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91578M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2023

91578M Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā roto i te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Whakaatuhia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi i ngā wāhi e kitea ai te kauruku whakahāngai ( ). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria ngā pārōnaki o te $y = \sqrt{3x - 2}$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) Whiriwhiria te pāpātanga o te whiti o te pānga $f(t) = t^2 e^{2t}$ mehemea ko te $t = 1.5$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION ONE

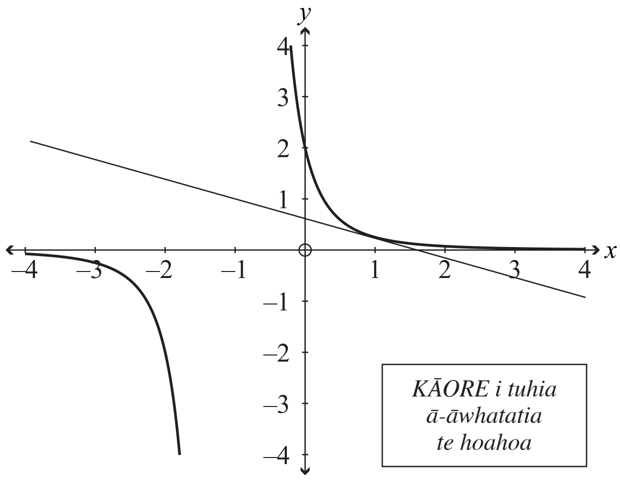
(a) Differentiate $y = \sqrt{3x - 2}$.

You do not need to simplify your answer.

(b) Find the rate of change of the function $f(t) = t^2 e^{2t}$ when $t = 1.5$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) E whakaaturia ana ki te kauwhata te ānau $y = \dfrac{2}{(x+1)^3}$, me te pātapa ki te kōpiko kua tuhia ki $x = 1$.

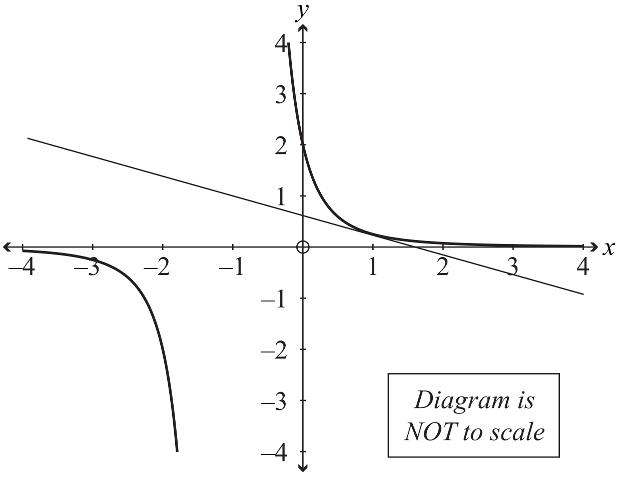


Ka tuhia tētahi pātapa tuarua ki tēnei kōpiko e whakarara ana ki te pātapa tuatahi e whakaaturia ana i te hoahoa.

Whiriwhiria te taunga-x o te pūwāhi e pā ai te pātapa tuarua ki te kōpiko.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) The graph shows the curve $y = \dfrac{2}{(x+1)^3}$, along with the tangent to the curve drawn at $x = 1$.

