SUPERVISOR'S USE ONLY

See back cover for an English translation of this cover

3

91579M

915795

Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

+

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2023

91579M Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā roto i te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Whakaatuhia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi i ngā wāhi e kitea ai te kauruku whakahāngai (). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria te $\int\left(3x+2+\frac{1}{3x+2}\right)\mathrm{d}x$.

(b) E taea nei te tere o tētahi mea te hoahoa ki te whārite $v(t) = \sec^2 t$, arā, ko te v te tere o te mea hei km hr^{-1}, ā, ko te t te wā ā-haora mai i te tīmatanga o te inenga o te wā.

I te tīmatanga, i 3 km te tawhiti o te mea i tētahi pūwāhi P.

Whiriwhiria te tawhiti o tēnei mea i te pūwāhi P ā muri i ngā haora $\frac{\pi}{4}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION ONE

(a) Find $\int\left(3x+2+\frac{1}{3x+2}\right)\mathrm{d}x$.

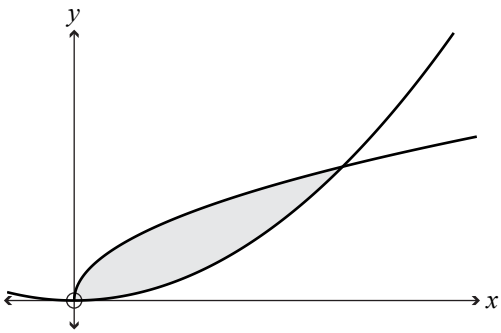
(b) An object's velocity can be modelled by the equation $v(t) = \sec^2 t$,
where v is the velocity of the object in km hr^{-1}, and t is the time in hours from the start of timing.

Initially the object was 3 km from a point P.

Find the distance of this object from the point P after $\frac{\pi}{4}$ hours.

You must use calculus and show the results of any integration needed to solve the problem.

(c) E whakaaturia ana i te kauwhata kei raro iho nei ngā pānga $y = \sqrt{x}$ me te $8y = x^2$.



Whiriwhiria te horahanga o te wāhi kua kaurukutia.

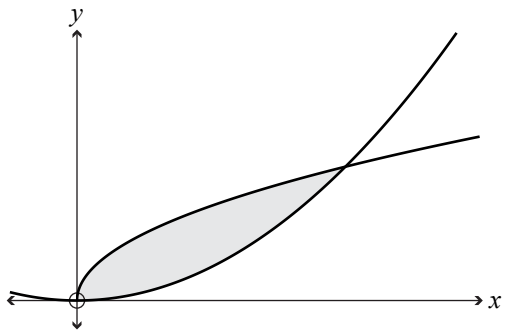
Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) Whakaarohia te whārite pārōnaki $\frac{dy}{dx} = y\left(2x - 3x^2\right)$.

Mehemea ko te $y = 1$ i te wā ko te $x = 2$, whiriwhiria te/ngā uara o te y i te wā ko te $x = 1$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) The graph below shows the functions $y = \sqrt{x}$ and $8y = x^2$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

