See back cover for an English translation of this cover

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91578M

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SUPERVISOR'S USE ONLY

Tohua tēnei pouaka mēnā KĀORE koe i tuhi kōrero ki tēnei pukapuka

Tuanaki, Kaupae 3, 2022

91578M Te whakahāngai i ngā tikanga pārōnaki i te wā e whakaoti rapanga ana

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki i te wā e whakaoti rapanga ana.	Te whakahāngai i ngā tikanga pārōnaki i te wā e whakaoti rapanga ana, mā roto i te whakaaro pānga.	Te whakahāngai i ngā tikanga pārōnaki i te wā e whakaoti rapanga ana, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi i ngā wāhanga e kitea ai te kauruku whakahāngai (). Ka poroa pea taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria te pārōnakitanga o te $y = (\ln x)^2$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) Whiriwhiria te/ngā uara-x o ngā pito whakaroau o te kauwhata i runga i te pānga o te $f(x) = \frac{x^2+1}{x}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

QUESTION ONE

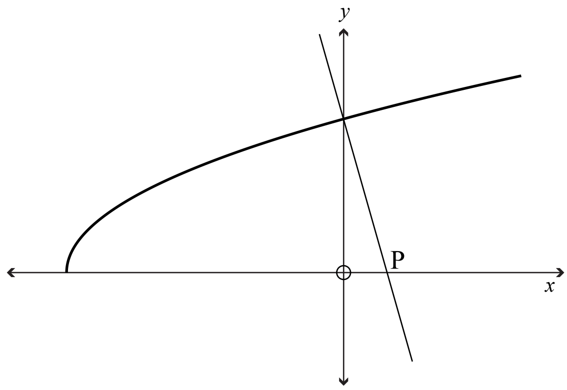
(a) Differentiate $y = (\ln x)^2$.

You do not need to simplify your answer.

(b) Find the x-value(s) of any stationary points on the graph of the function $f(x) = \frac{x^2 + 1}{x}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

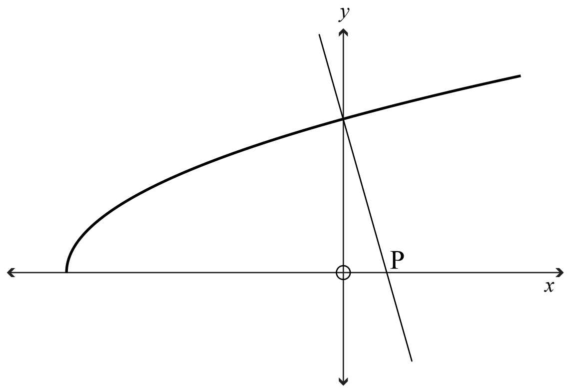
(c) E whakaaturia ana ki te kauwhata kei raro iho nei te pānga o te $y = \sqrt{x+2}$, me te rārangi hāngai ki te pānga i te whakawhitinga o te pānga i te tuaka pou (y).



Whiriwhiria ngā taunga o te pito P, arā, te whakawhitinga-x o te rārangi hāngai.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

(c) The graph below shows the function $y = \sqrt{x+2}$, and the normal to the function at the point where the function intersects the y-axis.



Find the coordinates of point P, the x-intercept of the normal.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) E tautuhia tawhātia ana tētahi kōpiko ki ngā whārite:

Ko te $x = 2 + 3t$ me te $y = 3t - \ln(3t - 1)$ mehemea $t > \frac{1}{3}$.

Whiriwhiria ngā taunga, (x,y), o t/ētahi o ngā pito i te kōpiko nā runga i te mōhio ko te $\frac{1}{2}$ te rōnaki o te pātapa.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

(d) A curve is defined parametrically by the equations:

$$x = 2 + 3t \text{ and } y = 3t - \ln(3t - 1) \text{ where } t > \frac{1}{3}.$$

Find the coordinates, (x,y), of any point(s) on the curve where the tangent to the curve has a gradient of $\frac{1}{2}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Mehemea he tau pūmau tōrunga e tūturu ana te p, hāponotia te whakaaro kāore he wāhi tupa o te $y = \mathrm{e}^{px^2}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

(e) If p is a positive real constant, prove that $y = e^{px^2}$ does not have any points of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

