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91579M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

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Tohua tēnei pouaka mēnā KĀORE koe i tuhi kōrero ki tēnei pukapuka

Tuanaki, Kaupae 3, 2022

91579M Te whakahāngai i ngā tikanga pāwhaitua i te wā e whakaoti rapanga ana

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua i te wā e whakaoti rapanga ana.	Te whakahāngai i ngā tikanga pāwhaitua i te wā e whakaoti rapanga ana, mā roto i te whakaaro pānga.	Te whakahāngai i ngā tikanga pāwhaitua i te wā e whakaoti rapanga ana, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–21 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

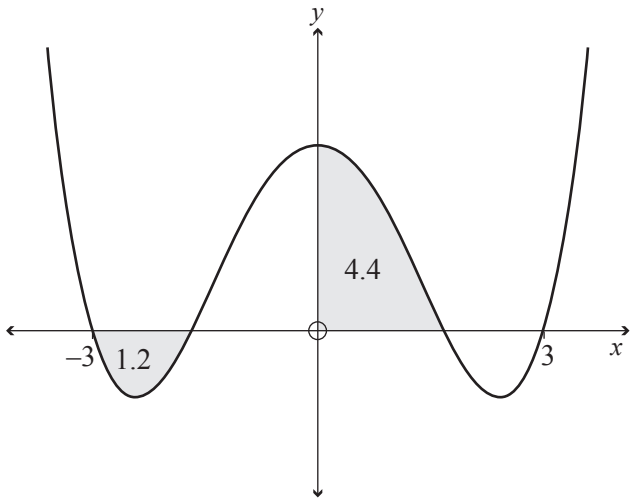
Kaua e tuhi i ngā wāhanga e kitea ai te kauruku whakahāngai (⁄⁄⁄). Ka poroa pea taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria te $\int \frac{4}{x} - \sec^2 x \, \mathrm{d}x$.

(b) E ōrite ana te pānga $y = f(x)$ mō te tuaka pou (y) o te kauwhata kei raro iho nei. Kua tuhi kētia ngā whiringa mō ngā wāhanga kua kaurukutia.



Whiriwhiria te $\int_{-3}^{3} f(x) \, \mathrm{d}x$.

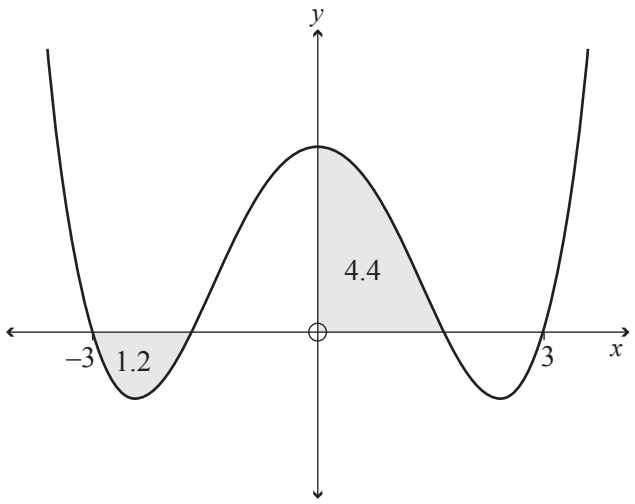
(c) Whiriwhiria te $\int_{0}^{\frac{\pi}{4}} \sin^2(2x) \, \mathrm{d}x$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION ONE

(a) Find $\int \frac{4}{x} - \sec^2 x \, dx$.

(b) The graph of the function $y = f(x)$ below is symmetrical about the y-axis. The areas of the shaded regions are given.

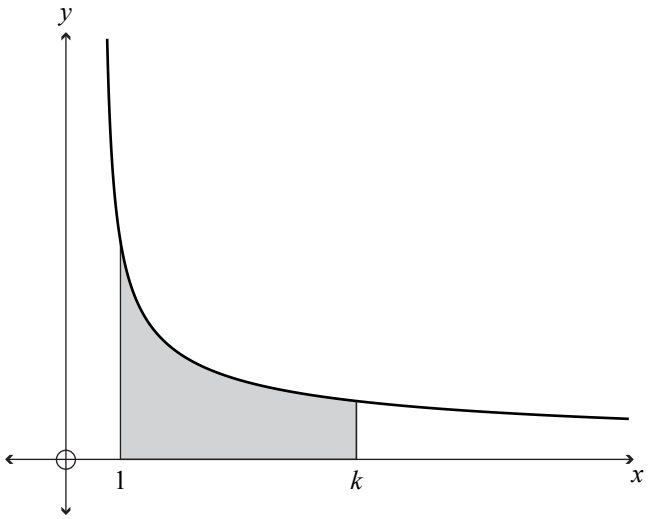


Find $\int_{-3}^{3} f(x) dx$.