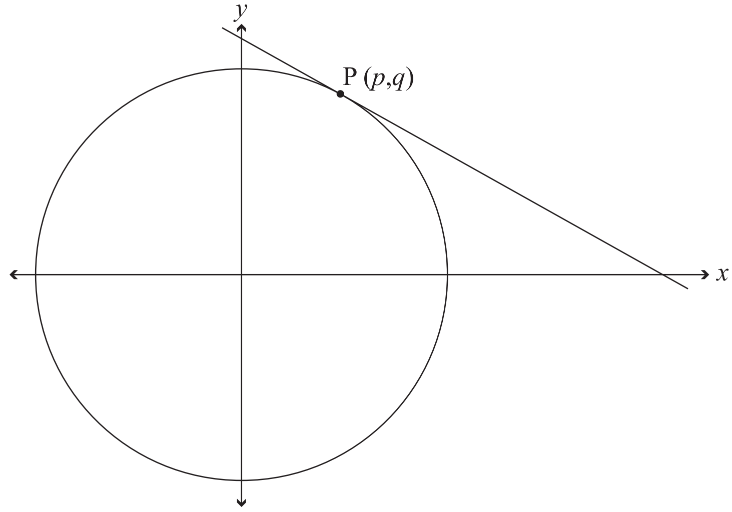


A second tangent to this curve is drawn which is parallel to the first tangent shown in the diagram.

Find the x-coordinate of the point where this second tangent touches the curve.

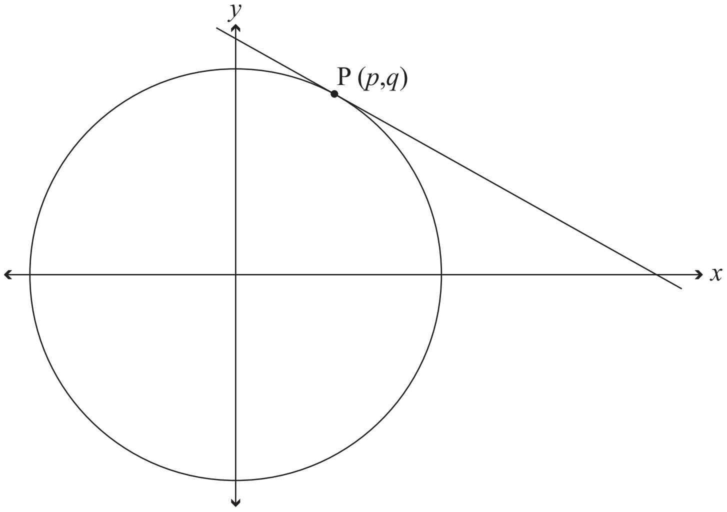
You must use calculus and show any derivatives that you need to find when solving this problem.

(d) E whakaaturia ana ki te hoahoa kei raro iho nei tētahi pātapa e whakawhiti ana mā te pūwāhi P (p, q) e noho ana ki te porowhita ko ōna whārite tawhā ko te $x = 4\cos\theta$ me te $y = 4\sin\theta$.



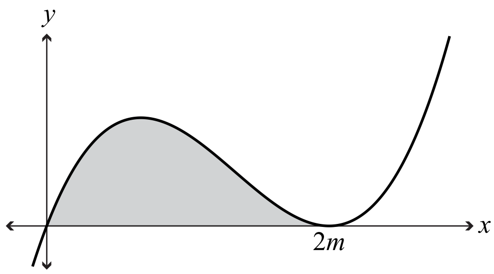
Whakaaturia ko te whārite o te rārangi pātapa ko te $px + qy = p^2 + q^2$.

(d) The diagram below shows a tangent passing through the point P (p, q) which lies on the circle with parametric equations $x = 4\cos\theta$ and $y = 4\sin\theta$.

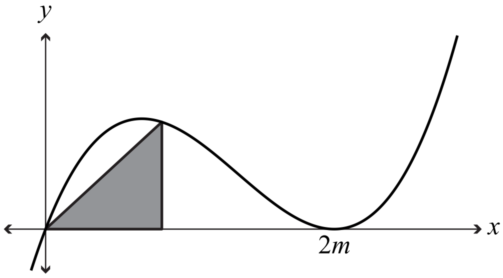


Show that the equation of the tangent line is $px + qy = p^2 + q^2$.

(e) E whakaaturia ana te kauwhata o te $y = x(x - 2m)^2$, mehemea ko te $m > 0$. Ko te tapeke o te wāhi kua kaurukutia i waenga i te kōpiko me te tuaka-x, mai i $x = 0$ ki $x = 2m$, ka tohua ki te $A = \frac{4m^4}{3}$.