TE TŪMAHI TUARUA

(a) Whiriwhiria te pārōnakitanga $f(x) = (5x - 3)\sin(4x)$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) Whiriwhiria te rōnakitanga o te pātapa ki te kōpiko $y = (3x^2 - 2)^3$ mehemea ko te $x = 2$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

QUESTION TWO

(a) Differentiate $f(x) = (5x - 3)\sin(4x)$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \left(3x^2 - 2\right)^3$ when $x = 2$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) E haere tōtika ana tētahi mea. Ā-mita nei, kei te tikanga tātai tana peihanga:

$d(t) = \dfrac{t^2 - 6}{2t^3}$ mehemea ko te $t > 0$, ko te t te wā ā-hēkona.

Whiriwhiria te/ngā wā kua tū taua mea.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

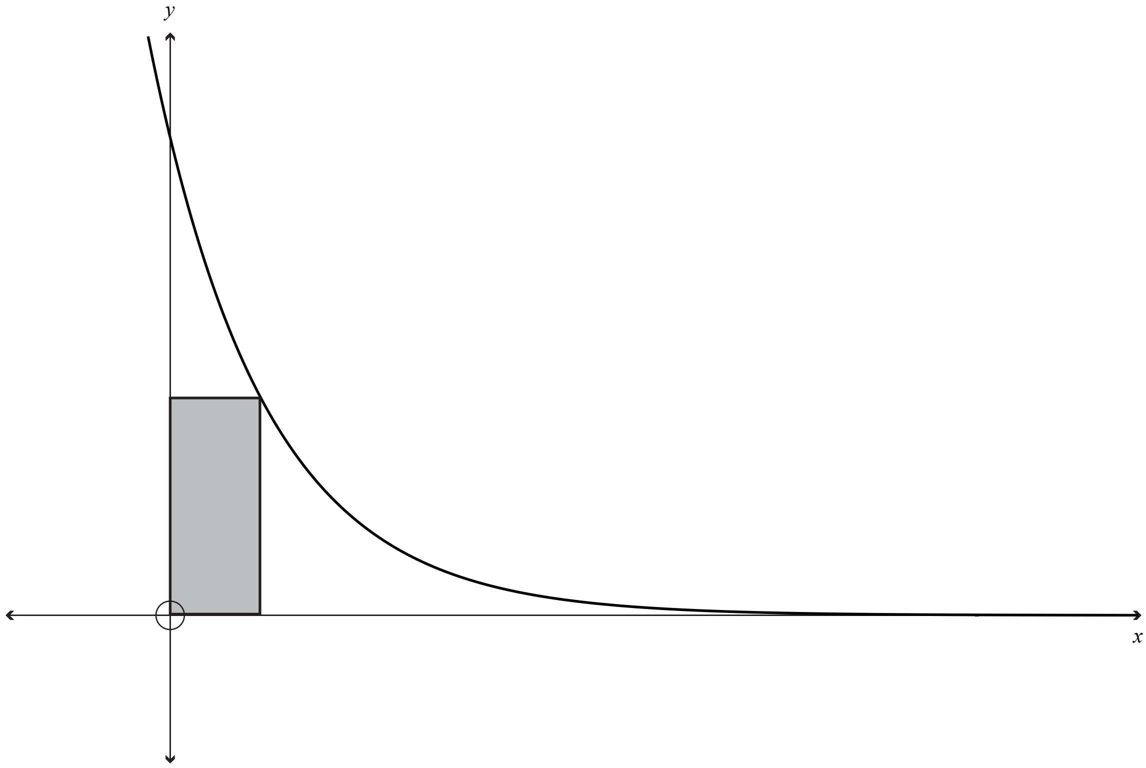
(c) An object is travelling in a straight line. Its displacement, in metres, is given by the formula:

$$d(t) = \frac{t^2 - 6}{2t^3} \text{ where } t > 0, \ t \text{ is time in seconds.}$$

Find the time(s) when the object is stationary.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Kei te (0,0) tētahi akitu o tētahi tapawhā hāngai, ka mutu, kei te pā te akitu tauaro ki te kōpiko $y = 6e^{1-0.5x}$, mehemea ko te $x > 0$, e whakaaturia ana ki te kauwhata kei raro iho nei.

