SUPERVISOR'S USE ONLY

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Draw a cross through the box (☒) if you have NOT written in this booklet

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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 2 Mathematics and Statistics 2024

91262 Apply calculus methods in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) A function f is given by $f(x) = 4x^3 - 6x^2 + 5x + 2$.

Use calculus to find the gradient of the graph of the function at the point where $x = 2$.

(b) For the function f:

$f'(x) = 12 - 4x + 6x^2$

The graph of $f(x)$ passes through the point (2,36).

Find the equation of the function f.

(c) A curve is given by $y = 3x^3 - 9x^2 - 27x + 4$.

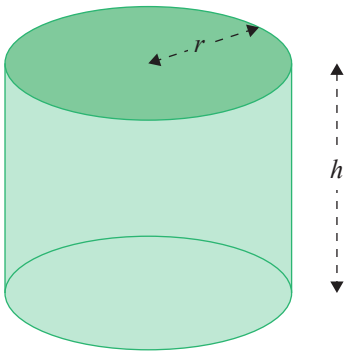
Using calculus methods:

(i) Find the x-coordinate of the local minimum.

(ii) Explain how you know this is a minimum point.

(d) A drink manufacturer would like to start a new line of cylindrical cups that are designed to keep hot liquids warm for as long as possible. To do this, they wish to minimise the surface area of the new cup, which includes a lid.

They would like the cup to hold a volume of 500 ml, and be cylindrical in shape.



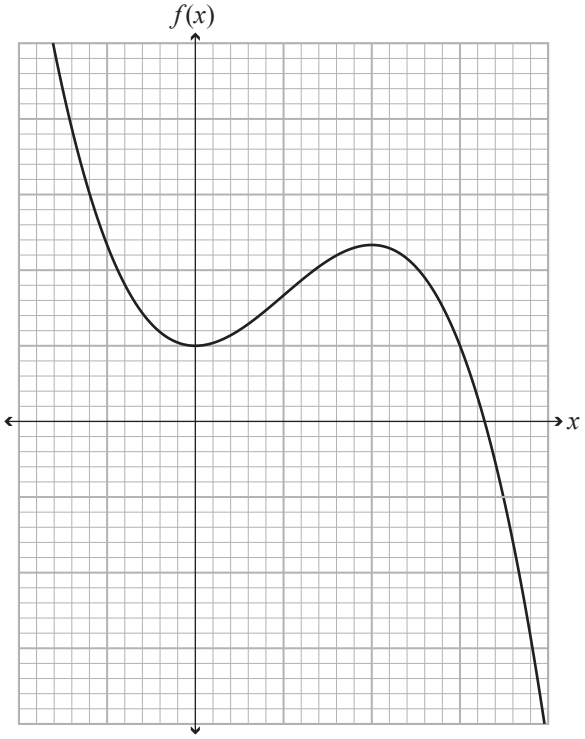
Calculate the dimensions of the cylinder that would satisfy the above conditions.

Volume of a cylinder: $V = \pi r^2 h$

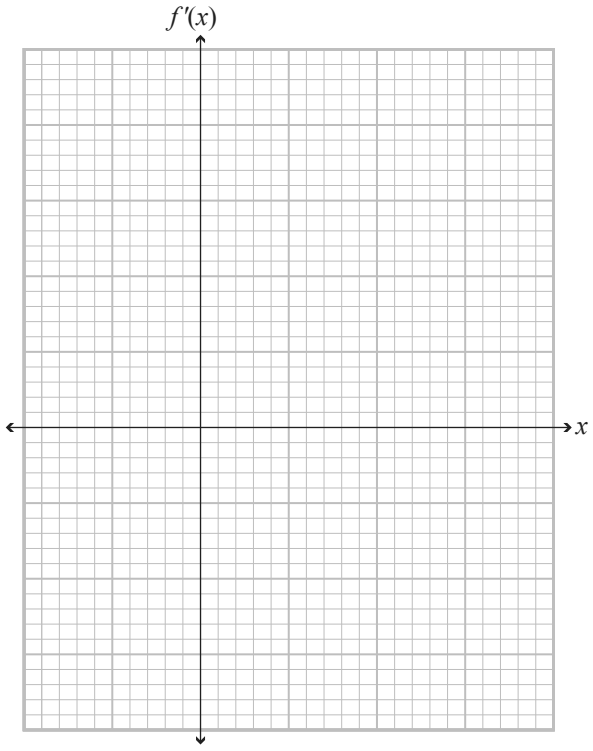
Surface area of a cylinder: $SA = 2\pi r^2 + 2\pi rh$

QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



If you need to redraw your graph, use the grid on page 11.