(c) Find $\int_{0}^{\frac{\pi}{4}} \sin^2(2x) dx$.

You must use calculus and show the results of any integration needed to solve the problem.

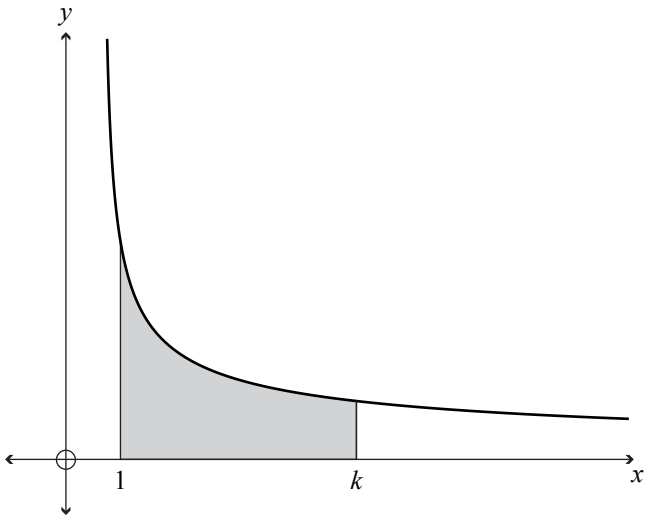
(d) E whakaaturia ana ki te kauwhata kei raro iho nei tētahi wāhanga o te pānga $y = \dfrac{4}{\sqrt{3x-2}}$.



Whiriwhiria te uara o te k nā runga i te mōhio ko te horahanga o te wāhanga kua kaurukutia, ko te 8.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

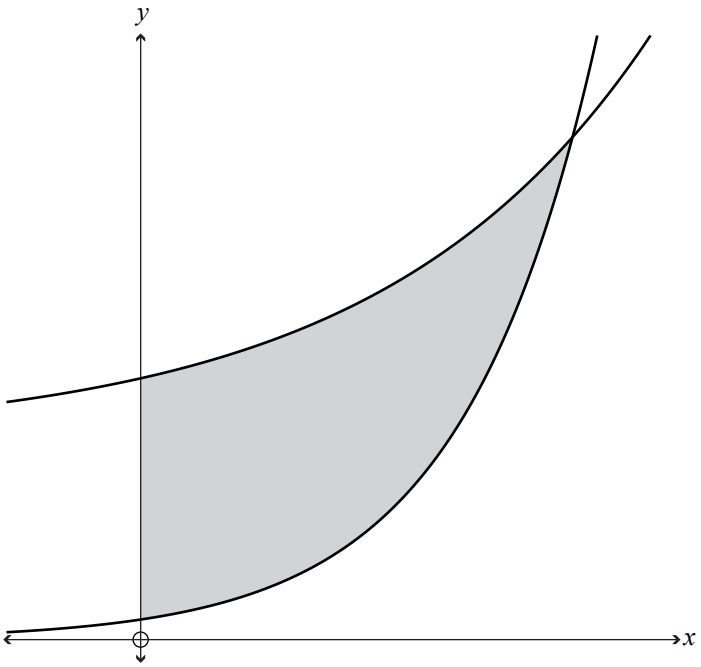
(d) The graph below shows part of the function $y = \dfrac{4}{\sqrt{3x-2}}$.



Find the value of k such that the shaded region has an area of 8.

You must use calculus and show the results of any integration needed to solve the problem.