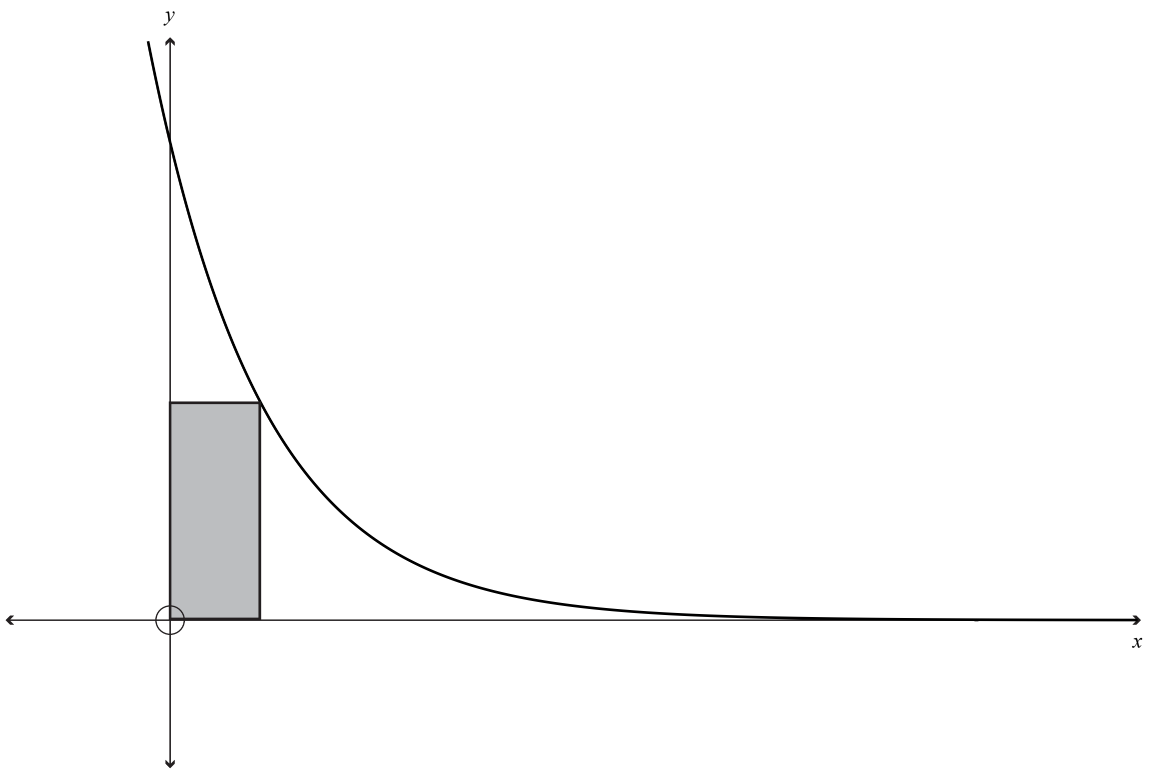


Whiriwhiria te horahanga nui katoa o te tapawhā hāngai ka taea.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

Ehara i te mea ko te horahanga i whiriwhiria ai e koe me hāpono e koe hei horahanga nui katoa ka taea.

(d) A rectangle has one vertex at (0,0) and the opposite vertex on the curve $y = 6e^{1-0.5x}$, where $x > 0$, as shown on the graph below.

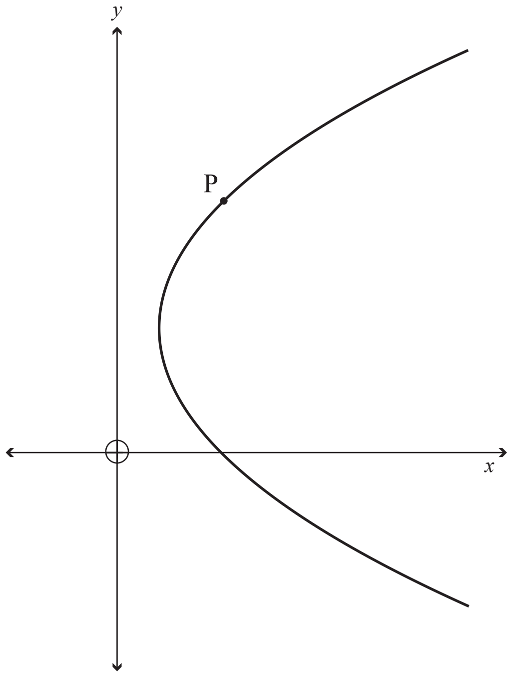


Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not have to prove that the area you have found is a maximum.

