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L3–CALCMF

993208

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2024

TE PUKAPUKA TIKANGA TĀTAI ME NGĀ TŪTOHI mō te 91577M, te 91578M me te 91579M

Tirohia tēnei pukapuka hei whakaoti i ngā tūmahi o ngā Pukapuka Tūmahi me ngā Tuhinga.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

E ĀHEI ANA TŌ PUPURI KI TĒNEI PUKAPUKA HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE PĀNGARAU – HE TIKANGA TĀTAI WHAI HUA

TE TAURANGI

Te Whārite Pūrua

Mehemea ko te $ax^2 + bx + c = 0$

ko te $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ngā Pūkōaro

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Ngā tau tuatini

$z = x + \mathrm{i}y$

$= r\ \mathrm{cis}\,\theta$

$= r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$= r\ \mathrm{cis}\,(-\theta)$

$= r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

mehemea ko te $\cos\theta = \dfrac{x}{r}$

me te $\sin\theta = \dfrac{y}{r}$

Te Ture a De Moivre

Mehemea he tau tōpū te n, ko te

$(r\ \mathrm{cis}\,\theta)^n = r^n\ \mathrm{cis}\,(n\theta)$

KO TE ĀHUAHANGA TAUNGA

Te Rārangi Tōtika

Te Whārite $y - y_1 = m(x - x_1)$

TE TUANAKI

Te Pārōnaki

$y = f(x)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\,x$	$-\mathrm{cosec}\,x \cot x$
$\cot x$	$-\mathrm{cosec}^2 x$

Te Pāwhaitua

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

Te Pānga Tawhā

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

MATHEMATICS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex numbers

$z = x + \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}\,(n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\cot x$	$-\mathrm{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\|x\| + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\|f(x)\| + c$

Parametric Function

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\cdot\frac{\mathrm{d}t}{\mathrm{d}x}$$

Te Ture Otinga

$(f.g)' = g.f' + f.g'$ mehemea rānei ko te $y = uv$, ko te $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Te Ture Huawehe

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$ mehemea rānei ko te $y = \frac{u}{v}$, ko te $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Ngā Pānga Hiato, ngā Ture Mekameka rānei

$\left(f(g)\right)' = f'(g).g'$

mehemea rānei ko te, $y = f(u)$ ko te $u = g(x)$, nō reira, ko te $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

NGĀ TIKANGA TAU

Te Ture Trapezium

$$\int_a^b f(x)\ dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + ... + y_{n-1})\right]$$

mehemea ko te $h = \frac{b-a}{n}$, ā, ko te $y_r = f(x_r)$

Te ture a Simpson

$$\int_a^b f(x)\ dx \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$$

mehemea ko te $h = \frac{b-a}{n}$, $y_r = f(x_r)$, ā, he taurua te n.

Product Rule

$(f.g)' = g.f' + f.g'$ or if $y = uv$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = v\dfrac{\mathrm{du}}{\mathrm{d}x} + u\dfrac{\mathrm{d}v}{\mathrm{d}x}$

Quotient Rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Composite Function or Chain Rule

$\left(f\left(g\right)\right)' = f'\left(g\right).g'$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u}\cdot\dfrac{\mathrm{d}u}{\mathrm{d}x}$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

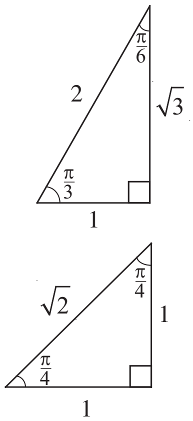
TE PĀKOKI

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$



Te Ture Aho

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Te Ture Whenu

$c^2 = a^2 + b^2 - 2ab\cos C$

Ngā Tuakiri

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Ngā Otinga Whānui

Mehemea ko te $\sin\theta = \sin\alpha$, ko te $\theta = n\pi + (-1)^n\alpha$

Mehemea ko te $\cos\theta = \cos\alpha$, ko te $\theta = 2n\pi \pm \alpha$

Mehemea ko te $\tan\theta = \tan\alpha$, ko te $\theta = n\pi + \alpha$

mehemea he tau tōpū te n

Ngā Koki Pūhui

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Ngā Koki Rearua

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

Ngā Otinga

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Ngā Tapeke

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

TE INENGA

Te Tapatoru

Te horahanga $= \dfrac{1}{2}ab\sin C$

Te Taparara

Te horahanga $= \dfrac{1}{2}(a+b)h$

Te Pewanga

Te horahanga $= \dfrac{1}{2}r^2\theta$

Te roa o te pewa $= r\theta$

Te Rango

Te horahanga $= \pi r^2 h$

Te horahanga mata o te kōpiko $= 2\pi rh$

Te Koeko

Te rōrahi $= \dfrac{1}{3}\pi r^2 h$

Te horahanga mata o te kōpiko $= \pi rl$ mēnā ko l = te teitei ā-taiuru

Te Poi

Te rōrahi $= \dfrac{4}{3}\pi r^3$

Te horahanga mata $= 4\pi r^2$

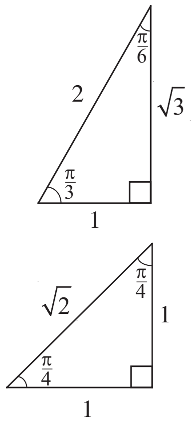
TRIGONOMETRY

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$



Sine Rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule

$c^2 = a^2 + b^2 - 2ab\cos C$

Identities

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Double Angles

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

Products

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Sums

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

MEASUREMENT

Triangle

Area $= \dfrac{1}{2}ab\sin C$

Trapezium

Area $= \dfrac{1}{2}(a+b)h$

Sector

Area $= \dfrac{1}{2}r^2\theta$

Arc length $= r\theta$

Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

Cone

Volume $= \dfrac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$ where $l =$ slant height

Sphere

Volume $= \dfrac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

English translation of the wording on the front cover

Mana Tohu Mātauranga o Aotearoa
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L3–CALCMF

Level 3 Calculus 2024

FORMULAE AND TABLES BOOKLET
for 91577M, 91578M, and 91579M

Refer to this booklet to answer the questions in your Question and Answer Booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.