SUPERVISOR'S USE ONLY

See back cover for an English translation of this cover.

2

91262M

912625

Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Pāngarau me te Tauanga, Kaupae 2, 2024

91262M Te whakamahi tikanga tuanaki i te whakaoti rapanga

Ngā whiwhinga: E rima

Paetae	Kaiaka	Kairangi
Te whakamahi tikanga tuanaki i te whakaoti rapanga.	Te whakamahi tikanga tuanaki i te whakaoti rapanga, mā te whakaaro ā-pānga.	Te whakamahi tikanga tuanaki i te whakaoti rapanga, mā te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Whakaaturia ngā whiriwhiringa KATOA.

Tirohia kia kitea ai kei a koe te Pepa Ture Tātai L2–MATHMF.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapa o ngā whārangi 2–27, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai ( ). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Ko te pānga, ko f, e whakaataria ana ki te whārite o $f(x) = 4x^3 - 6x^2 + 5x + 2$.

Whakamahia te tuanaki hei whiriwhiri i te rōnaki o te kauwhata o te pānga i te pūwāhi o $x = 2$.

(b) Mō te pānga, mō f:

$$f'(x) = 12 - 4x + 6x^2$$

Ka whakawhiti te kauwhata o $f(x)$ mā te pūwāhi o (2,36).

Whiriwhiria te whārite o te pānga, o f.

QUESTION ONE

(a) A function f is given by $f(x) = 4x^3 - 6x^2 + 5x + 2$.

Use calculus to find the gradient of the graph of the function at the point where $x = 2$.

(b) For the function f:

$$f'(x) = 12 - 4x + 6x^2$$

The graph of $f(x)$ passes through the point (2,36).

Find the equation of the function f.

(c) E whakaataria ana tētahi kōpiko ki te whārite o $y = 3x^3 - 9x^2 - 27x + 4$.

Mā te whakamahi tikanga tuanaki:

(i) Whiriwhiria te taunga-x o te whāruarua.

(ii) Whakamāramatia te take i mōhio ai koe he pūwāhi whāruarua tēnei.

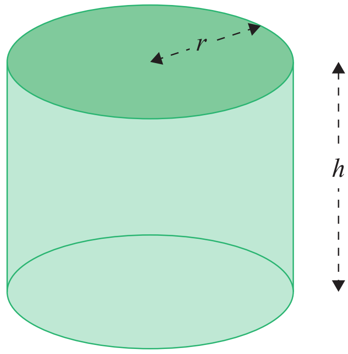
(c) A curve is given by $y = 3x^3 - 9x^2 - 27x + 4$.

Using calculus methods:

(i) Find the x-coordinate of the local minimum.

(ii) Explain how you know this is a minimum point.

(d) E hiahia ana tētahi kamupene whakanao inu ki te waihanga i tētahi momo kapu rango e hou ana kua hoahoatia kia mahana tonu ngā wē wera mō te wā roa katoa e taea ana. E pēnei ai, e hiahia ana te kamupene ki te whakaheke i te horahanga mata o te kapu hou, ā, he taupoki tō te kapu.



E hiahiatia ana kia 500 ml te rōrahi o te kapu, ā, kia rango te āhua.

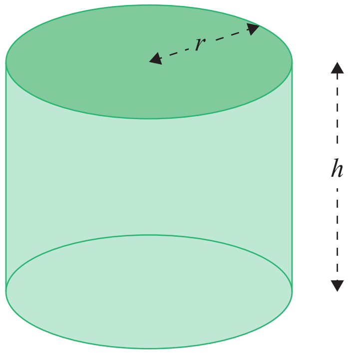
Tātaihia te roa o ngā tapa o te rango e ea ai ngā paearu o runga ake nei.

Te rōrahi o tētahi rango: $V = \pi r^2 h$

Te horahanga mata o tētahi rango: $SA = 2\pi r^2 + 2\pi rh$

(d) A drink manufacturer would like to start a new line of cylindrical cups that are designed to keep hot liquids warm for as long as possible. To do this, they wish to minimise the surface area of the new cup, which includes a lid.

They would like the cup to hold a volume of 500 ml, and be cylindrical in shape.



