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91586M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Pāngarau me te Tauanga (Te Tauanga), Kaupae 3, 2023

91586M Te whakahāngai tuari tūponotanga i te whakaoti rapanga

Ngā whiwhinga: E whā

Paetae	Kaiaka	Kairangi
Te whakahāngai tuari tūponotanga i te whakaoti rapanga.	Te whakahāngai tuari tūponotanga, mā roto i te whakaaro pānga, i te whakaoti rapanga.	Te whakahāngai tuari tūponotanga, mā roto i te whakaaro waitara e whānui ana, i te whakaoti rapanga.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te Puka mō ngā Ture Tātai me ngā Tūtohi L3–STATMF.

Whakaaturia ngā whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapa o ngā whārangi 2–27, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (AE TUHI KI TEW KAUAE TUHI KI). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) He nui ngā pāmu hipi i Aotearoa kei reira rā ngā kararehe e tītongi mauti ana, puta noa i te tau. Ko te pāpātanga kararehe, ko te nui o ngā kararehe e tītongi ana i tētahi rohenga whenua. Mō ngā hipi e pāmuhia ana i ngā hiwi me ngā maunga o Aotearoa, **e 9 hipi i ia heketea** te toharite o ngā pāpātanga kararehe.

(i) Whakamahia tētahi tuari tūponotanga e tika ana hei whakatau tata i te tūponotanga ka nui ake i te 10 ngā hipi i tētahi rohenga heketea i ngā hiwi me ngā maunga o Aotearoa.

Tuhia te ingoa me te/ngā tawhā o te tuari tūponotanga i kōwhiria ai e koe hei wāhanga mō tō tuhinga.

(ii) E mōhiotia whānuitia ana, i te nuinga o te wā, ka piri ngā hipi hei kāhui (ka piri hei whakataka).

Homai kia RUA ngā take e whakaaweawe ai pea tēnei kōrero i te tuari tūponotanga ka kōwhiria e koe, me ōna āhuatanga me mātua whakarite mō te whakatauira i te nui o ngā hipi i ia heketea i ngā hiwi me ngā maunga o Aotearoa.

Te take tuatahi: _____

Te take tuarua: _____

QUESTION ONE

(a) There are many sheep farms in New Zealand, where animals graze on pasture all year round. The stocking rate is defined as the number of animals grazing on a given amount of land. For sheep being farmed on New Zealand hills and mountains, average stocking rates are **9 sheep per hectare**.

(i) Use a suitable probability distribution to estimate the probability that there are more than 10 sheep on a given hectare of New Zealand hills and mountains.

State the name and parameter(s) for your chosen probability distribution as part of your answer.

(ii) It is commonly known that sheep tend to flock together (form groups).

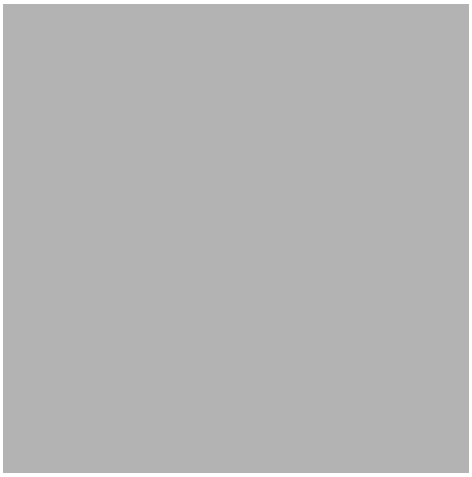
Give TWO reasons why this piece of information may affect your choice of probability distribution and its required conditions for modelling the number of sheep per hectare on New Zealand hills and mountains.

Reason one:

Reason two:

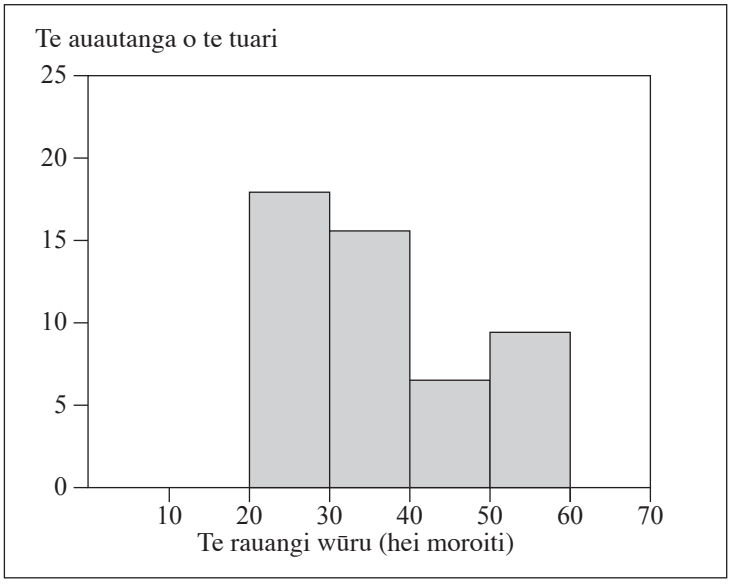
(b) He momo hipi mokorea ngā hipi Awassi o Aotearoa, ā, nō te Rāwhiti Waenga ōna whakapapa. Ko te nuinga he hipi miraka, heoi ka puta hoki i tēnei momo he wūru pai mō te ahumahi whāriki.