(e) E whai ana te kōpiko me te whārite o te $(y-5)^2 = 16(x-2)$ i te pātapa me te rōnakitanga o te 1 i te pito P.

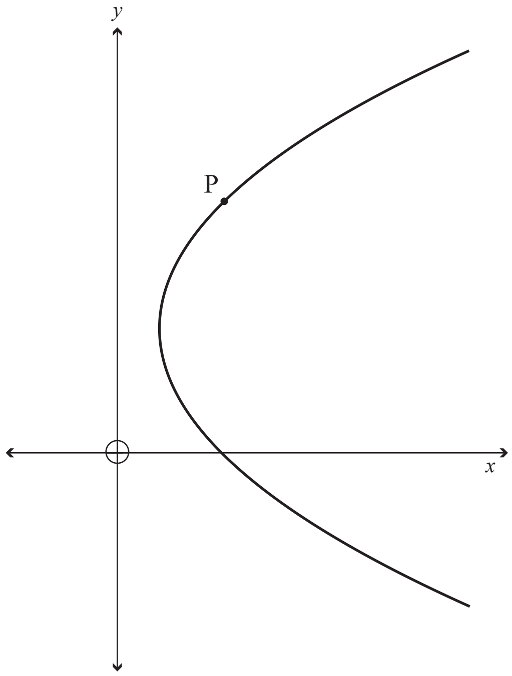


Kei te whakawhitia tēnei pātapa i te tuaka x me te tuaka y i te pito R me te pito S i tēnā raupapatanga .

Hāponotia ko te roa o te RS, ko te $7\sqrt{2}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

(e) The curve with the equation $(y-5)^2 = 16(x-2)$ has a tangent of gradient 1 at point P.



This tangent intersects the *x* and *y* axes at points R and S respectively.

Prove that the length RS is $7\sqrt{2}$.

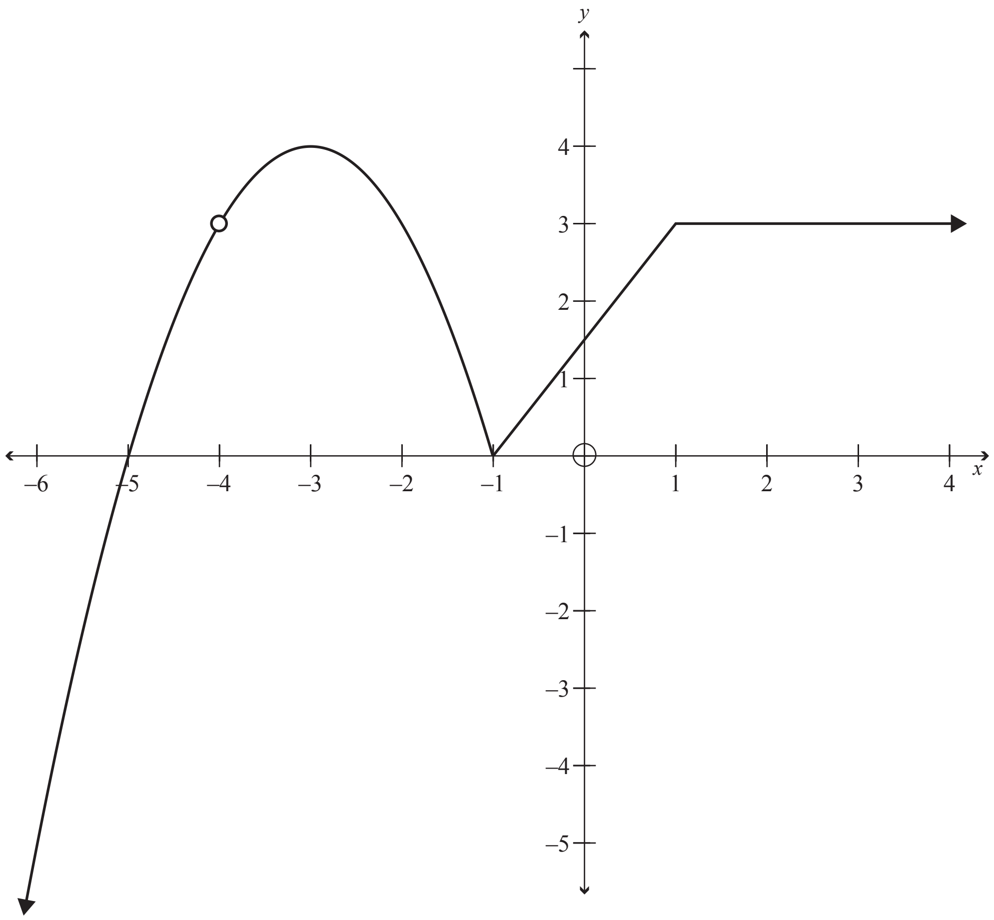
You must use calculus and show any derivatives that you need to find when solving this problem.