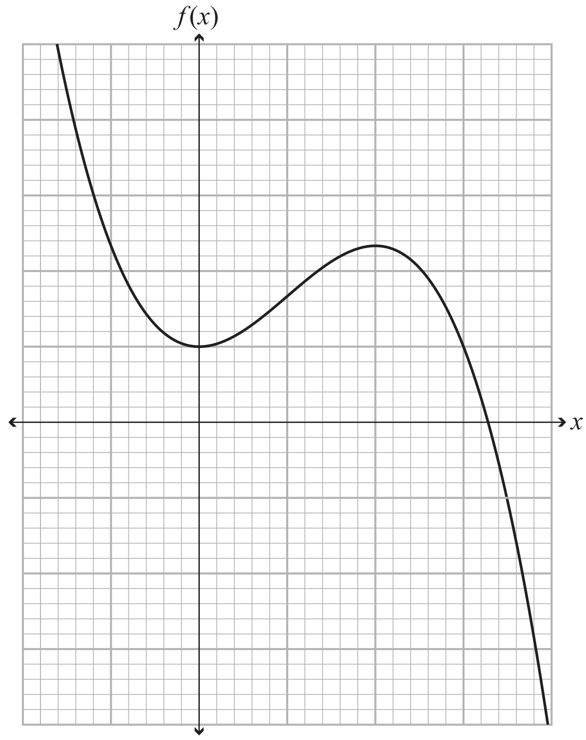
Calculate the dimensions of the cylinder that would satisfy the above conditions.

Volume of a cylinder: $V = \pi r^2 h$

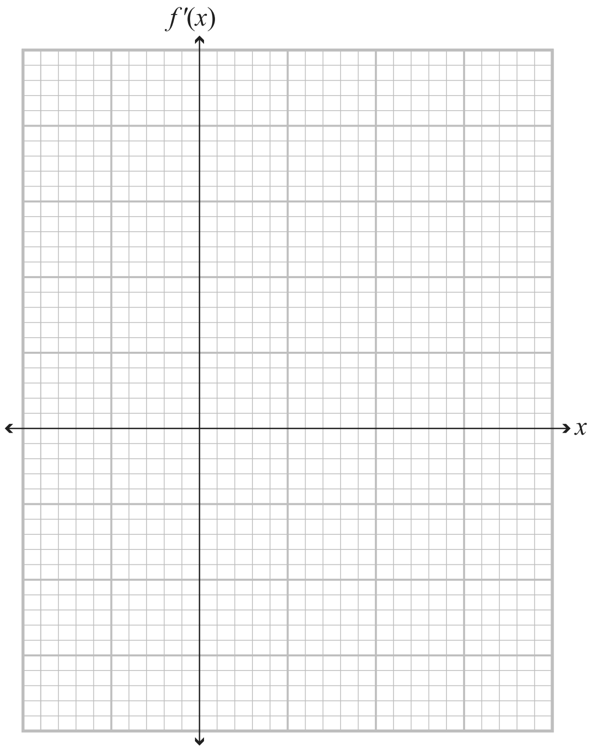
Surface area of a cylinder: $SA = 2\pi r^2 + 2\pi rh$

TE TŪMAHI TUARUA

(a) E whakaaturia ana i ngā tuaka i raro nei te kauwhata o tētahi pānga, o $y = f(x)$.



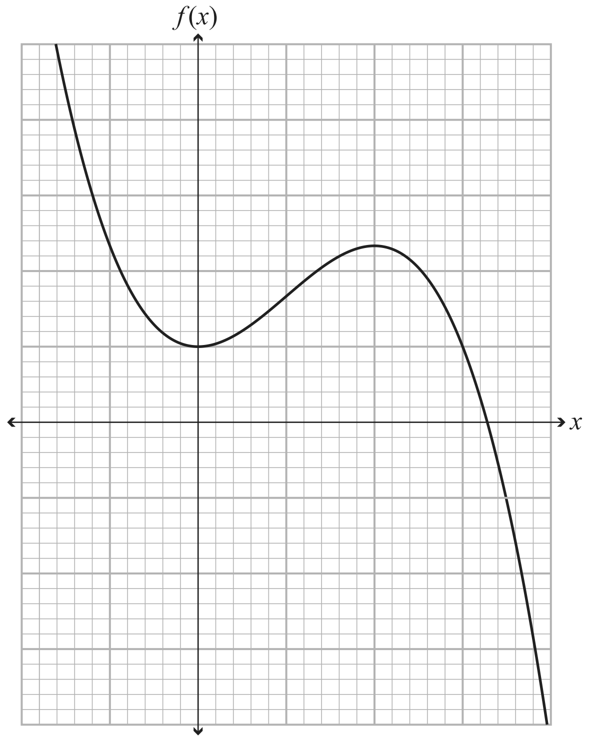
Huahuatia te kauwhata o te pānga rōnaki, o $y = f'(x)$, i ngā tuaka i raro nei.