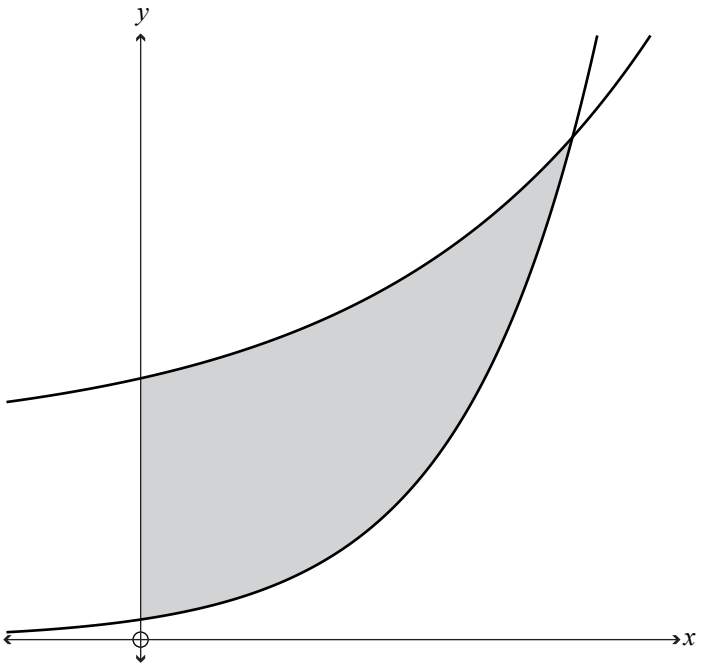
(e) E whakaaturia ana i te kauwhata kei raro iho nei te pānga $y = (e^x)^2$ me te $y = 3e^x + 10$.



Whiriwhiria te uara tika katoa o te wāhi kua kaurukutia.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) The graph below shows the functions $y = (e^x)^2$ and $y = 3e^x + 10$.



Find the exact value of the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

TE TŪMAHI TUARUA

(a) Whiriwhiria te $\int\left(\mathrm{e}^{3x}-\sqrt{x}\right)\mathrm{d}x$.

(b) Whiriwhiria te uara o te k, mehemea ko te $\int_1^k \frac{2}{\sqrt{x}}\,\mathrm{d}x = 8$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) Whakaarohia te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2(x-1)}$, mehemea $x > 1$.

Mehemea ko te $y = -1$ i te wā ko te $x = 2$, whiriwhiria te/ngā uara o te x i ahu ai te uara o te 1 ki te y.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION TWO

(a) Find $\int \left(e^{3x} - \sqrt{x} \right) dx$.

(b) Find the value of k, given that $\int_1^k \frac{2}{\sqrt{x}}\, dx = 8$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Consider the differential equation $\frac{dy}{dx} = \frac{1}{3y^2(x-1)}$, where $x > 1$.

Given that $y = -1$ when $x = 2$, find the value(s) of x which give a y value of 1.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Ka taea te whakaterenga o tētahi mea te hoahoa ki te whārite $a(t) = 0.9\mathrm{e}^{0.3t}$

arā, ko te a te whakaterenga o te mea ki te m s^{-2}, ā, ko te t te wā ā-hēkona mai i te tīmatanga o te inenga o te wā.

Ko te 10 m s^{-1} te tere o te mea rā i muri i te paunga o ngā hēkona e 2.

I pēhea te tawhiti o te haere a taua mea i te hēkona tuarima o tana haere?

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) An object's acceleration can be modelled by the equation $a(t) = 0.9e^{0.3t}$

where a is the acceleration of the object in m s^{-2}, and t is the time in seconds from the start of timing.

The object had a velocity of 10 m s^{-1} after 2 seconds.

How far did the object travel during the 5th second of its motion?

You must use calculus and show the results of any integration needed to solve the problem.

(e) Tērā tētahi kura hinu rango, 150 cm tana teitei, ā, kī ana i te hinu i te tīmatanga.

Ka tīmata te māturuturu atu i tētahi kōwhao i te taha o taua kura.

Ka taea te hoahoa te teitei h, ā-cm nei, o te hinu e toe ana i muri i tōna māturuturu mō te t meneti ki te whārite pārōnaki $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{-1}{4}\sqrt{(h-6)^3}$.

Whiriwhiria te roa kia tae atu te hinu i roto i te kura hinu ki te wāhi 15cm i runga ake o te takere o te kura hinu.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) A cylindrical tank of height 150 cm is originally full of oil.

The tank starts to leak out of a hole in its side.

The height h, in cm, of the oil left in the tank after it has been leaking for t minutes can be modelled by the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-1}{4}\sqrt{(h-6)^3}$.

Find how long it takes for the oil in the tank to be 15 cm above the bottom of the tank.

You must use calculus and show the results of any integration needed to solve the problem.

TE TŪMAHI TUATORU

(a) Whiriwhiria te $\int (2x+5)^3 \, \mathrm{d}x$.

(b) Whakamahia ngā uara i te papatau kei raro hei whiriwhiri i tētahi āwhiwhitanga mō te $\int_0^2 f(x) \mathrm{d}x$ mā te whakamahi i te Ture Trapezium.

x	0	0.4	0.8	1.2	1.6	2.0
$f(x)$	3.6	4.2	4.8	5.4	4.5	3.2

(c) Whiriwhiria te $\int_5^8 \frac{4x-5}{x-3} \mathrm{d}x$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION THREE

(a) Find $\int (2x+5)^3 \, \mathrm{d}x$.