Ka waihangatia ināianei tētahi tapatoru koki-hāngai ko tētahi akitu kei te (0,0) me tētahi anō kei te kōpiko o te $y = x(x - 2m)^2$, e whakaaturia ana ki raro iho nei.

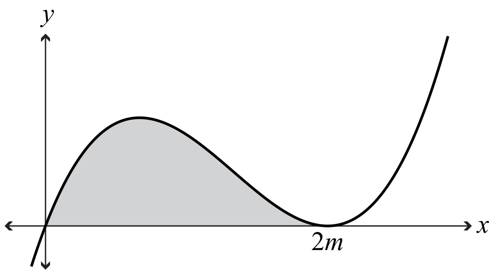


Whakaaturia ko te horahanga mōrahi o tētahi tapatoru pēnei, ko te $\frac{3}{8}$ o te wāhanga kua kaurukutia.

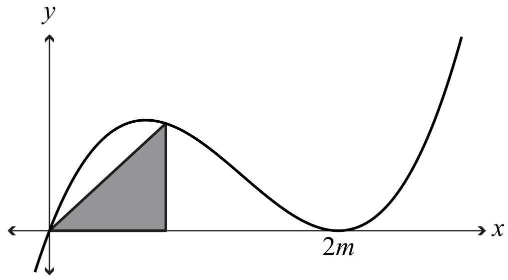
Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

Ehara i te mea me hāpono rawa e koe, ko te horahanga i whiriwhiria ai e koe, he mōrahi.

(e) The graph of $y = x(x - 2m)^2$, where $m > 0$, is shown. The total shaded area between the curve and the x-axis from $x = 0$ to $x = 2m$ is given by $A = \frac{4m^4}{3}$.



A right-angled triangle is now constructed with one vertex at (0,0) and another on the curve $y = x(x - 2m)^2$, as shown below.



Show that the maximum area of such a triangle is $\frac{3}{8}$ of the total shaded area.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not have to prove that the area you have found is a maximum.

TE TŪMAHI TUARUA

(a) Whiriwhiria ngā pārōnaki o te $f(x)=\dfrac{x^2}{\cos x}$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) Whiriwhiria te rōnaki o te pātapa ki te kōpiko $y=\cot(2x)$ i te pūwāhi ko $x=\dfrac{\pi}{12}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION TWO

(a) Differentiate $f(x)=\dfrac{x^2}{\cos x}$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y=\cot(2x)$ at the point where $x=\dfrac{\pi}{12}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) E tautuhia ana tētahi kōpiko ki te whārite $f(x)=\frac{e^x}{x^2+2x}$.

Whiriwhiria te/ngā uara-x o te/ngā pūwāhi i te kōpiko ko te pātapa ki te kōpiko e whakarara ana ki te tuaka-x.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) Whiriwhiria te/ngā uara-x o ngā pūwāhi tūpā i te kauwhata o te pānga $f(x) = 3x^2\ln(x)$.

Me whakapono noa koe he pūwāhi tūpā mārika te/ngā pūwāhi e kitea ana e koe.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) A curve is defined by the equation $f(x)=\dfrac{e^x}{x^2+2x}$.

Find the x-value(s) of any point(s) on the curve where the tangent to the curve is parallel to the x-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Find the x-value(s) of any points of inflection on the graph of the function $f(x) = 3x^2\ln(x)$.