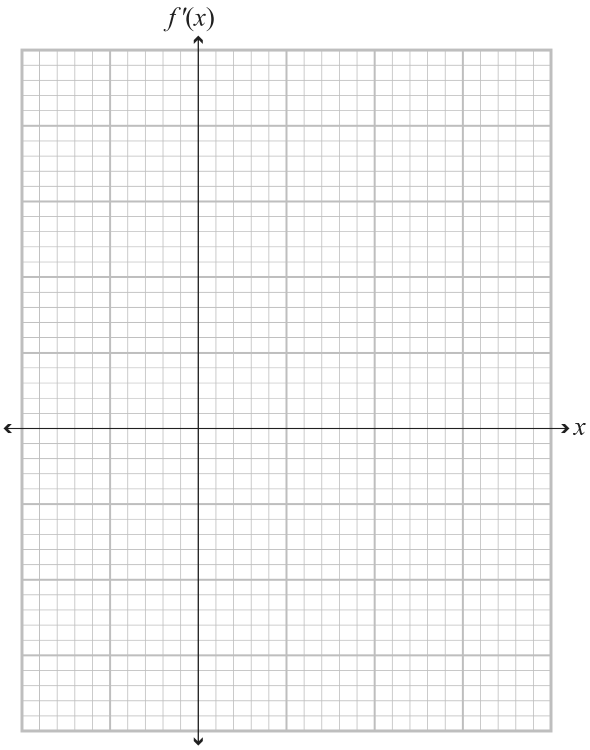
Ki te hiahia koe ki te tā anō i tō kauwhata, whakamahia te tukutuku kei te whārangi 20.

QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



If you need to redraw your graph, use the grid on page 21.

(b) Whiriwhiria te whārite o te pātapa ki te kauwhata o te pānga, o $f(x) = 3 - 12x + 3x^3$ i te pūwāhi o $(-2,3)$.

(c) E whakatipu ana tētahi kaimātai koiora moroiti i te huakita i tētahi pae porowhita i tētahi taiwhanga, ā, e aroturukitia ana te huinga o te huakita i te porowhita.

Ā muri i ētahi rangi, ka pau katoa i te huinga huakita te katoa o ngā taiora i te pae porowhita. Ka pēnei ana, ka eke te huinga ki tōna nui katoa, ā, ka tīmata te hekenga iho. Ka āpiti ana te kaimātai koiora moroiti i ētahi anō taiora ki te pae porowhita, ka tipu haere anō te huinga o te huakita.

Ka whakaaro te kaimātai koiora moroiti ka taea te huinga te whakatauira ki tēnei whārite:

$$P(t) = t^3 - 60t^2 + 768t + 40\,960$$

arā, ko P te huinga o te huakita, ā, ko t te nui o ngā rā mai i te whakaritenga o te tauira.

(i) He aha te pāpātanga o te hekenga iho o te huinga i te rā tuangahuru?

(ii) Whakamahia ngā tikanga tuanaki hei whakatau i te **rerekētanga** o te nui o te huinga huakita i te wā i pau katoa rā ngā taiora i te pae porowhita, ki te wā i āpiti anō ai te kaimātai koiora moroiti i ētahi anō taiora.

(b) Find the equation of the tangent to the graph of the function $f(x) = 3 - 12x + 3x^3$ at the point $(-2,3)$.

(c) A microbiologist is growing bacteria in a Petri dish in a lab, and monitoring the population of bacteria in the dish.

After a certain number of days the bacteria colony will have used up all the nutrients in the Petri dish. At this time, the population will be at a maximum and it will then begin to decrease. When the microbiologist adds more nutrients to the Petri dish, the population of bacteria will begin to increase again.

The microbiologist thinks the population could be modelled by this equation:

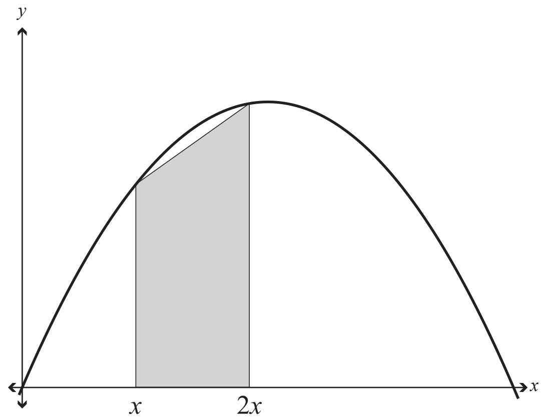
$$P(t) = t^3 - 60t^2 + 768t + 40\,960$$

where P is the population of the bacteria, and t is the number of days since the sample was prepared.

(i) What is the rate at which the population is decreasing on day 10?