Ko te rauangi wūru te inenga o te whitianga o ngā weu wūru (hei moroiti) o te hipi. E whakaatu ana te Ata 1(a) i te tuari auau o te rauangi wūru mō tētahi tīpakonga o ngā hipi Awassi, e 49.



Te mātāpuna: https://nzsheep.co.nz/awassi/

Te Ata 1(a): Te kauwhata pouhere auau o te rauangi wūru i te wūru Awassi (n = 49)



Te mātāpuna: Lewin, Horowitz & Zacks, 1957

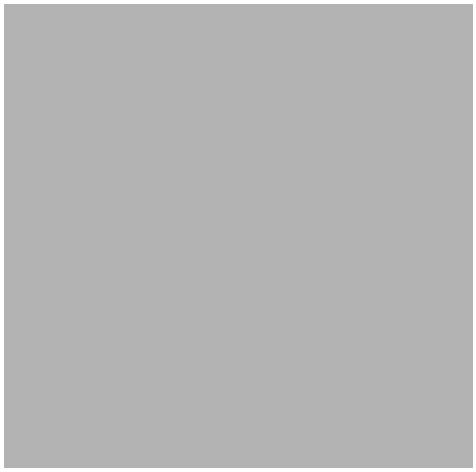
(i) Me kī, ka kōwhiria tētahi tuari ōrite hei whakatauira i te rauangi wūru o ngā hipi Awassi.

Whakamahia taua tuari tūponotanga hei tātai i te tūponotanga ka nui ake i te 35 moroiti te rauangi wūru o tētahi hipi Awassi ka kōwhiria matapōkeretia.

Tuhia ngā tawhā i whakamahia, hei wāhanga mō tō whakautu.

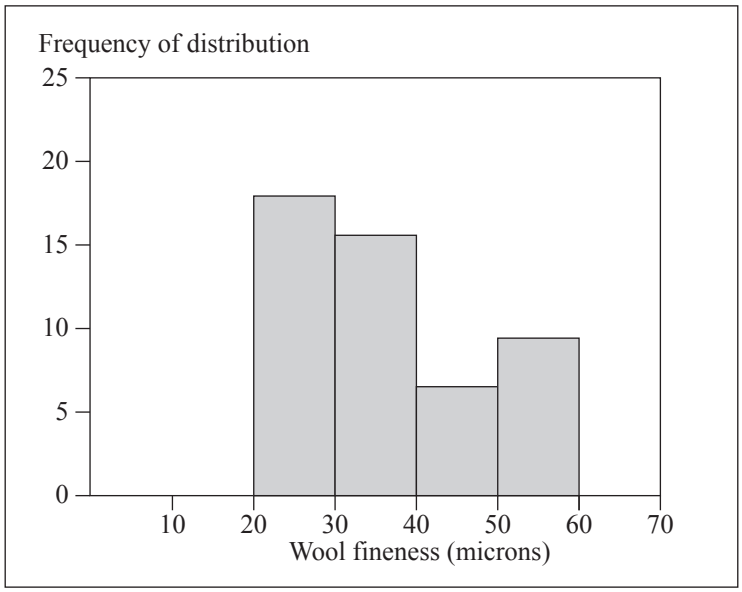
(b) Awassi sheep are a rare New Zealand sheep breed with origins in the Middle East. They are predominantly milking sheep, although they also produce wool that is suited to the carpet industry.

Wool fineness is a measure of the diameter of the sheep wool fibres (in microns). Figure 1(a) shows the frequency distribution of wool fineness for a sample of 49 Awassi sheep.



Source: https://nzsheep.co.nz/awassi/

Figure 1(a): Frequency histogram of wool fineness in Awassi wool (n = 49)



Source: Lewin, Horowitz & Zacks, 1957

(i) Suppose a uniform distribution is chosen to model the wool fineness of Awassi sheep.