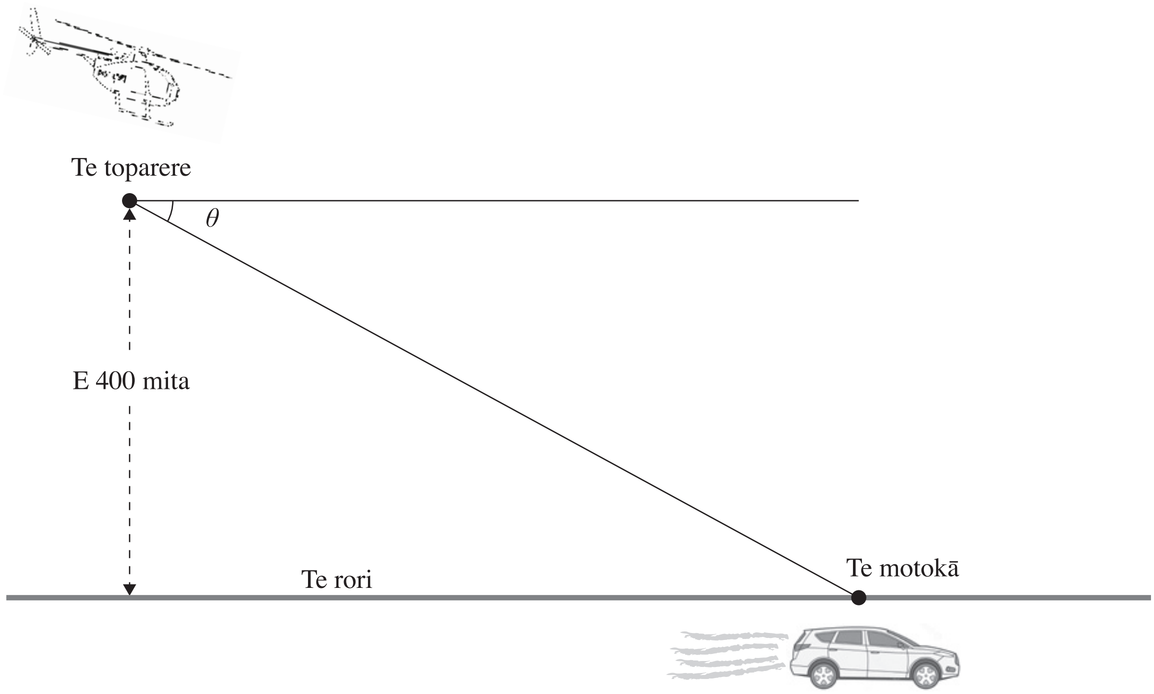
You can assume that your point(s) found are actually point(s) of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) E rere ana tētahi toparere pirihimana i runga ake i tētahi wāhanga huapae torotika o te huanui matua e whaiwhai ana i tētahi motokā e tere ana te haere.

E pūmau ana ki te 72 m s^{-1} te tere o te rere o te toparere, ā, e pūmau ana te teitei ki te 400 mita i runga ake i te papa. E whai nei te toparere kia tae atu ia ki te motokā.

Ka 2500 mita ana te tawhiti tōtika o te toparere i te motokā, ko te 0.002 rad s^{-1} te pāpātanga o te tupu o te koki tāheke, θ, i waenga i te huapae me te rārangi titiro, atu i te toparere ki te motokā.



He mea whakahāngai i: https://animalia-life.club/qa/pictures/police-helicopter-drawing, https://www.freepik.com/premium-vector/car-sedan-suv-drawing-outlines-converted-objects_17981778.htm

Tātaitia te tere o te **motokā** i tēnei wā.