(b) Find the equation of the tangent to the graph of the function $f(x) = 3 - 12x + 3x^3$ at the point $(-2,3)$.

(c) A microbiologist is growing bacteria in a Petri dish in a lab, and monitoring the population of bacteria in the dish.

After a certain number of days the bacteria colony will have used up all the nutrients in the Petri dish. At this time, the population will be at a maximum and it will then begin to decrease. When the microbiologist adds more nutrients to the Petri dish, the population of bacteria will begin to increase again.

The microbiologist thinks the population could be modelled by this equation:

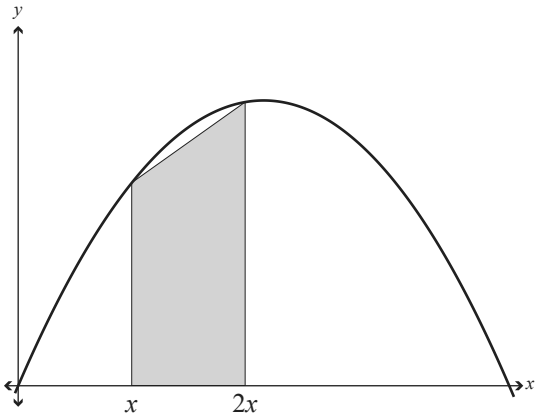
$$P(t) = t^3 - 60t^2 + 768t + 40\,960$$

where P is the population of the bacteria, and t is the number of days since the sample was prepared.

(i) What is the rate at which the population is decreasing on day 10?

(ii) Use calculus methods to determine the **difference** in bacteria population size from the time all the nutrients in the Petri dish had been used up, to the time the microbiologist added more nutrients.

(d) A trapezium is drawn within a parabola given by $y = 8x - x^2$ and the x-axis, such that the two parallel sides are positioned at x and $2x$, as shown in the graph below, where $0 \leq x \leq 4$.



Given that the area of a trapezium is given by $A = \frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides, and h is the perpendicular distance between them, use calculus to find the maximum area of the trapezium.

QUESTION THREE

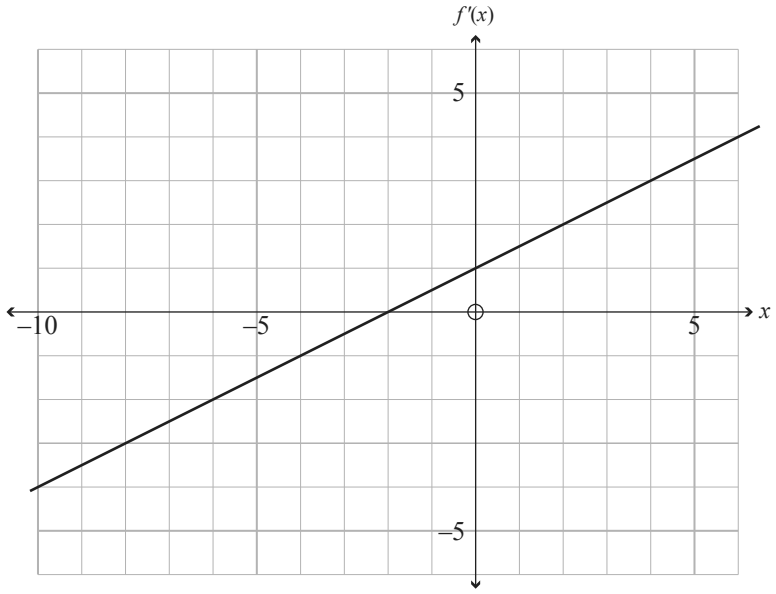
(a) A function is given by $h(x) = 0.25x^2 - 2x + 4$.

Find the coordinate(s) of the point on the graph of this function where the gradient is equal to 3.

(b) A truck is waiting at a red traffic light. When the traffic light turns green, the truck begins to accelerate at 0.64 m s^{-2} until it reaches its top speed of 25 m s^{-1}.