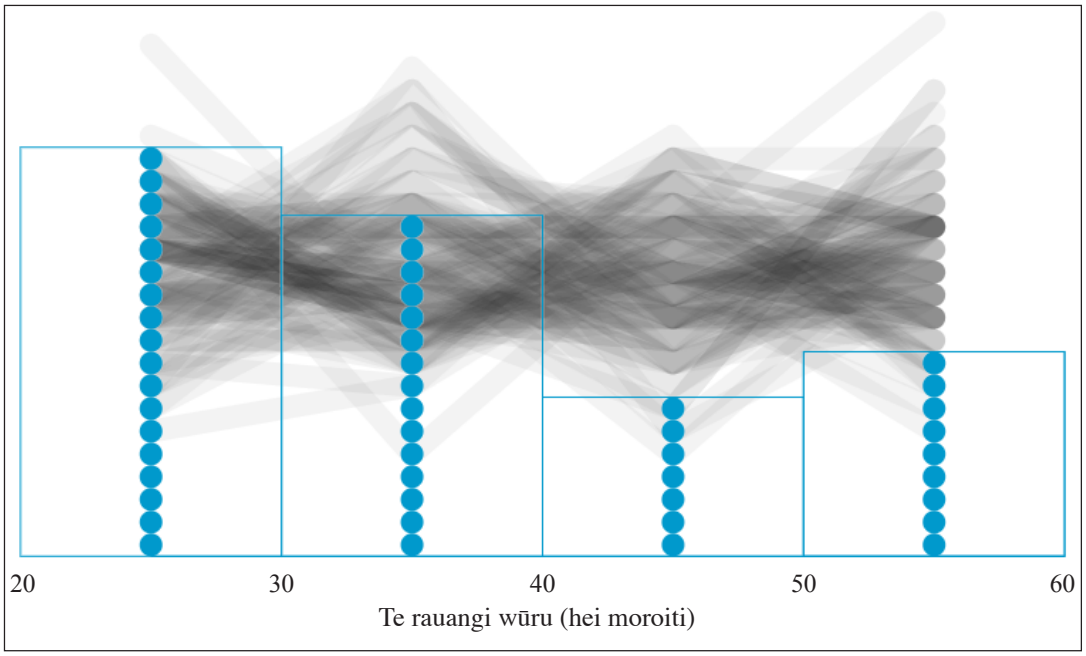
Use this probability distribution to calculate the probability that for a randomly selected Awassi sheep, the wool fineness is greater than 35 microns.

As part of your answer, state the parameters used.

(ii) Ka 1000 ngā kuhunga o ngā tīpakonga raraunga e whakaaturia ana i te Ata 1(a) ki tētahi tauira whaihanga, ā, e whakapaetia ana e ōrite ana te tuaritanga o te rauangi wūru mō ngā hipi Awassi, waihoki, e 20 moroiti te mōkito, ā, e 60 moroiti te mōrahi.

E whakaaturia ana i te Ata 1(b) ngā hua o te tauira whaihanga (he hanga nui rawa mō ngā raraunga i puta rā i te hoahoa) me ngā raraunga taketake e tūturu ana i tirohia. E whakaatu ana ngā ira kahurangi i te auau o te putanga o ngā momo rauangi wūru rerekē. E whakaatu ana ngā rārangi kiwikiwi i te taurangirangi e whakapaetia ana mō ngā momo rauangi wūru, e hāngai ana ki ngā hipi e 49 me te tūponotanga ¼ mō ia momo wūru.

Te Ata 1(b): Ngā hua o te tauira whaihanga me ngā raraunga taketake e tūturu ana



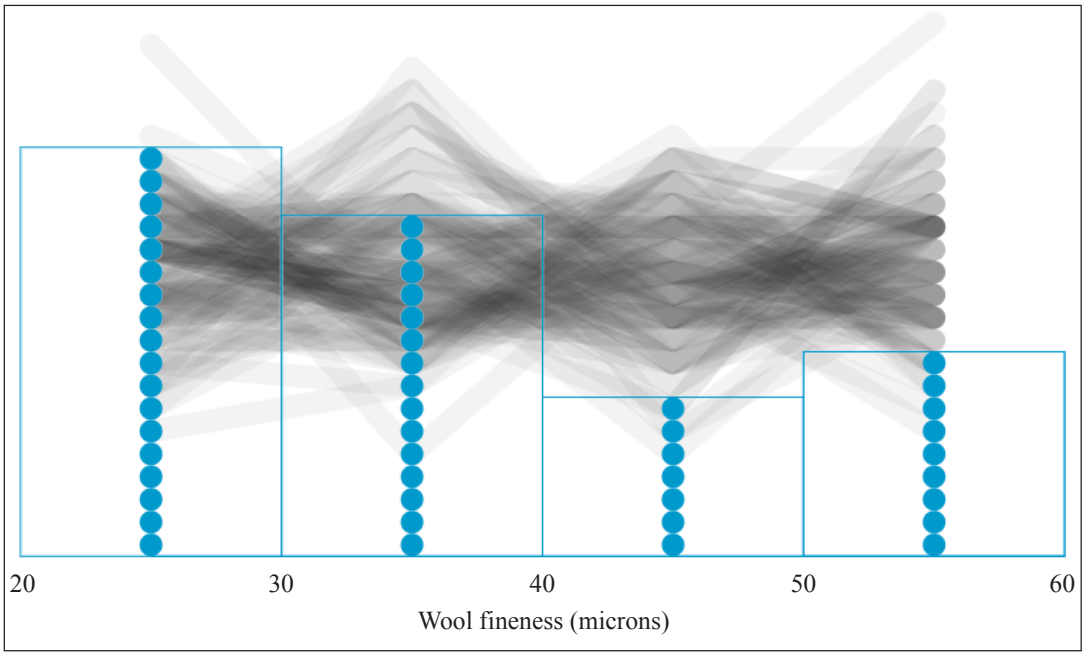
E ai ki ngā hua o te tauira whaihanga me ngā raraunga taketake e tūturu ana i tirohia (Te Ata 1(b)), matapakina mēnā rānei e tika ana te āhua o te tauira tuari ōrite e whakaaturia ana i runga ake nei, hei whakatauira i te rauangi o ngā wūru o ngā hipi Awassi.