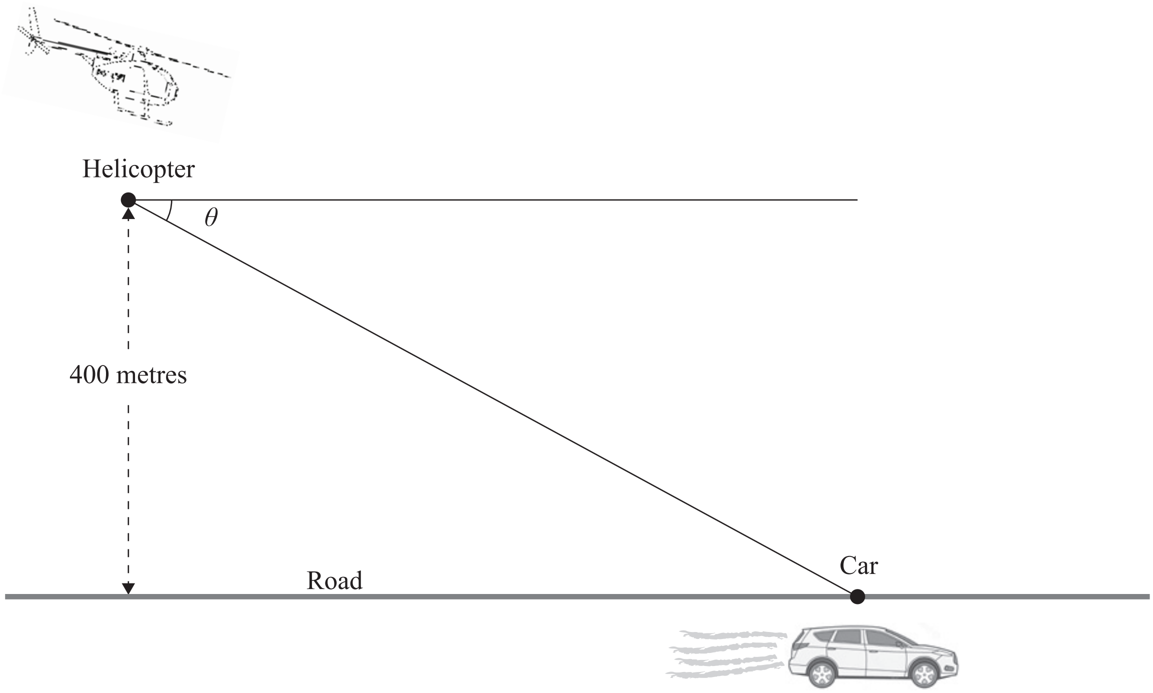
Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

He wāhi anō mō tō tuhinga ki tēnei tūmahi i te whārangi 16.

(e) A police helicopter is flying above a straight horizontal section of motorway chasing a speeding car.

The helicopter is flying at a constant speed of 72 m s^{-1} and at a constant height of 400 metres above the ground. The helicopter is attempting to catch up with the car.

When the direct distance from the helicopter to the car is 2500 metres, the angle of depression, θ, between the horizontal and the line of sight from the helicopter to the car is increasing at a rate of 0.002 rad s^{-1}.



Adapted from: https://animalia-life.club/qa/pictures/police-helicopter-drawing, https://www.freepik.com/premium-vector/car-sedan-suv-drawing-outlines-converted-objects_17981778.htm

Calculate the speed of the **car** at this instant.

You must use calculus and show any derivatives that you need to find when solving this problem.