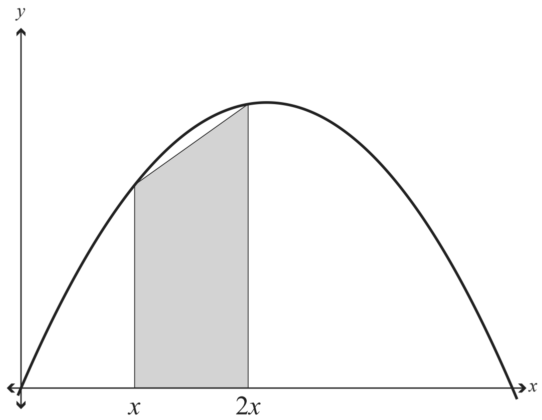
(ii) Use calculus methods to determine the **difference** in bacteria population size from the time all the nutrients in the Petri dish had been used up, to the time the microbiologist added more nutrients.

(d) Kua tāngia tētahi taparara i roto i tētahi unahi e whakaataria ana ki te whārite o $y = 8x - x^2$ me te tuaka-x, e taunga ai ngā tapa whakarara ki x me $2x$, e whakaaturia nei i te kauwhata i raro nei, ā, ko $0 \leq x \leq 4$.



Mēnā e whakaataria ana te horahanga o tētahi taparara ki te whārite o $A = \frac{1}{2}(a+b)h$, arā, ko a me b ngā roa o ngā tapa whakarara, ā, ko h te tawhiti hāngai i waenga i ngā tapa, whakamahia te tuanaki hei whiriwhiri i te horahanga mōrahi o te taparara.

(d) A trapezium is drawn within a parabola given by $y = 8x - x^2$ and the x-axis, such that the two parallel sides are positioned at x and $2x$, as shown in the graph below, where $0 \leq x \leq 4$.



Given that the area of a trapezium is given by $A = \frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides, and h is the perpendicular distance between them, use calculus to find the maximum area of the trapezium.

TE TŪMAHI TUATORU

(a) E whakaataria ana tētahi pānga ki te whārite o $h(x) = 0.25x^2 - 2x + 4$.

Whiriwhiria te/ngā taunga o te pūwāhi i te kauwhata o tēnei pānga e rite ai te rōnaki ki te 3.

(b) E tatari ana tētahi taraka i tētahi rama ārahi whero. Ka kākāriki ana te rama ārahi, ka 0.64 m s^{-2} te tere haeretanga o te taraka, kia tae rā anō ki te tere mōrahi o te 25 m s^{-1}.

Whakamahia te tuanaki hei whiriwhiri i te tawhiti o te haere a te taraka kia tae rā anō ki tana tere mōrahi.

QUESTION THREE

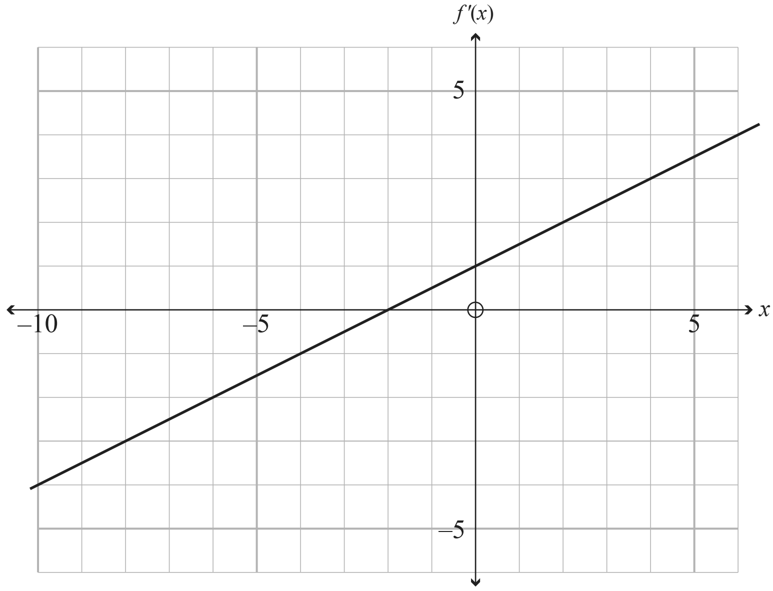
(a) A function is given by $h(x) = 0.25x^2 - 2x + 4$.

Find the coordinate(s) of the point on the graph of this function where the gradient is equal to 3.

(b) A truck is waiting at a red traffic light. When the traffic light turns green, the truck begins to accelerate at 0.64 m s^{-2} until it reaches its top speed of 25 m s^{-1}.