Taunakitia tō tuhinga ki ngā whakaaro ā-tauanga.

(ii) The sample data shown in Figure 1(a) is run through a simulation model 1000 times, assuming that the wool fineness for Awassi sheep is uniformly distributed, with a minimum of 20 microns and a maximum of 60 microns.

Figure 1(b) shows the results of the simulation model (overfitted-shape for the model generated data) and original real observed data. The blue dots show the relative frequencies of the different classes of wool fineness. The grey band shows the variation expected for classes of wool fineness, based on a total of 49 sheep and ¼ probability for each wool class.

Figure 1(b): Results of the simulation model and original real data



Based on the results of the simulation model and the original real observed data (Figure 1(b)), discuss whether the uniform distribution model presented above appears to be appropriate for modelling the wool fineness for Awassi sheep.

Support your answer with statistical reasoning.

(iii) Tātaia tētahi whakataunga tata, mā te whakamahi i tētahi tauira tuari tūponotanga e tika ana, mō te tūponotanga kei waenganui i te 25 me te 40 moroiti te rauangi wūru mō tēnā, mō tēnā o ngā hipi Awassi e whā.

Tuhia te ingoa me ngā tawhā mō te tuari tūponotanga i kōwhiria ai e koe hei wāhanga mō tō tuhinga.

(iv) Ko te motuhake o ngā pāpono tētahi o ngā whakapae me whai wāhi ki te tātainga i te wāhanga (iii).

Tautohua te pāpono e tika ana, ka parahau ai i tēnei whakapae i tōna horopaki.

(iii) Using an appropriate probability distribution model, calculate an estimate for the probability that the wool fineness is between 25 and 40 microns for each of four Awassi sheep.

State the name and parameters for your chosen probability distribution as part of your answer.

(iv) One of the assumptions needed for the calculation in part (iii) is independence of events.

Identify the appropriate event and justify this assumption in context.

TE TŪMAHI TUARUA

(a) Ka taea te whakatauira te pāpātanga o te whānau hēki a te heihei, mā te tuari Poisson, ā, i te 99.73% o ngā whānautanga, ka kotahi, ka neke atu rānei ngā hēki ka puta, i ia wiki.

https://www.thestatesman.com/supplements/8thday/quality-egg-issue-1502756802.html

(i) Tātaia tētahi whakataunga tata mō te nui ā-toharite o ngā hēki ka puta i ngā heihei i ia wiki.

(ii) E hia ngā hēki kāore e kore ka puta i te heihei i ia wiki?

Taunakitia tō whakautu ki ngā whakaaro ā-tauanga, ki ngā tātainga rānei.

(iii) Ka taka te wā, ka 10 ngā hēki ka puta i te heihei. Ka paopao rānei, kāore rānei e paopao ia hēki motuhake, ā, ko te 0.65 te tūponotanga o te paopao.

Tātaia te tūponotanga ka paopao ngā hēki e 9 rānei, 10 rānei o te 10.

QUESTION TWO

https://www.thestatesman.com/supplements/8thday/quality-egg-issue-1502756802.html

(a) The rate at which a hen lays eggs can be modelled by a Poisson distribution, and 99.73% of the time she lays at least one egg each week.

(i) Calculate an estimate for the average number of eggs the hen lays each week.

(ii) What is the most likely number of eggs that the hen will lay each week?