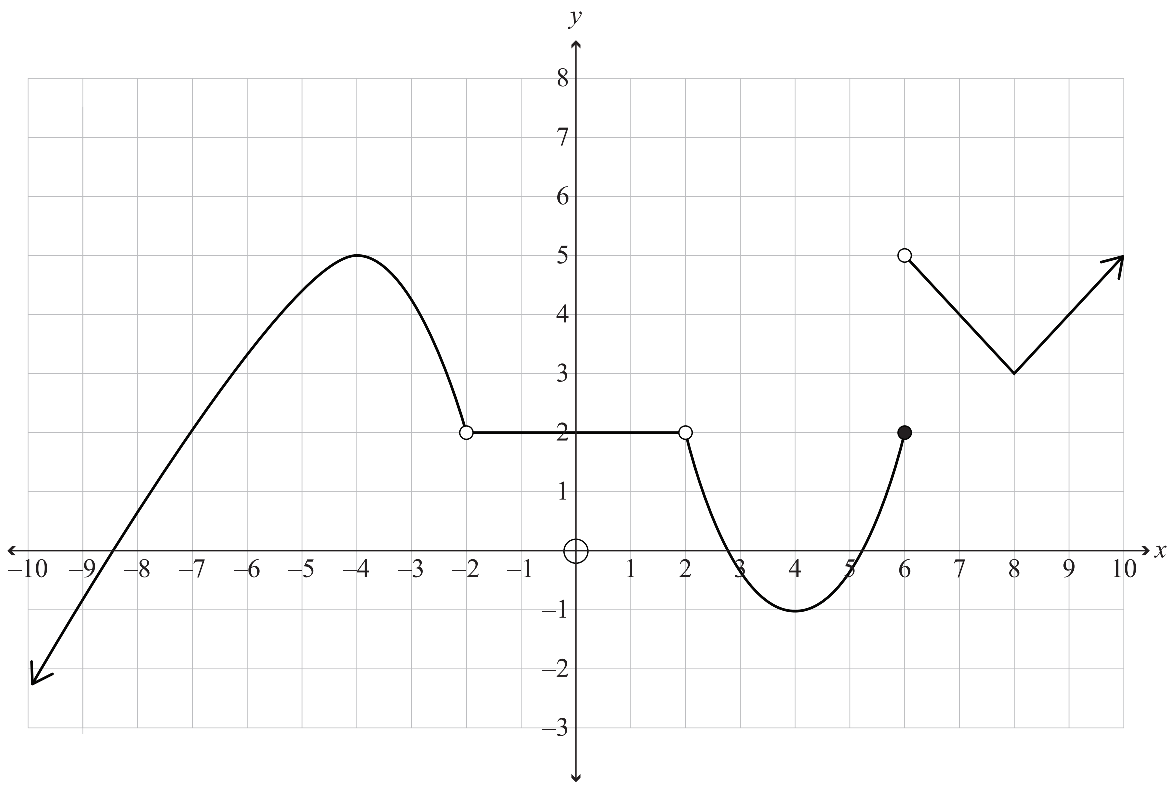
There is more space for your answer to this question on the following page.

TE TŪMAHI TUATORU

(a) Whiriwhiria ngā pārōnaki o te $y = \ln(x^2 - x^4 + 1)$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) E whakaaturia ana i te kauwhata kei raro iho nei te pānga $y = f(x)$.



Mō te pānga kei runga ake nei:

(i) Whiriwhiria te/ngā uara o te x mehemea e motukore ana te $f(x)$, engari kāore e taea te whiriwhiri ōna pārōnaki.

(ii) Whiriwhiria te/ngā uara o te x mehemea e motuhenga ana te $f'(x) = 0$ me te $f''(x) < 0$.

(iii) Ko te aha te uara o te $\lim_{x \to 6} f(x)$?

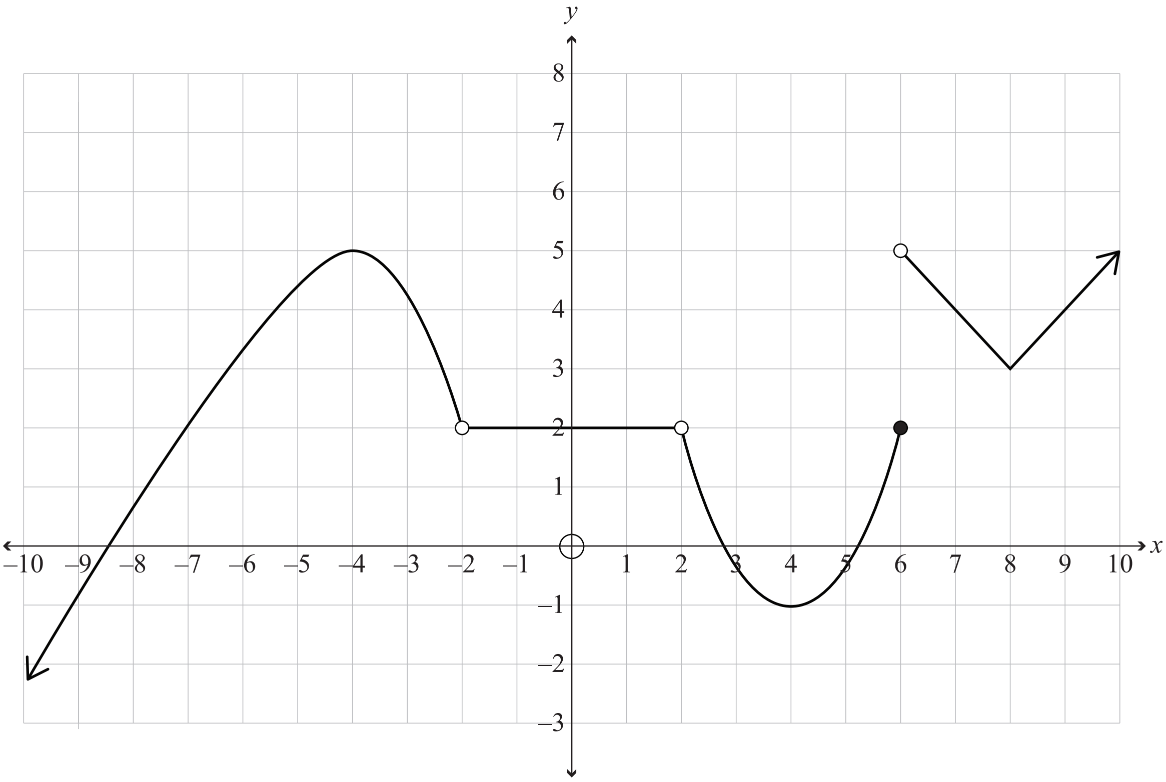
Me āta whakamārama mehemea karekau he uara.