TE TŪMAHI TUATORU

(a) Whiriwhiria te pārōnakitanga o te $y = e^{4\sqrt{x}}$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) E whakaaturia ana ki te kauwhata o raro iho nei te pānga $y = f(x)$.



Mō te pānga kei runga ake nei:

(i) Whiriwhiria te/ngā uara o te x mehemea kāore e taea te kimi pārōnaki i roto i te $f(x)$.

(ii) Whiriwhiria te/ngā uara o te x mehemea ko te $f'(x) = 0$.

(iii) Whiriwhiria te/ngā uara o te $\lim_{x \to -4} f(x)$?

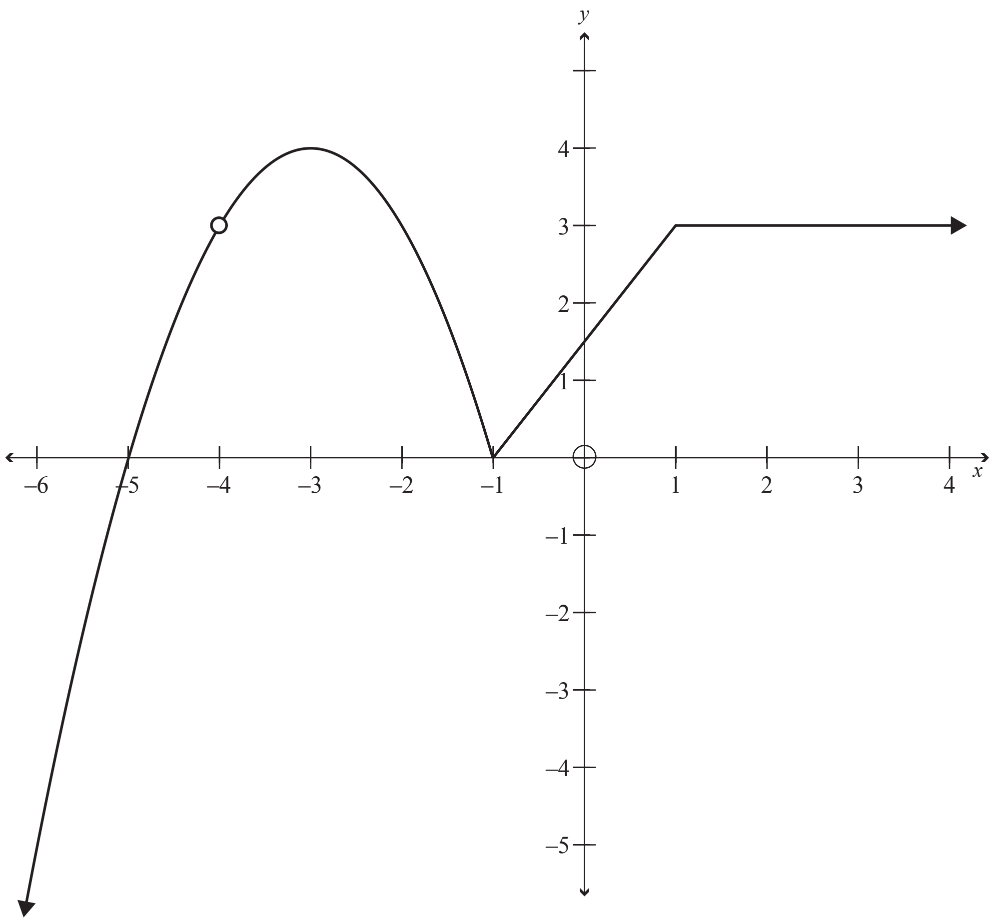
(Me āta whakamārama mehemea kārekau he uara.)