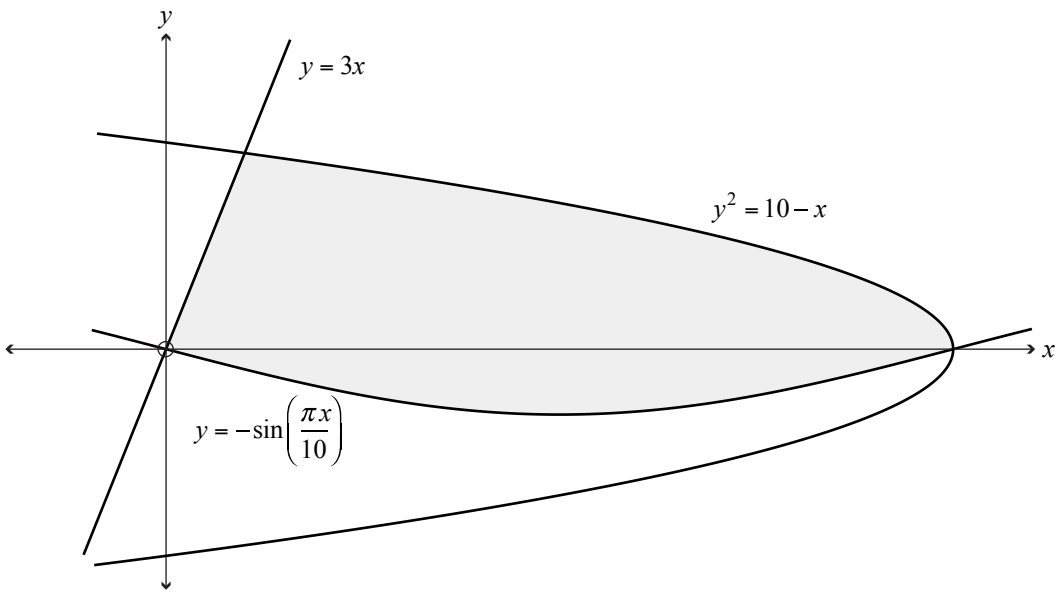
(d) Consider the differential equation $\frac{dy}{dx} = y\left(2x - 3x^2\right)$.

Given that $y = 1$ when $x = 2$, find the value(s) of y when $x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) E rohea ana e ngā kauwhata e toru nei te wāhi kua kaurukutia i te hoahoa kei raro iho nei:

$$y^2 = 10 - x$$
$$y = 3x$$
$$y = -\sin\left(\frac{\pi x}{10}\right)$$



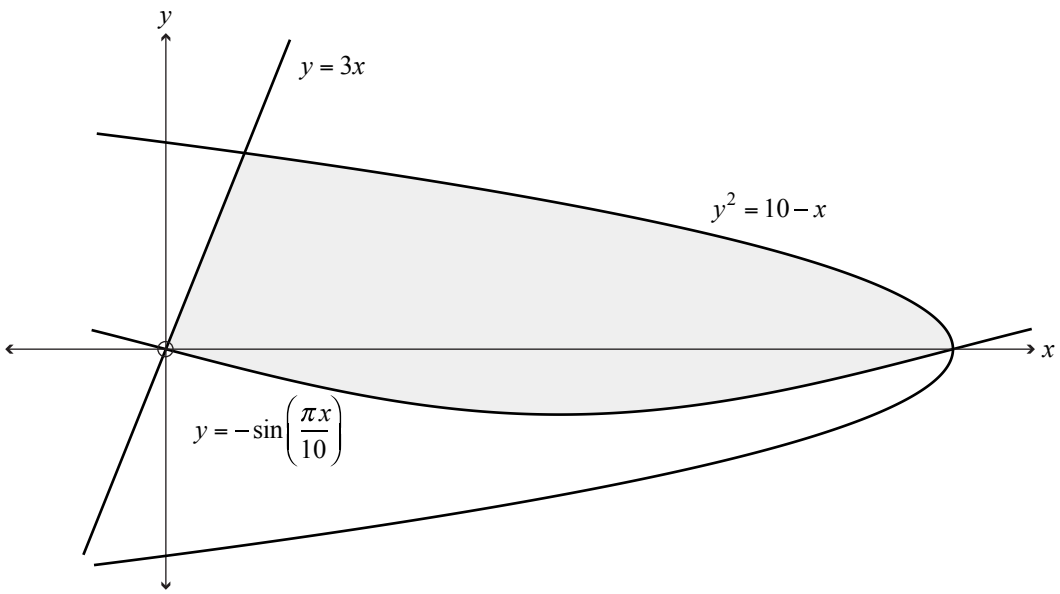
Whiriwhiria te horahanga o te wāhi kua kaurukutia.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

He wāhi anō mō tō tuhinga ki tēnei tūmahi i te whārangi 8.

(e) The shaded region in the diagram below is bounded by the three graphs:

$$y^2 = 10 - x$$
$$y = 3x$$
$$y = -\sin\left(\frac{\pi x}{10}\right)$$



Find the area of the shaded region.