QUESTION THREE

(a) Differentiate $y = \ln(x^2 - x^4 + 1)$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x where $f(x)$ is continuous but not differentiable.

(ii) Find the value(s) of x where $f'(x) = 0$ and $f''(x) < 0$ are both true.

(iii) What is the value of $\lim_{x \to 6} f(x)$?

State clearly if the value does not exist.

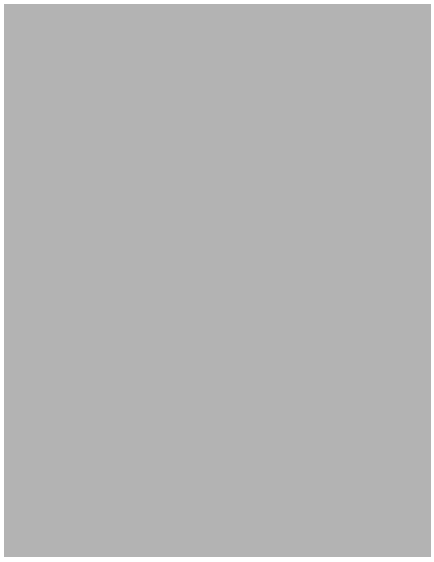
(c) Ka eke a Char i tētahi porowhita hurihuri tino nui. I a ia e huri haere ana, e taea ana tana taunga te whakaahua ki ēnei whārite tawhā e rua:

$$x = 5\sqrt{2}\sin\left(\frac{\pi t}{5}\right) \text{ me te } y = 10 - 5\sqrt{2}\cos\left(\frac{\pi t}{5}\right)$$

mehemea ko te t te wā, ā-hēkona, mai i te tīmatanga o te ekenga.

Whiriwhiria te rōnaki o te rārangi hāngai ki tēnei ānau i te wā ko te $t = 6.25$ hēkona, i muri i te tīmatanga o te ekenga.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.



Te mātāpuna: www.greenbaypressgazette.com/story/news/2019/06/27/bay-beachs-new-big-wheel-rolls-into-action-july-2/1582740001/