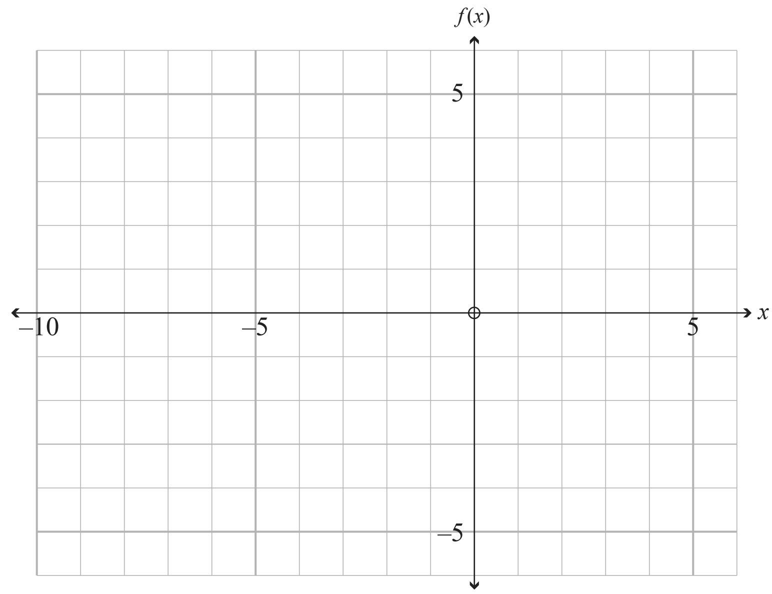
Use calculus to find the distance the truck travelled until it reached its top speed.

(c) E whakaaturia ana i te kauwhata i raro nei tētahi pānga rōnaki, te $f'(x)$:



(i) Whiriwhiria te whārite o $f(x)$ ka whakawhiti i te pūwāhi o (1,0).

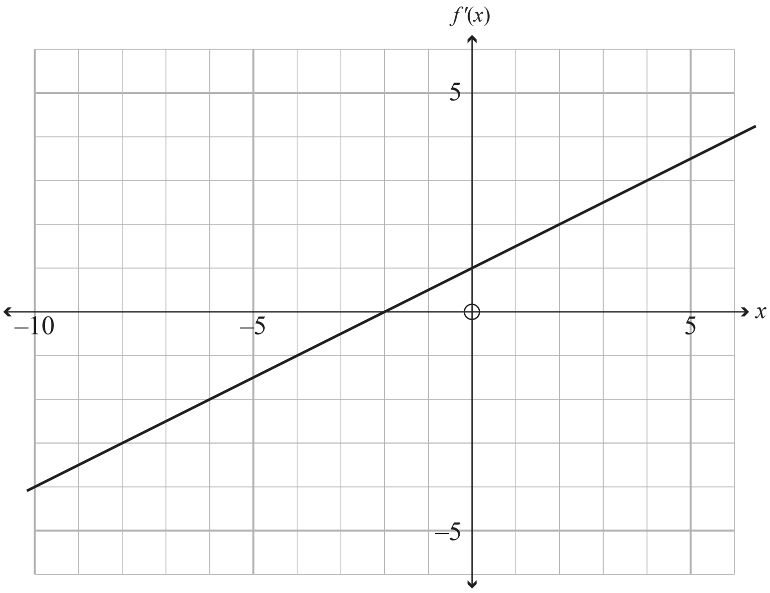
(ii) Tāngia te kauwhata o $f(x)$ i te tuaka i raro nei, me tapa ngā taunga o te/ngā haukoti, o te/ ngā pūwāhi tū rānei.



Ki te hiahia koe ki te tā anō i tō kauwhata, whakamahia te tukutuku kei te whārangi 22.

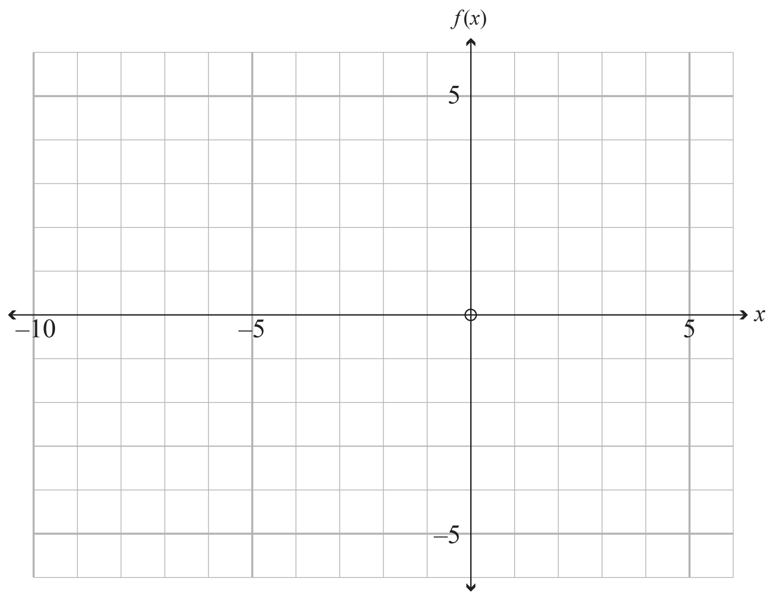
E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(c) Shown on the graph below is a gradient function $f'(x)$:



(i) Find the equation for $f(x)$ which passes through (1,0).