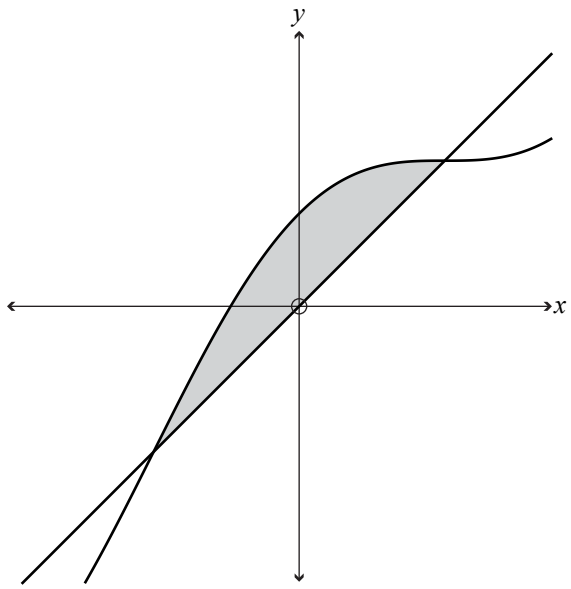
(b) Use the values given in the table below to find an approximation to $\int_0^2 f(x) \mathrm{d}x$ using the Trapezium Rule.

x	0	0.4	0.8	1.2	1.6	2.0
$f(x)$	3.6	4.2	4.8	5.4	4.5	3.2

(c) Find $\int_5^8 \frac{4x-5}{x-3} \mathrm{d}x$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaaturia ana ki te kauwhata kei raro nei tētahi wāhanga o te kōpiko $y = x + \cos x$ me te rārangi $y = x$.

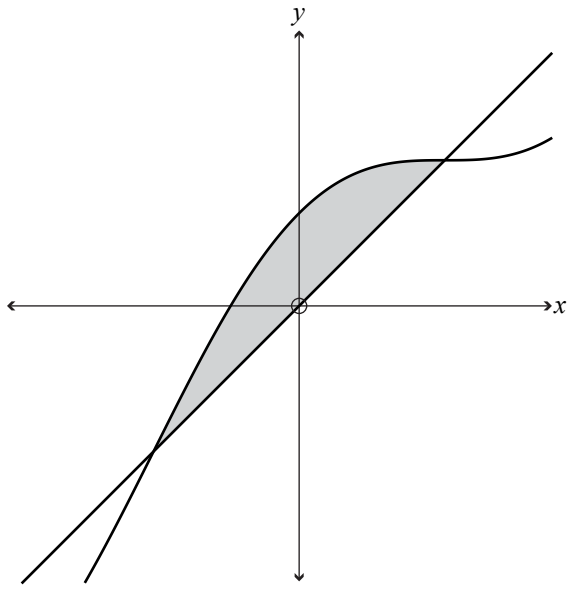


Whiriwhiria te horahanga o te wāhi kua kaurukutia.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(d) The graph below shows part of the curve $y = x + \cos x$ and the line $y = x$.