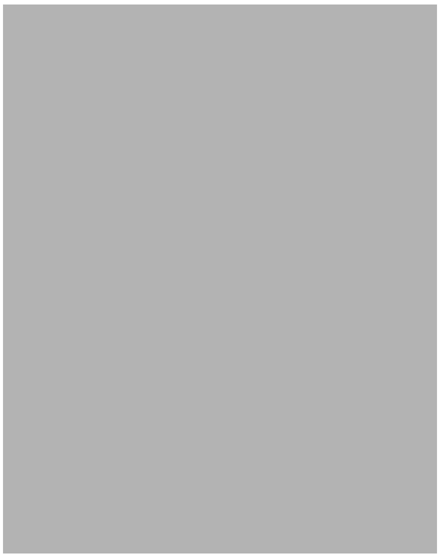
(c) Char goes for a ride on a Ferris wheel. As she rotates around, her position can be described by the pair of parametric equations:

$$x = 5\sqrt{2}\sin\left(\frac{\pi t}{5}\right) \text{ and } y = 10 - 5\sqrt{2}\cos\left(\frac{\pi t}{5}\right)$$

where t is time, in seconds, from the start of the ride.

Find the gradient of the normal to this curve at the point when $t = 6.25$ seconds, after the start of the ride.

You must use calculus and show any derivatives that you need to find when solving this problem.



Source: www.greenbaypressgazette.com/story/news/2019/06/27/bay-beachs-new-big-wheel-rolls-into-action-july-2/1582740001/

(d) Whiriwhiria ngā taunga o ngā pūwāhi tūnoa i te kauwhata o te pānga $f(x)=\frac{1}{x}-\frac{2}{x^3}$, me te tautohu anō i te āhua o aua pūwāhi.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) Find the co-ordinates of any stationary points on the graph of the function $f(x) = \frac{1}{x} - \frac{2}{x^3}$, identifying their nature.

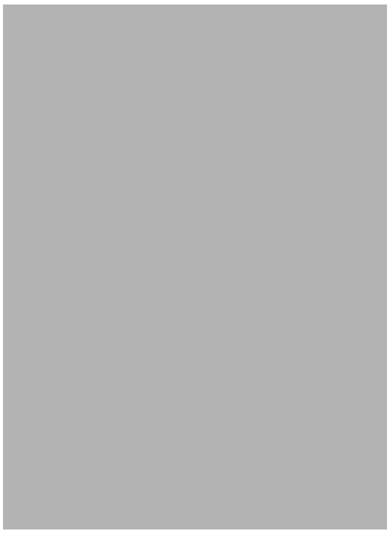
You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Ka tautau tētahi waea hiko i waenga i ngā pou e rua.

Ka taea te whiriwhiri te whārite o te kōpiko $y = f(x)$ e whakatauira ana i te āhua o te waea hiko mā te whakaoti i te whārite pārōnaki:

$$a\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$$

Whakamahia te pārōnaki hei hāpono i te whakaea a te pānga $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ i te whārite pārōnaki o runga, mehemea he tau pūmau tōrunga te a.



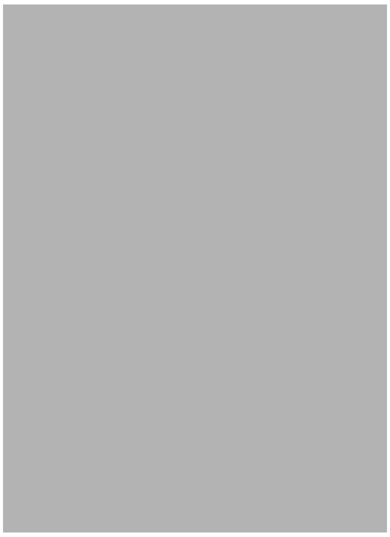
Te mātāpuna: www.thelocalelectrician .com.au/power-lines-power-pole-who-is-responsible/

(e) A power line hangs between two poles.

The equation of the curve $y = f(x)$ that models the shape of the power line can be found by solving the differential equation:

$$a\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$$

Use differentiation to verify that the function $y = \frac{a}{2}\left(\mathrm{e}^{\frac{x}{a}} + \mathrm{e}^{-\frac{x}{a}}\right)$ satisfies the above differential equation, where a is a positive constant.



Source: www.thelocalelectrician.com.au/power-lines-power-pole-who-is-responsible/

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2023

91578M Apply differentiation methods in solving problems

Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.