New Zealanders were both chosen from an area that often experiences water shortages, how would you expect this to change your answer to part (i) above?

Explain your reasoning clearly.

(c) Suzanne lives with her family of 3 children and her partner. She is conscious of water-saving efforts suggested by the council, and so encourages her family to have shorter showers. Suzanne reads that the average shower time of New Zealanders is 8 minutes, with a standard deviation of 2 minutes. She times the showers taken by her family over the next 2 weeks and records the results in the graph below.

Shower times in Suzanne's household



How do Suzanne's household results compare to a normal distribution model with a mean of 8 minutes and a standard deviation of 2 minutes?

You should discuss at least TWO of centre, shape, and spread in your response.

(d) After four more weeks, Suzanne did another survey and found the shower times in her household approximated a normal distribution model, with a mean of 8 minutes and a standard deviation of 2 minutes. Suzanne considers only 15% of the shower times to be acceptable.

What is the longest shower time that Suzanne considers acceptable?

(e) Businesses use about a quarter of Auckland's water. Auckland Council states that, "About 85 per cent of businesses use less than 2000 litres of water per day."

(i) If the standard deviation for business water use is 450 litres, calculate the mean water usage for business customers, assuming that a normal distribution is an appropriate model.

(ii) The Council also stated "A small number of businesses in Auckland (1.5%) use more than 15 000 litres per day."

What issues does this raise for your answer to part (i), and why?

QUESTION TWO

(a) LAWA (Land, Air, Water Aotearoa) conducted a survey and tested some New Zealand rivers in different land types to see if they were safe for swimming. The results from the survey are presented in Table 1.

Table 1: Numbers of rivers in four land categories which are safe or unsafe for swimming

	Native vegetation	Exotic forest	Pasture	Urban	Total
Unsafe for swimming	48	11	424	57	540
Safe for swimming	146	15	104	5	270
Total	194	26	528	62	810

Data sourced from LAWA River Water Quality 2020 Report

(i) What is the probability that a randomly chosen river out of those tested is an urban river and is unsafe for swimming?

(ii) Is a river in native vegetation more likely to be safe for swimming than a river in an exotic forest area?

The actual proportion of New Zealand rivers in each land type is different to those in the survey summarised in Table 1. In fact:

- 48% of rivers are in native vegetation
- 5% are in exotic forest
- 46% are in pasture areas
- 1% are in urban areas.

(iii) Use Table 1 and these proportions to estimate the percentage of all New Zealand rivers that are unsafe for swimming.

(iv) How confident do you feel that this probability would be correct for **all** New Zealand rivers?

(b) Mia and Joe want to go on holiday near a swimmable river in New Zealand. They find this summary table of popular river sites in the North and South Island that are monitored regularly to check if they are swimmable.

Table 2: Popular river sites which are safe or unsafe for swimming

	North Island	**South Island**
Safe for swimming	105	65
Unsafe for swimming	74	34

Data sourced from lawa.org.nz, January 2022.

(i) What is the probability that a river chosen at random from these monitored sites is safe for swimming?

(ii) Mia looks at the data and says, "It is more than twice as likely for a river in the North Island to be unsafe for swimming compared to a South Island river."

Joe says, "It's **only** 20% more likely for a North Island river to be unsafe for swimming compared to a South Island river."

Explain **each** person's reasoning AND decide if either reasoning is supported mathematically.

Use relevant calculations to support your answer.

QUESTION THREE

(a) A large online survey was carried out, asking 2500 New Zealanders for their views about water use.