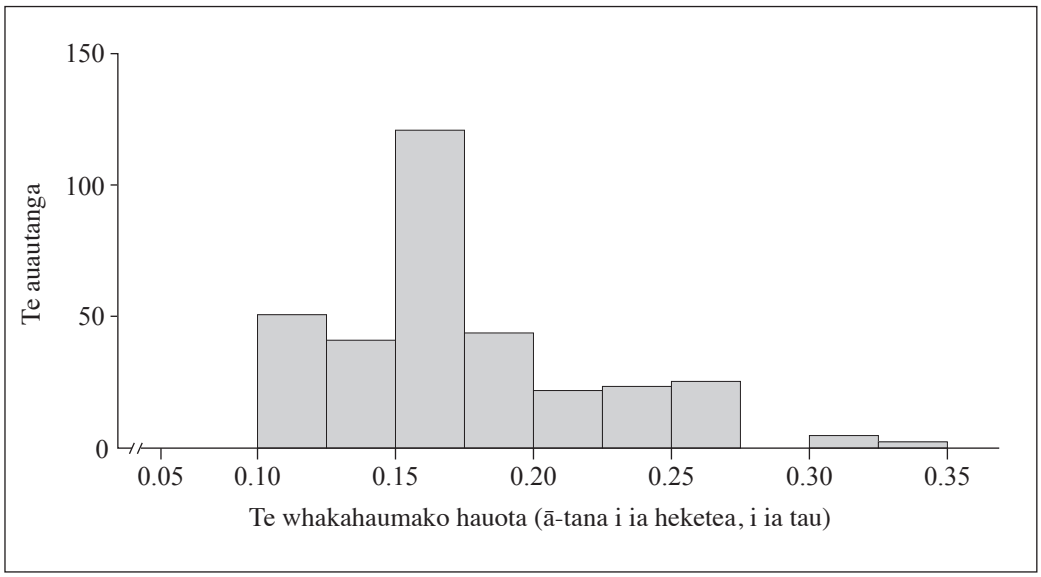
Justify your answer with statistical reasoning or calculations.

(iii) Over a period of time, the hen lays 10 eggs. Each egg independently does or doesn't hatch, with probability 0.65 of hatching.

Calculate the probability that 9 or more of the 10 eggs will hatch.

(b) E whakaaturia ana i te ata e whai ake nei te tuaritanga o te nui o te whakahaumako hauota (ā-tana) ka tukuna ki ia heketea i ia tau, i ngā pāmu miraka e 332 i Aotearoa.

Te Ata 2: Te whakahaumako hauota (ā-tana) ka tukuna ki ia heketea i ia tau, i ngā pāmu miraka i Aotearoa



Te mātāpuna (kua whakahāngaitia): Doole, G. J., Marsh, D., and Ramilan, T. (2013). Evaluation of agri-environmental policies for reducing nitrate pollution from New Zealand dairy farms accounting for firm heterogeneity. *Land Use Policy*, 30 (1), 57–66. 10.1016/j.landusepol.2012.02.007.

(i) Mā te whakamahi i ngā raraunga i te Ata 2, tautapahia tētahi tuari tūponotanga e tika ana hei whakatauira i te nui o te whakahaumako ka tukuna ki ia heketea, i ia tau, i ngā pāmu miraka i Aotearoa.

Me whakapae koe, ko te nui ā-pātahi o te whakahaumako pākawa ota ka tukuna ki ia heketea, i ia tau, ko te 0.16 tana.

Tuhia te ingoa me te/ngā tawhā mō te tuari tūponotanga i kōwhiria ai e koe, Ā, parahautia te tuaritanga me te/ngā tawhā i kōwhiria ai e koe hei wāhanga mō tō whakautu.