QUESTION THREE

(a) Differentiate $y = e^{4\sqrt{x}}$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



For the function above:

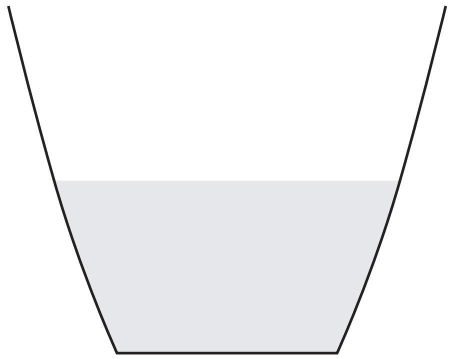
(i) Find the value(s) of x where $f(x)$ is not differentiable.

(ii) Find the value(s) of x for which $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to -4} f(x)$?

(State clearly if the value does not exist.)

(c) E whakaaturia ana ki raro iho nei te motuhanga o tētahi kumete me te wai kei roto.



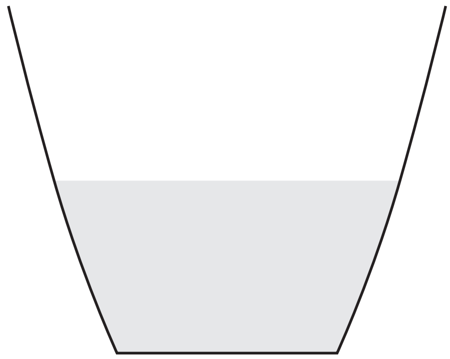
Mehemea ko te taumata o te pae wai i te kumete, ko te h cm, ka tohua te rōrahi, arā te V cm^3, o te wai i te kumete e te $V = \pi\left(\frac{3}{2}h^2 + 3h\right)$.

Ka riringia te wai ki te kumete i te pāpātanga pūmau o te 20 cm^3 s^{-1}.

Whiriwhiria te pāpātanga, mā roto i ngā cm s^{-1}, e piki haere ana te taumata o te pae wai ka eke ana te taumata o te pae wai ki te 3 cm.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

(c) The diagram below shows the cross-section of a bowl containing water.



When the height of the water level in the bowl is h cm, the volume, V cm^3, of water in the bowl is given by $V = \pi\left(\frac{3}{2}h^2 + 3h\right)$.

Water is poured into the bowl at a constant rate of 20 cm^3 s^{-1}.

Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

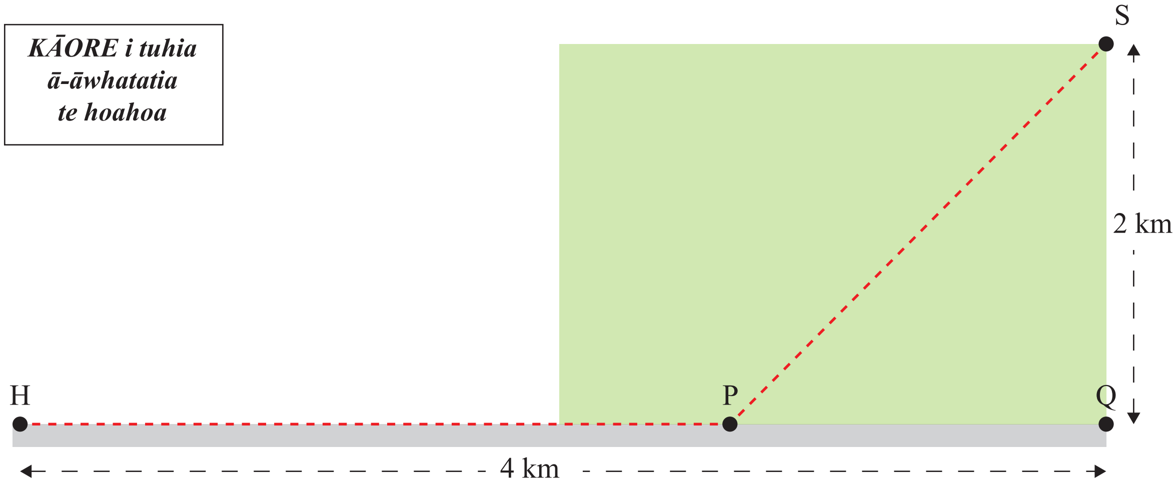
(d) Whiriwhiria te/ngā uara-x o tētahi o ngā pito whakaroau o te kauwhata o te pānga $y = 9x - 2 + \frac{3}{3x-1}$ ka whakatauria ai t/ōna āhua.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