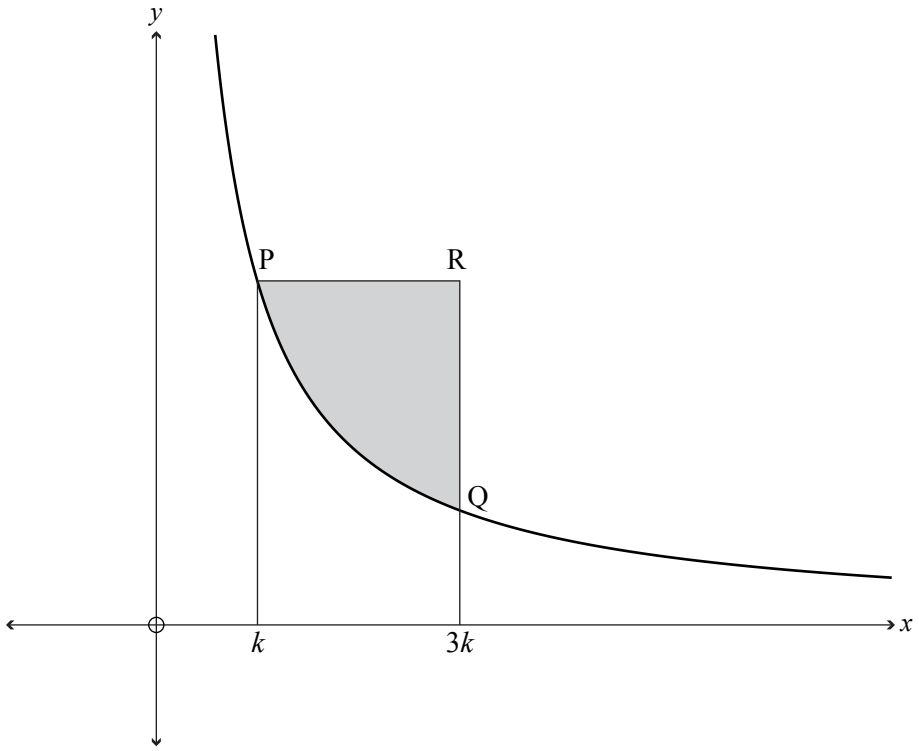


Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the next page.

(e) E whakaaturia ana ki te kauwhata kei raro nei tētahi wāhanga o te kōpiko i te whārite $y = \frac{2}{x}$.



E takoto ana te pito P me te pito Q ki runga o te kōpiko me ngā taunga-x k me te $3k$ i tēnā raupapa, mēnā ko te $k > 0$.

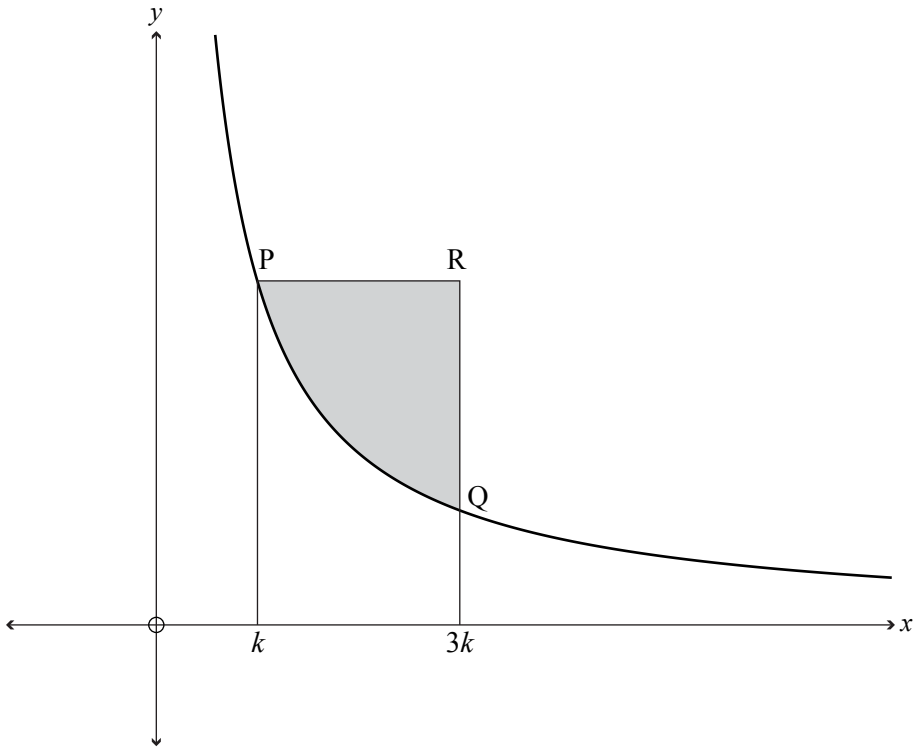
Kua takoto te R kia whakarara ai te PR ki te tuaka pae (x), ā, e whakarara ana te QR ki te tuaka pou (y).

Ka taea te tuhi te wāhanga kua kaurukutia ki te takotoranga o te $a + b \ln c$, mēnā he tau tōpū te a, te b, me te c.

Whiriwhiria te uara o te a, o te b, o te c hoki.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) The graph below shows part of the curve given by the equation $y = \frac{2}{x}$.



Points P and Q lie on the curve with x-coordinates k and $3k$ respectively, where $k > 0$.

Point R is such that PR is parallel to the x-axis and QR is parallel to the y-axis.

The shaded area can be written in the form $a + b \ln c$, where a, b, and c are integers.

Find the values of a, b, and c.

You must use calculus and show the results of any integration needed to solve the problem.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2022

91579M Apply integration methods in solving problems

Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 21 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.