Te ingoa me ngā tawhā o te tuaritanga: _____

_____

Te parahautanga: _____

_____

_____

_____

_____

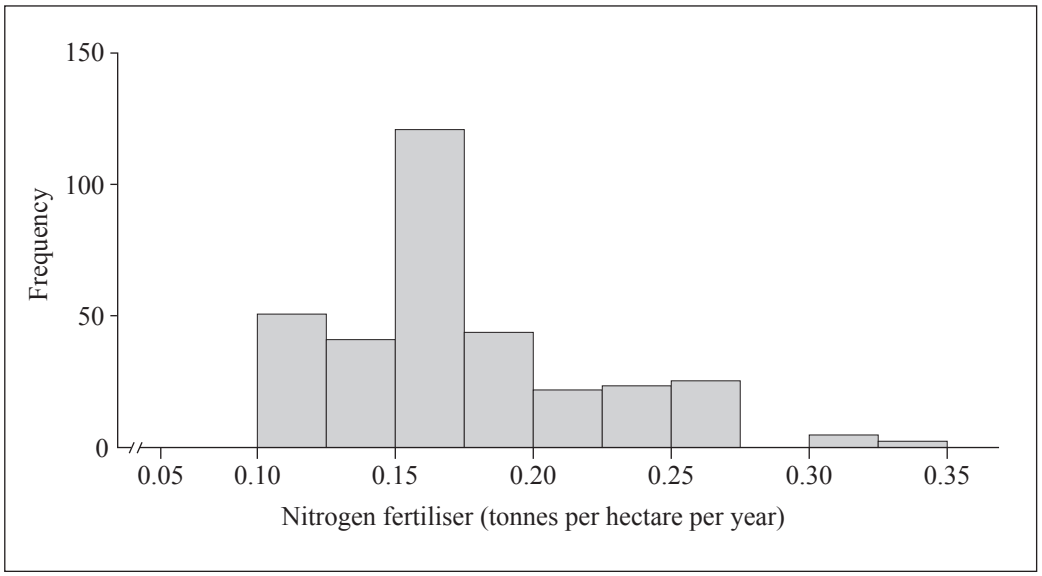
_____

_____

_____

(b) The following figure shows the distribution of the amount of nitrogen fertiliser (tonnes) applied per hectare per year on 332 New Zealand dairy farms.

Figure 2: Nitrogen fertiliser (tonnes) applied per hectare per year on New Zealand dairy farms



Source (adapted): Doole, G. J., Marsh, D., and Ramilan, T. (2013). Evaluation of agri-environmental policies for reducing nitrate pollution from New Zealand dairy farms accounting for firm heterogeneity. *Land Use Policy*, 30 (1), 57–66. 10.1016/j.landusepol.2012.02.007.

(i) Using the data in Figure 2, suggest a suitable probability distribution for modelling the amount of fertiliser used per hectare per year on New Zealand dairy farms.

Assume that the most common amount of nitrate fertiliser applied is 0.16 tonnes per hectare per year.

State the name and parameter(s) for your chosen probability distribution AND justify your choice of distribution and parameter(s) as part of your answer.

Name and parameters of distribution: _____

Justification: _____

(ii) Tekau ngā pāmu miraka i Aotearoa ka kōwhiria matapōkeretia.

Whakamahia te tuari tūponotanga i marohitia rā e koe i te wāhanga (i) hei whakatau tata i te tūponotanga ka neke atu i te haurua o ngā pāmu tekau nei ka iti iho i te 0.16 tana te tukunga o te whakahaumako hauota i ia heketea, i ia tau.

(ii) Ten New Zealand dairy farms are chosen at random.

Use the probability distribution you proposed in part (i) to estimate the probability that more than half of these 10 farms applied less than 0.16 tonnes of nitrogen fertiliser per hectare per year.

TE TŪMAHI TUATORU

https://agfundernews.com/new-zealand-dairy-appeals-to-pension-funds-as-firm-launches-second-fund-despite-milk-price-lows-exclusive010

(a) Ka rerekē te tapeke o te miraka ka puta i tēnā, i tēnā kau i te kaupeka miraka. Mō ngā kau miraka i Aotearoa, me kī, ka taea te whakatauira te tapeke ā-kirokaramu o te miraka o ia kau, i ia kaupeka, mā tētahi tuari māori, ā, ka 4370 kirokaramu te toharite, ā, ka 1350 kirokaramu te ine mahora.

(i) E mōhiotia ana i tōna 30% o ngā kau miraka i **Te Waipounamu** ka iti iho i te 4290 ngā kirokaramu miraka ka puta i ia kau, i ia kaupeka.

Matapakina mēnā rānei e tika ana te āhua o te tauira tuari māori i whakaahuatia rā i runga ake, hei whakatauira i te nui o te miraka ka puta i ia kau miraka nō Te Waipounamu, i ia kaupeka.

Taunakitia tō tuhinga ki ngā tātainga ā-tauanga.

QUESTION THREE

https://agfundernews.com/new-zealand-dairy-appeals-to-pension-funds-as-firm-launches-second-fund-despite-milk-price-lows-exclusive010

(a) Individual cows produce different total amounts of milk over the milking season. For New Zealand dairy cows, suppose the total kilograms of milk per season per cow can be modelled by a normal distribution, with a mean of 4370 kg and standard deviation of 1350 kg.

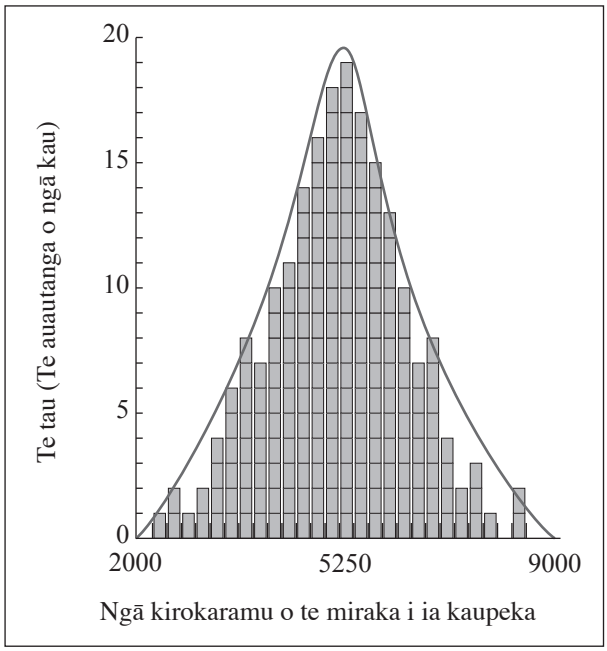
(i) It is known that for **South Island** dairy cows, approximately 30% of the cows produce less than 4290 kg of milk per season per cow.