You must use calculus and show the results of any integration needed to solve the problem.

There is more space for your answer to this question on the following page.

TE TŪMAHI TUARUA

(a) Whiriwhiria te $\int 4e^{2x-1}\,dx$.

(b) Whakaotia te whārite pārōnaki $\frac{dy}{dx} = (4x+1)^{-\frac{1}{2}}$, mēnā ko te $x \geq 0$, mehemea ko te $x = 6$, i te wā ko te $y = 7.5$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION TWO

(a) Find $\int 4e^{2x-1}\,dx$.

(b) Solve the differential equation $\frac{dy}{dx} = (4x+1)^{-\frac{1}{2}}$, where $x \geq 0$, given that when $x = 6$, $y = 7.5$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Whiriwhiria te uara o te k, mehemea ko te $\int_2^k \left(\frac{6x-3}{2x-3}\right) \mathrm{d}x = 3k$.

(d) Whiriwhiria te $\int \frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x} \mathrm{d}x$.

(c) Find the value of k, given that $\int_2^k \left(\frac{6x-3}{2x-3}\right) \mathrm{d}x = 3k$.

(d) Find $\int \frac{\cos 2x + \sin 2x}{\cos 2x - \sin 2x} \mathrm{d}x$.

(e) Kei tētahi wheketere keke tētahi puoto tiakarete wē ka whakamahia i te tununga o ngā keke tiakarete.

Ka papua atu te tiakarete wē i te puoto e riterite ai te pāpātanga o te whiti o te tiakarete wē e toe ana i te puoto ki te pūrua o te rōrahi o te tiakarete wē e toe ana.

Ā muri i tētahi haora kotahi i tētahi rangi, ko te rōrahi o te tiakarete e toe ana ko te p rita, ā, he tau pūmau tōrunga te p.

Ā muri i tētahi haora anō, ko ētahi $\frac{4}{5}p$ rita noa o te tiakarete e toe ana i te puoto.

Tuhia mai he whārite pārōnaki e whakatauira ana i tēnei tūāhua, ka whakaotihia ai hei tātai i te nui o te tiakarete wē i te puoto i te tīmatanga o te rā, mā te whakaatu i tō whakautu e ai ki te p.

He wāhi anō mō tō tuhinga ki tēnei tūmahi i te whārangi 16.

(e) A cake factory has a container of liquid chocolate that is used in the manufacture of chocolate cakes.

The liquid chocolate is pumped out of the container so that the rate of change of the volume of liquid chocolate remaining in the container is proportional to the square of the volume of liquid chocolate remaining.

After one hour of use on a particular day, the volume of chocolate remaining is p litres, where p is a positive constant.

After a further one hour, there are only $\frac{4}{5}p$ litres of chocolate remaining in the container.

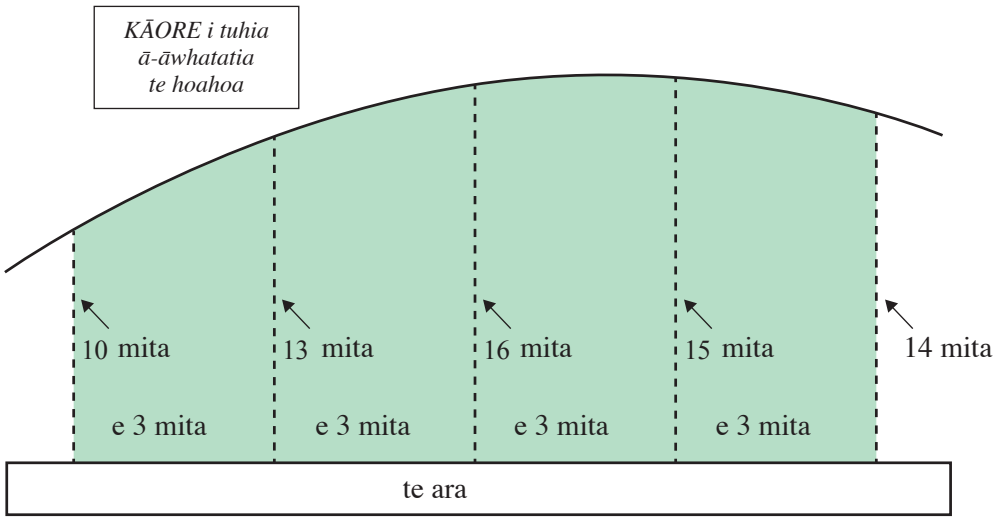
Write a differential equation that models this situation, and solve it to calculate how much liquid chocolate was in the container at the start of the day, giving your answer in terms of p.