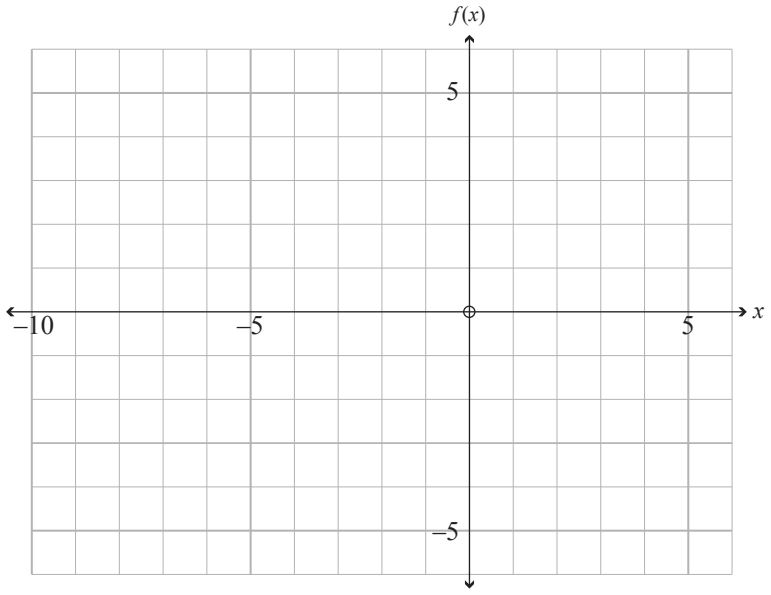
Use calculus to find the distance the truck travelled until it reached its top speed.

(c) Shown on the graph below is a gradient function $f'(x)$:



(i) Find the equation for $f(x)$ which passes through (1,0).

(ii) Draw the graph of $f(x)$ on the axis below, labelling the coordinates of any intercept or stationary point(s).



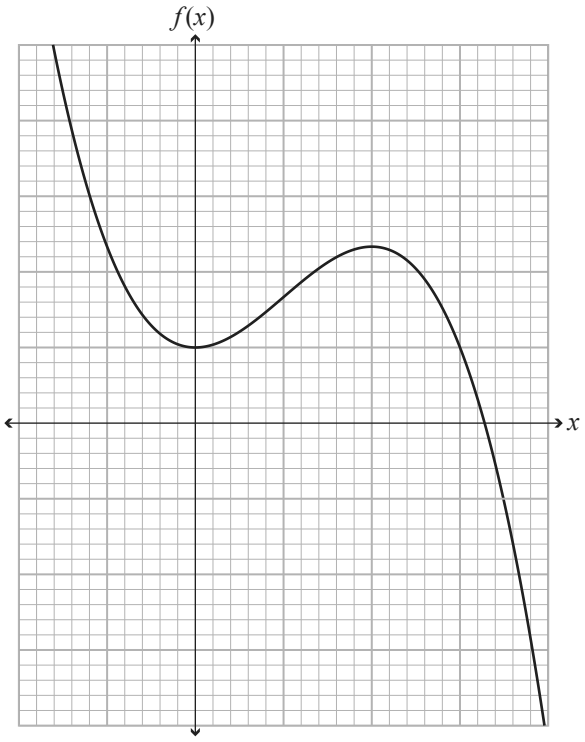
If you need to redraw your graph, use the grid on page 12.

Question Three continues on the following page.

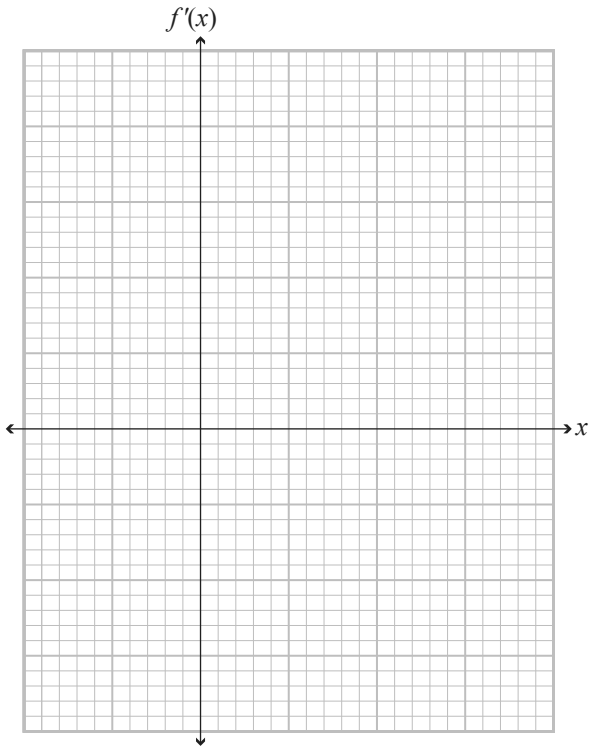
(d) If $y = k - x$, where k is a number, use calculus to show that the minimum value of $x^2 + 2y^2$ is equal to $\frac{2k^2}{3}$.