87% were concerned about water shortages, and:

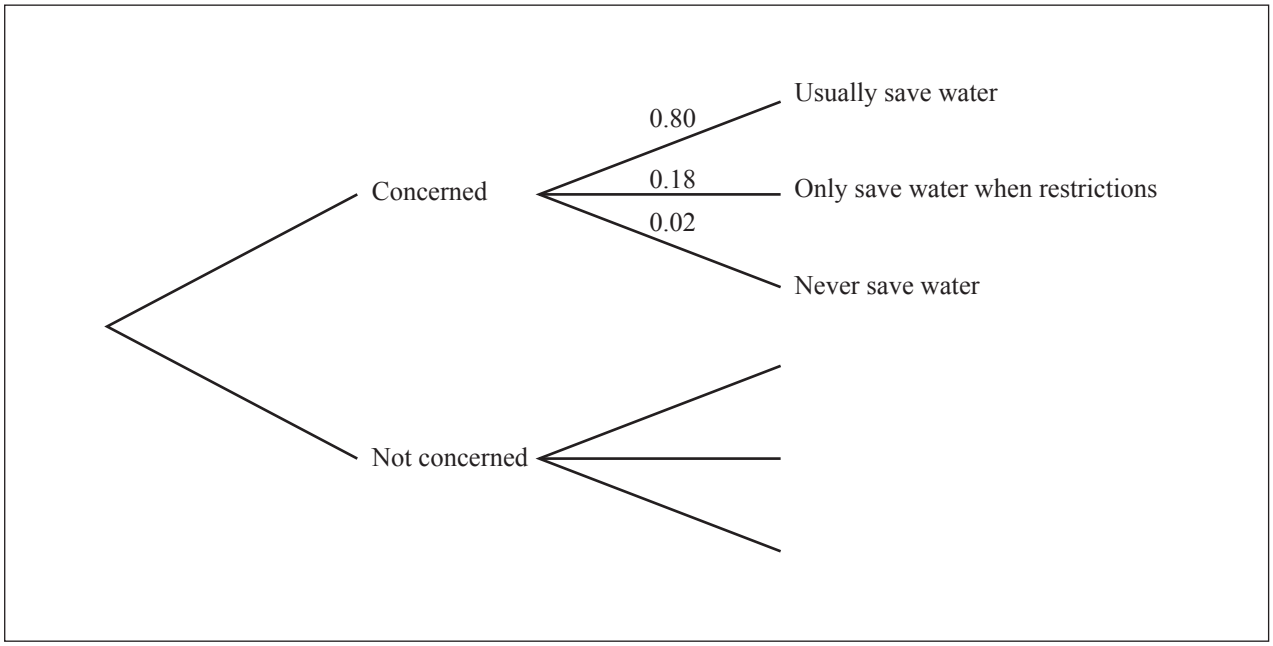
- 80% usually try to save water (such as taking shorter showers).
- 18% only try to save water when restrictions are in place.
- 2% never try to save water.

Of those not concerned about water shortages:

- ½ usually try to save water.
- ¼ only try to save water when restrictions are in place.
- The rest never try to save water.

Use this information to complete the probability tree below.

Figure 1



(i) What is the probability that a respondent chosen at random is concerned about water shortages and usually saves water?

(ii) Of 2500 survey respondents, how many said they "only save water when restrictions are in place"?

(b) A secondary school teacher wondered if young people's views would be similar to those seen in the online survey described in part (a). She surveyed 200 students from years 9 to 13 at two local high schools.

She found:

- 70% of students were concerned about climate change.
- Of those who were concerned about climate change, 85% were also concerned about water shortages.
- 55% of those who were **not** concerned about climate change were concerned about water shortages.

(i) Draw a probability tree of this situation and use it to find the probability that a student chosen at random is concerned about water shortages.

(ii) The teacher also found that:

- The probability that a student saved water was 2 times greater if they were concerned about water shortages than if they were not concerned.
- The probability that a student saved water was the same whether they were concerned about climate change or not.
- The overall probability that a student in this survey saved water was 0.5632.

Find the probability that a student is **not** concerned about climate change, is **not** concerned about water shortages, and does **not** try to save water.

Question Three continues on the next page.

(iii) The teacher used the findings of both surveys (from part (a) and part (b) on previous pages) to claim that New Zealand secondary school students are about 25% **less** likely to save water in their homes than the general population.

Discuss the validity of this claim.

Use relevant calculations and statistical considerations to support your answer.

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