There is more space for your answer to this question on the following page.

TE TŪMAHI TUATORU

(a) E hiahia ana tētahi kaihoahoa māra ki te whiriwhiri i tētahi āwhiwhitanga o te horahanga o tētahi wāhanga o tētahi māra, kua kaurukutia ki raro iho nei.

E whakaaturia ana i te hoahoa kei raro iho nei ētahi o āna inenga.



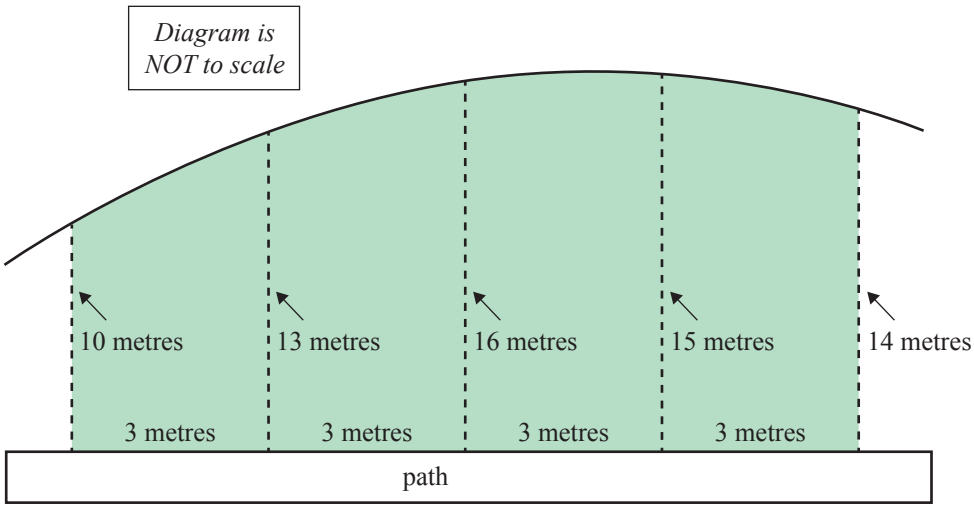
Mā te whakamahi i ēnei inenga, me te Ture a Simpson, whiriwhiria tētahi āwhiwhitanga o te horahanga o te wāhanga māra nei.

(b) Whiriwhiria te $\int \left(\frac{\sqrt{x} - 3}{\sqrt{x}} \right) \mathrm{d}x$.

QUESTION THREE

(a) A garden designer wants to find an approximation of the area of a section of a garden, shaded below.

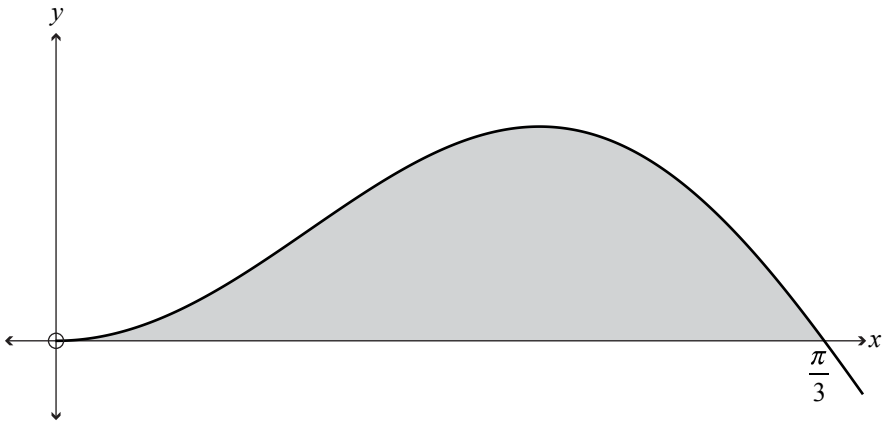
They take some measurements and these are shown in the diagram.