(d) Find the x-value(s) of any stationary point(s) on the graph of the function $y = 9x - 2 + \frac{3}{3x-1}$ and determine their nature.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Eke paihikara ai a Megan i tōna kāinga, H, ki tōna kura, S, i ia rā.



Ka pūmau te tere o tana eke, arā, te tekau manomita i ia hāora mā te ara i tōna kāinga ki te pito P.

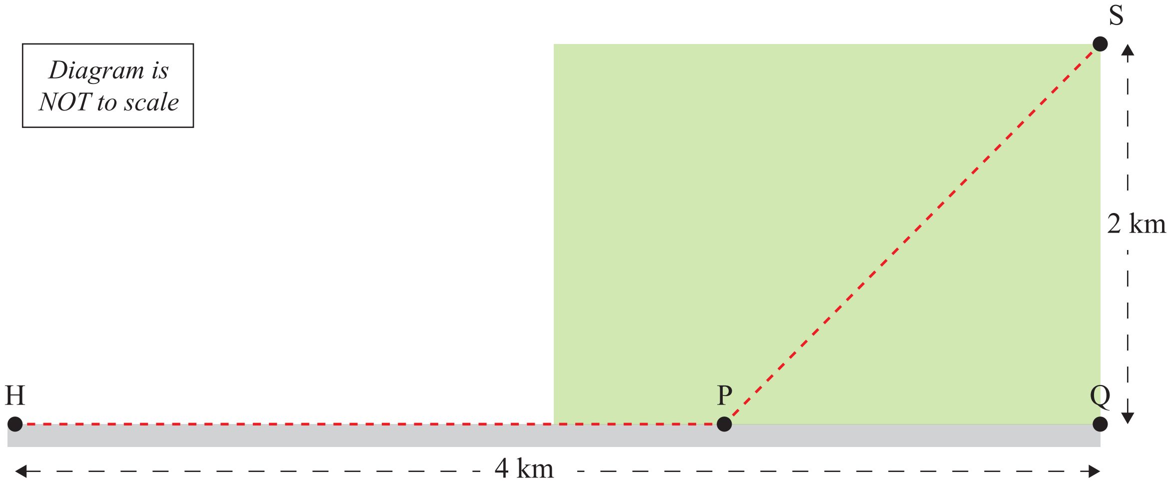
I te pito P, ka whakawhiti a Megan i te pāka e kotahi atu ai ia ki te kura. Ka whakawhiti ana ia i te pāka, e 6 manomita anake te tere o tana eke paihikara.

Me whiriwhiri ia i tēhea wāhanga o te tawhiti atu i tōna kāinga kia whakawhiti i te pāka e poto katoa ai tana haere?

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā otinga o te mahi pārōnaki i te wā e whakaotihia ana tēnei rapanga.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(e) Megan cycles from her home, H, to school, S, each day.



She rides along a path from her home to point P at a constant speed of 10 kilometres per hour.

At point P, Megan cuts across a park, heading directly to school. When cycling across the park, Megan can only cycle at 6 kilometres per hour.

At what distance from her home should she choose to cut across the park in order to make her travelling time a minimum?

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the next page.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2022

91578M Apply differentiation methods in solving problems

Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.