(ii) Draw the graph of $f(x)$ on the axis below, labelling the coordinates of any intercept or stationary point(s).



If you need to redraw your graph, use the grid on page 23.

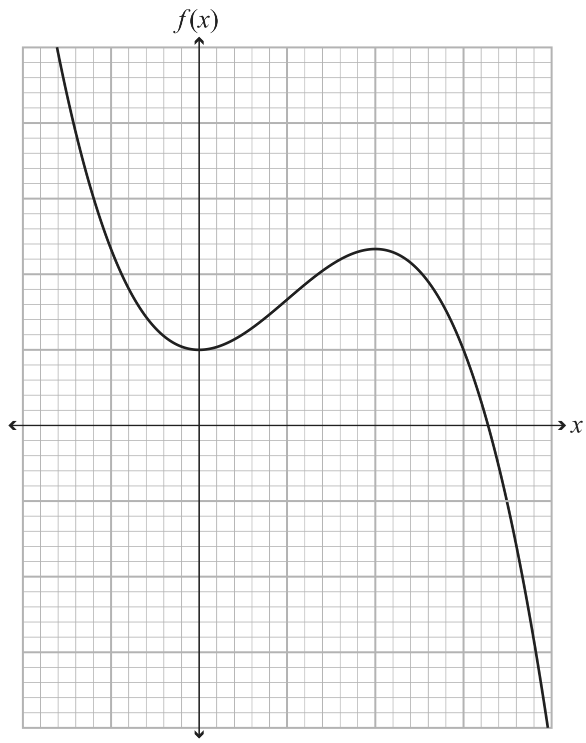
Question Three continues on the following page.

(d) Mēnā ko $y = k - x$, arā, ko k tētahi tau, whakamahia te tuanaki hei whakaatu i te rite o te uara mōkito o $x^2 + 2y^2$ ki te $\frac{2k^2}{3}$.

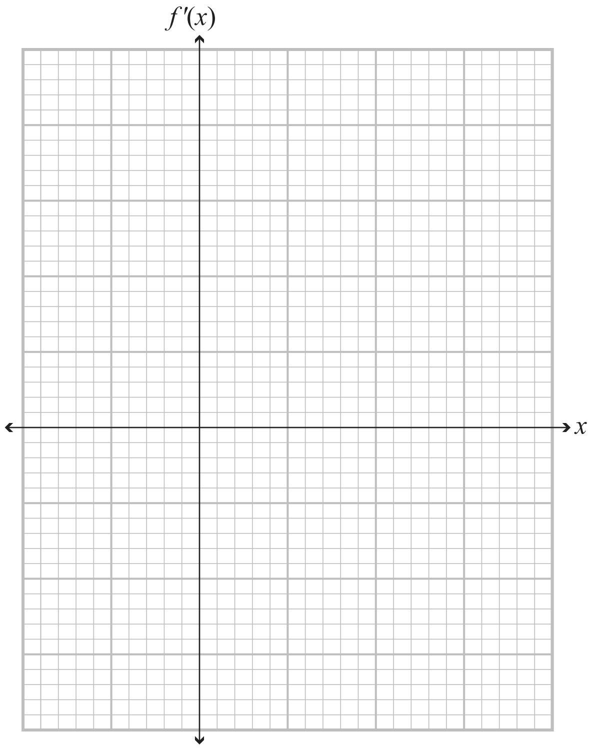
(d) If $y = k - x$, where k is a number, use calculus to show that the minimum value of $x^2 + 2y^2$ is equal to $\frac{2k^2}{3}$.

HE HOAHOA WĀTEA

Ki te hiahia koe ki te tā anō i tō urupare ki te Tūmahi Tuarua (a), whakamahia te tukutuku kei raro nei. Me mārama tō tohu mai i te tuhinga e hiahia ana koe kia mākahia.