Discuss whether the normal distribution model described above appears to be appropriate for modelling milk production per season per cow for South Island dairy cows.

Support your answer with statistical calculations.

(ii) Ka whakaemihia ngā raraunga putanga miraka i tētahi kāhui kau miraka e 200 (Te Ata 3 kei raro nei).

Te Ata 3: Ngā kirokaramu o te miraka i ia kaupeka, i ia kau, i te kāhui kau miraka e 200



Te mātāpuna (kua whakahāngaitia):/ /bizplan-uz.com/learning/course/?COURSE_ID=6&LESSON_ID=439&LESSON_PATH=8.436.438.439

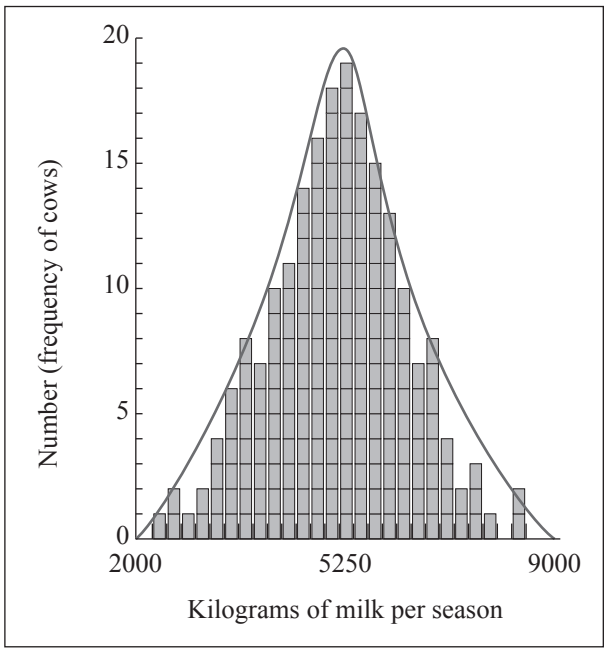
I tēnei kāhui, e iti iho ana i te 3600 kirokaramu te nui o te miraka ka puta i te 10% o ngā kau miraka, i ia kaupeka.

Tautapahia ētahi uara e tika ana mō ngā tawhā o tētahi tauira tuari māori, e taea ai te whakamahi hei whakatauira i ngā putanga miraka i ia kaupeka, mō tēnei kāhui tonu.

Taunakitia tō tuhinga ki ngā tātainga me ngā whakaaro ā-tauanga.

(ii) Milk production data is collected from a herd of 200 dairy cows (Figure 3 below).

Figure 3: Kilograms of milk per season per cow for 200 dairy cows



Source (adapted):/ /bizplan-uz.com/learning/course/?COURSE_ID=6&LESSON_ID=439&LESSON_PATH=8.436.438.439

For this herd, 10% of dairy cows have milk production of less than 3600 kg per season.

Suggest suitable values of the parameters of a normal distribution model that could be used for modelling milk production per season for this particular herd.

Support your answer with statistical calculations and reasoning.

(iii) Whakatauritea ngā tauira tuari māori mō te tapeke o ngā kirokaramu miraka i ia kau, i ia kaupeka, ka puta i ngā kau e 200 i te wāhanga (ii), ki te tapeke o ngā kirokaramu miraka i ia kau miraka, i ia kaupeka, ka puta i ngā kau miraka katoa o Aotearoa.

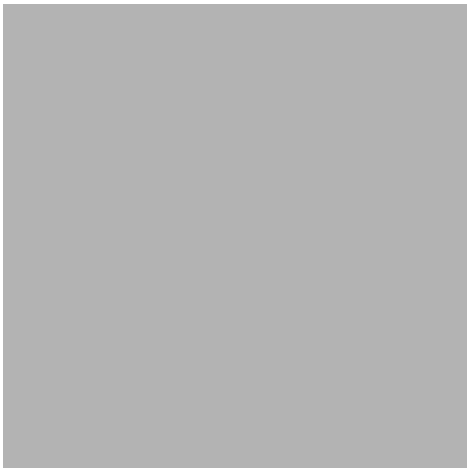
Tuhia kia RUA, kia neke atu rānei, ngā whakaaro, ā, tautokona tō matapakinga ki ngā tātainga me ngā huahuanga i ngā wāhi e tika ana.