Using these measurements, and Simpson's Rule, find an approximation of the area of the garden section.

(b) Find $\int \left(\frac{\sqrt{x} - 3}{\sqrt{x}} \right) \mathrm{d}x$.

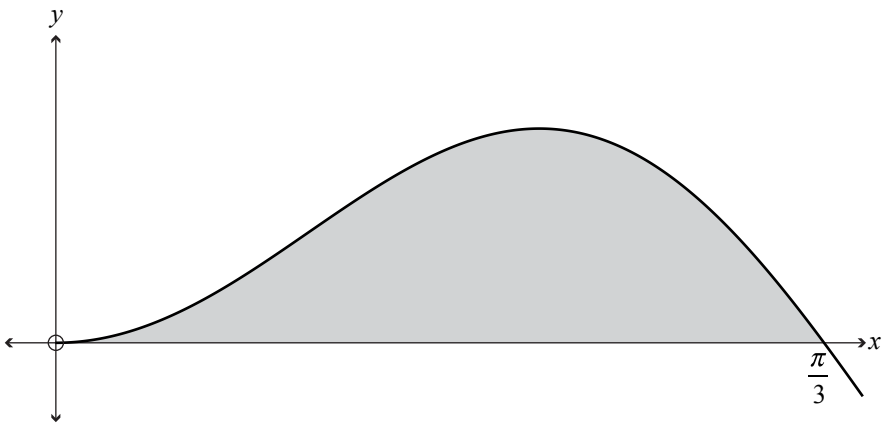
(c) E whakaaturia ana i te kauwhata kei raro iho nei te pānga $y = 5\sin(3x)\sin(x)$.



Whiriwhiria te horahanga o te wāhi kua kaurukutia.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) The graph below shows the function $y = 5\sin(3x)\sin(x)$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Ka taea te whakaterenga o tētahi mea te hoahoa ki te whārite $a = \dfrac{e^{2t}}{4e^{2t} - 3}$.

arā, ko te $t \geq 0$, ā, ko te a te whakaterenga o te mea hei m s^{-2}, ā,
ko t te wā, ā-hēkona, mai i te tīmatanga o te inenga o te wā.
I te $t = 0$ hēkona, ko te 5 m s^{-1} te tere o te mea.

Whiriwhiria te tere o te mea i te wā ko $t = 4$ hēkona.

(d) An object's acceleration can be modelled by the equation $a = \dfrac{e^{2t}}{4e^{2t} - 3}$.

where $t \geq 0$ and a is the acceleration of the object in m s^{-2}
and t is the time, in seconds, from the start of timing.

At $t = 0$ seconds, the object had a velocity of 5 m s^{-1}.

Find the object's velocity when $t = 4$ seconds.

(e) Whakaarohia te whārite pārōnaki o te $\left(1-x^2\right)\left(1+y\right)\frac{\mathrm{d}y}{\mathrm{d}x}+\left(1-x\right)\left(1-y^2\right)=0$.

Mehemea ko te $y = 0$ i te wā ko te $x = 2$, whiriwhiria te/ngā uara o te y i te wā ko te $x = 6$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) Consider the differential equation $\left(1-x^2\right)\left(1+y\right)\frac{\mathrm{d}y}{\mathrm{d}x}+\left(1-x\right)\left(1-y^2\right)=0$.

Given that $y = 0$ when $x = 2$, find the value(s) of y when $x = 6$.

You must use calculus and show the results of any integration needed to solve the problem.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2023

91579M Apply integration methods in solving problems

Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.