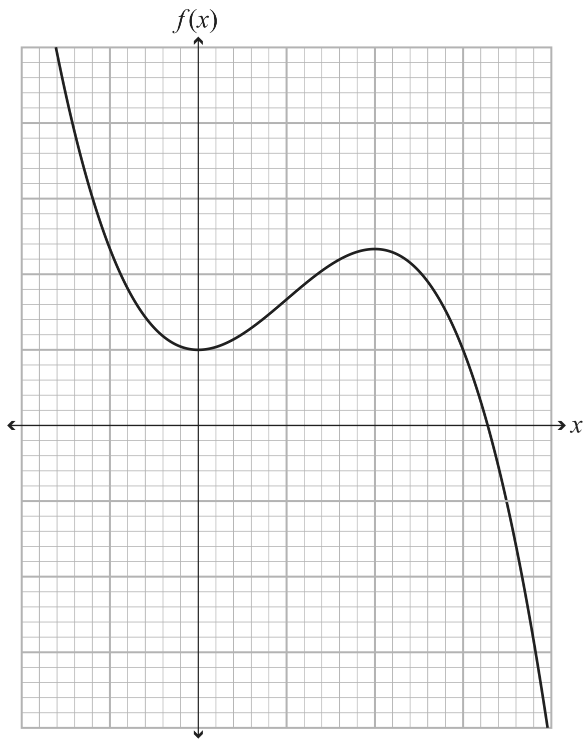


Huahuatia te kauwhata o te pānga rōnaki o $y = f'(x)$ i ngā tuaka i raro nei.

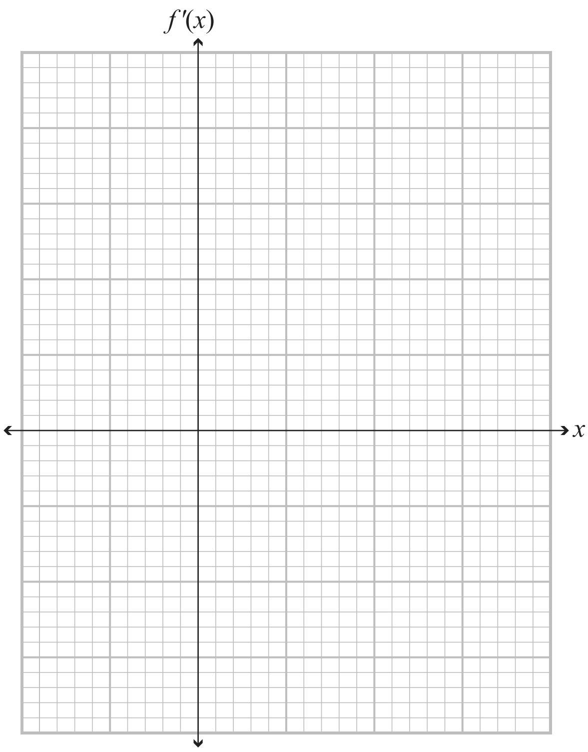


SPARE DIAGRAMS

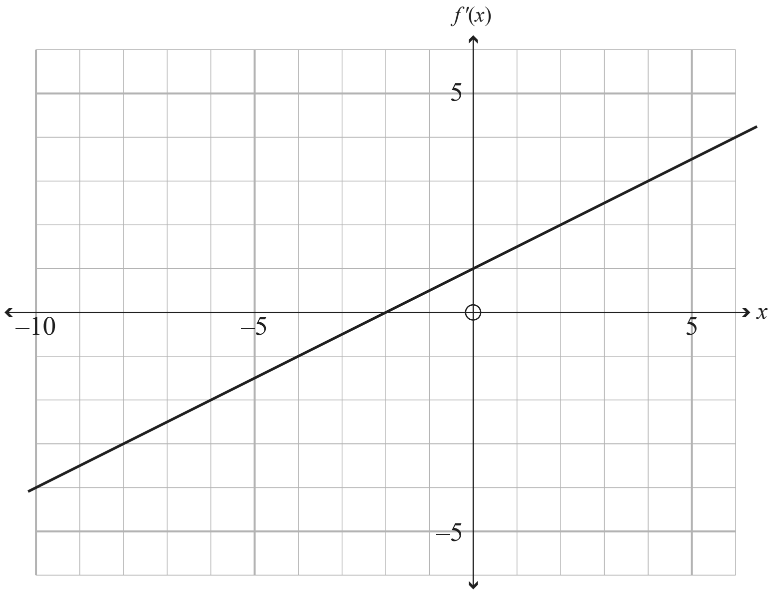
If you need to redraw your response to Question Two (a), use the lower grid. Make sure it is clear which answer you want marked.