SPARE DIAGRAMS

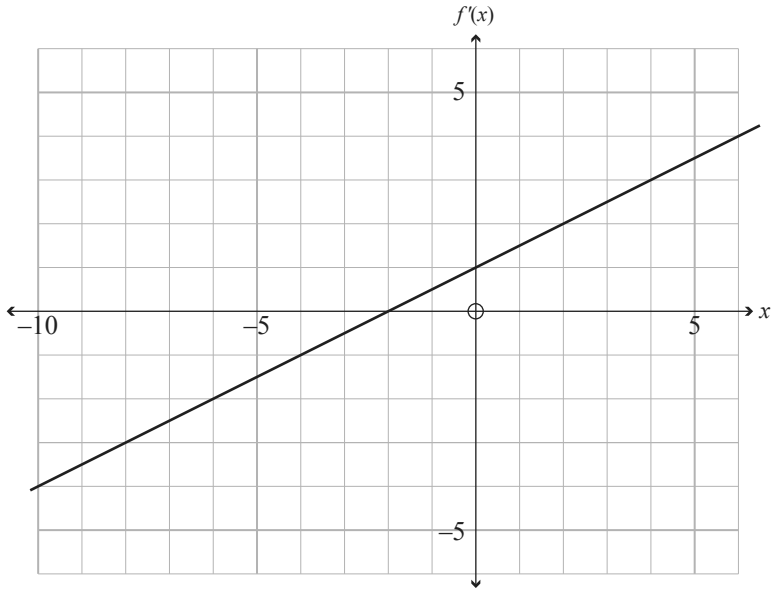
If you need to redraw your response to Question Two (a), use the lower grid. Make sure it is clear which answer you want marked.



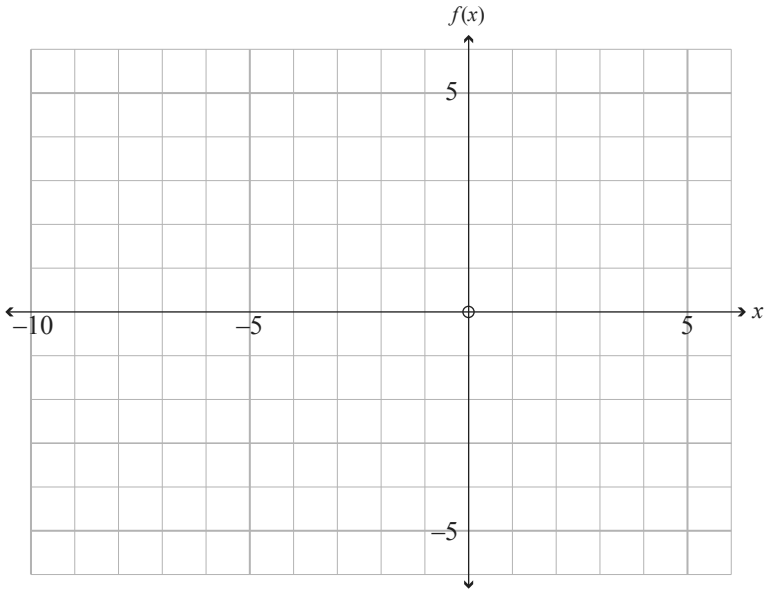
Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



If you need to redraw your response to Question Three (c)(ii), use the lower grid. Make sure it is clear which answer you want marked.



Draw the graph of $f(x)$ on the axis below, labelling the coordinates of any intercept or stationary point(s).



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Write the question number(s) if applicable.

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