(iii) Compare the normal distribution models for the total kilograms of milk per season per cow produced by the 200 cows in part (ii), and the total kilograms of milk per season per cow produced by all New Zealand dairy cows.

Make at least TWO comments and support your discussion with calculations and sketches where appropriate.

(b) E whakaaturia ana i te tūtohi o raro nei te tuari tūponotanga o te taurangi matapōkere N, arā, te nui o ngā punua koti ka whānau mai i ia hapūtanga mō tēnei momo nanenane miraka.

n	0	1	2	3	4
$P(N = n)$	0.1	0.12	0.75	0.02	0.01



https://www.nzdgba.co.nz/

(i) Tātaia te toharite me te ine mahora mō te nui o ngā punua koti i whānau mai i ia hapūtanga o tēnei momo nanenane.

Te toharite: _____

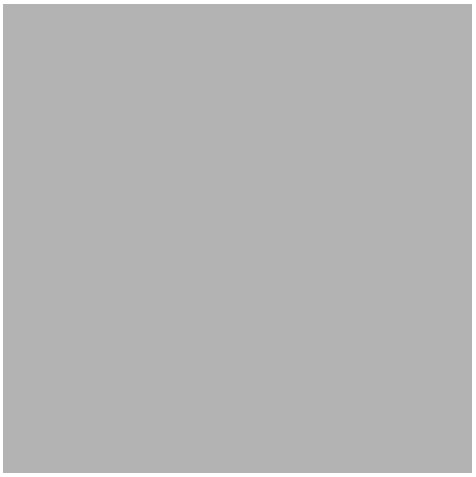
Te ine mahora: _____

(ii) He pai ake ki ngā kaipāmu nanenane miraka ngā momo nanenane miraka ka rua te toharite o ngā punua koti ka puta i a ia i ia hapūtanga.

Tuhia mēnā rānei e tohu ana ēnei raraunga i te pai o tēnei momo nanenane miraka hei kōwhiringa mā ngā kaipāmu nanenane miraka, kāore rānei.

(b) The table below shows the probability distribution of the random variable N, the number of goat kids (baby goats) born per pregnancy for this type of dairy goat.

n	0	1	2	3	4
$P(N = n)$	0.1	0.12	0.75	0.02	0.01



https://www.nzdgba.co.nz/

(i) Calculate the mean and standard deviation of the number of goat kids born per pregnancy for this breed of goat.

Mean: _____

Standard deviation: _____

(ii) Dairy goat farmers prefer types of dairy goat that produce an average of two goat kids per pregnancy.

Comment on whether or not this data suggests that this type of dairy goat will be a good choice for dairy goat farmers.

(iii) Me kī, ka pai ake ana te taioranga o ngā nanenane, puta noa i te tau, ka rerekē te hautau o ngā hapūtanga e puta ai ngā punua koti e rua, ka neke atu rānei i tēnā.

E whakaaturia ana i te tūtohi o raro nei te tuari tūponotanga o te taurangi matapōkere N, arā, te nui o ngā punua koti ka whānau i ia hapūtanga mō tēnei momo nanenane miraka, mēnā e pai ake ana te taioranga o ngā nanenane, puta noa i te tau.

n	0	1	2	3	4
$P(N = n)$	0.03	0.07	0.53	0.25	0.12

Whakamahia ngā whakaaaro ā-tauanga hei whakamārama i te pānga o te rerekētanga o tēnei tūponotanga ka pā pea ki te uara e matapaetia ana, me te ine mahora o te nui o ngā punua koti i ia hapūtanga o ngā nanenane miraka, ā, kaua e āpitihia he tātainga anō.

Te pānga ki te uara e matapaetia ana: _____

Te pānga ki te ine mahora: _____

(iii) Suppose that, with improved nutrition of goats throughout the year, the proportion of pregnancies that result in two or more goat kids changes.

The table below shows the probability distribution of the random variable N, the number of goat kids born per pregnancy for this type of dairy goat with improved nutrition of goats throughout the year.

n	0	1	2	3	4
$P(N = n)$	0.03	0.07	0.53	0.25	0.12

Without further calculations, use statistical reasoning to explain what effect this change in probabilities would have on the expected value and standard deviation of the number of goat kids per pregnancy for the dairy goats.

Effect on expected value:

Effect on standard deviation:

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER	

English translation of the wording on the front cover

Level 3 Mathematics and Statistics (Statistics) 2023

91586M Apply probability distributions in solving problems

Credits: Four

91586M

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE IN THIS AREA DO NOT WRITE). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.