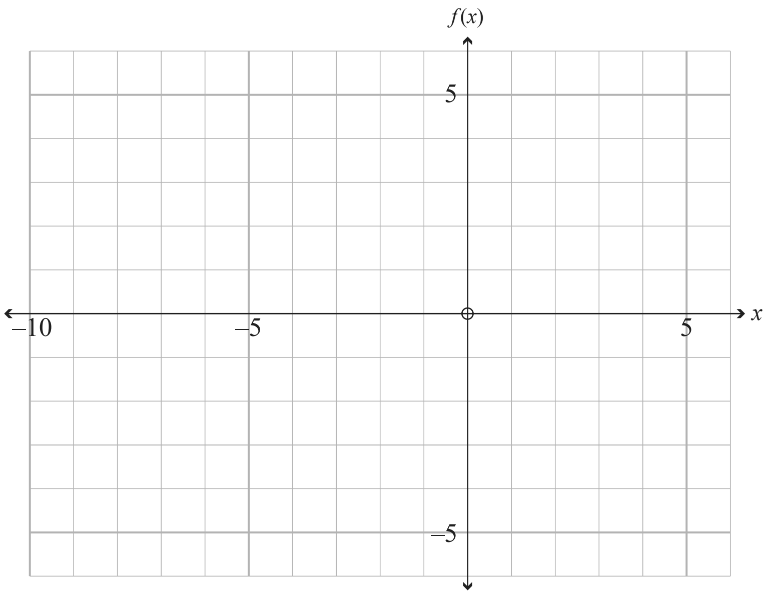
Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



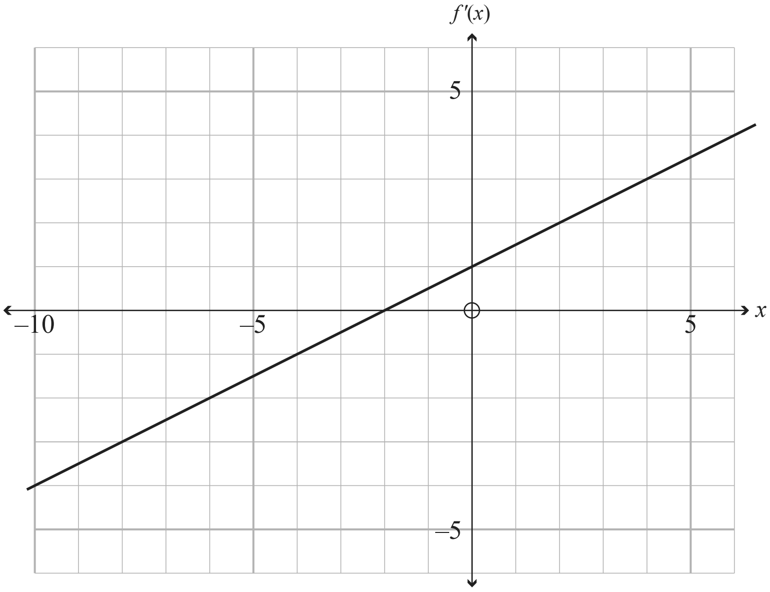
Ki te hiahia koe ki te tā anō i tō urupare ki te Tūmahi Tuatoru (c)(ii), whakamahia te tukutuku kei raro nei. Me mārama tō tohu mai i te tuhinga e hiahia ana koe kia mākahia.



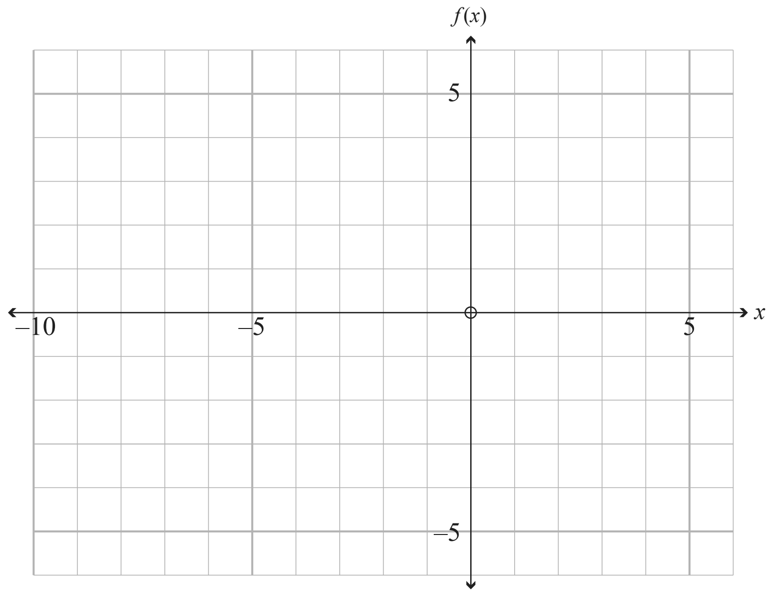
Tāngia te kauwhata o $f(x)$ i te tuaka i raro nei, me tapa ngā taunga o te/ngā haukoti, o te/ngā pūwāhi tū rānei.



If you need to redraw your response to Question Three (c)(ii), use the lower grid. Make sure it is clear which answer you want marked.



Draw the graph of $f(x)$ on the axis below, labelling the coordinates of any intercept or stationary point(s).



He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 2 Mathematics and Statistics 2024

91262M Apply calculus methods in solving problems

Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHMF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area. This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.