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91578M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2024

91578M Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā roto i te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Whakaaturia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–31 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (⁄⁄⁄). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria ngā pārōnaki o te $f(x) = \sqrt{\left(4 - 9x^4\right)}$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) E tautuhia ana tētahi kōpiko ki te whārite o te $y = (x^2 + 3x + 2)\sin x$.

Whiriwhiria te rōnaki o te pātapa ki tēnei kōpiko i te wā ko te $x = 0$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) Mō te pānga kei raro nei, whiriwhiria te whānuitanga o ngā uara o te x, mehemea kei te heke haere te pānga.

$y = 3(2x - 7)^2 + 60\ln x + 12,\ x > 0$

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION ONE

(a) Differentiate $f(x) = \sqrt{\left(4 - 9x^4\right)}$.

You do not need to simplify your answer.

(b) A curve is defined by the equation $y = (x^2 + 3x + 2)\sin x$.

Find the gradient of the tangent to this curve when $x = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) For the function below, find the range of values of x for which the function is decreasing.

$y = 3(2x - 7)^2 + 60\ln x + 12, \ x > 0$

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Whiriwhiria te/ngā uara-x o ngā pūwāhi tūnoa i te kauwhata o te pānga kei raro nei, **me whakatau hoki i te āhua o aua pūwāhi**.

$$y = (2x - 1)\,e^{-2x}$$

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) Find the x-value(s) of any stationary points on the graph of the function below, **and determine their nature**.

$$y = (2x - 1)\,e^{-2x}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) E tautuhia ana tētahi kōpiko ki te whārite o te $y = \dfrac{2x^2 - 1 - 2x\ln x}{x}$, arā, ko te $x > 0$.

He pūwāhi tūpā tō te kōpiko i te pūwāhi o te P.

Whiriwhiria te whārite o te pātapa ki te kōpiko i te pūwāhi o te P.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(e) A curve is defined by the equation $y = \dfrac{2x^2 - 1 - 2x\ln x}{x}$, where $x > 0$.

The curve has a point of inflection at the point P.

Find the equation of the tangent to the curve at the point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

TE TŪMAHI TUARUA

(a) E tautuhia tawhātia ana tētahi pānga ki ngā whārite e rua:

Ko te $x = 3t^2 + 1$ me te $y = \cos t$.

Whiriwhiria tētahi kīanga mō te $\frac{dy}{dx}$.

(b) E haere torotika ana tētahi mea. Ko tana peinga, ā-mita nei, e tohua ana ki te ture o te

$s(t) = \ln(3t^2 + 5t + 2)$, arā, ko $t > 0$, ā, ko t te wā, ā-hēkona nei.

Whiriwhiria te tere o tēnei mea mehemea ko te $t = 1$ hēkona.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION TWO

(a) A function is defined parametrically by the pair of equations:

$x = 3t^2 + 1$ and $y = \cos t$.

Find an expression for $\frac{dy}{dx}$.

(b) An object is travelling in a straight line. Its displacement, in metres, is given by the formula

$s(t) = \ln(3t^2 + 5t + 2)$, where $t > 0$ and t is time, in seconds.

Find the velocity of this object when $t = 1$ second.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Whakaaturia ko te $y = \sin(x^2) - \cos(x)$ tētahi otinga ki te whārite o te

$$\frac{d^2y}{dx^2} + 4x^2y = 2\cos\left(x^2\right) + \left(1 - 4x^2\right)\cos x.$$

(c) Show that $y = \sin(x^2) - \cos(x)$ is a solution to the equation

$$\frac{d^2y}{dx^2} + 4x^2y = 2\cos\left(x^2\right) + \left(1 - 4x^2\right)\cos x.$$

(d) Whakaarotia te pānga o te $f(x) = \frac{\ln x}{x}, x > 0$.

Whiriwhiria ngā taunga o te pūwāhi tūpā i te kauwhata o te pānga.

Me whakapono noa koe he pūwāhi tūpā mārika te pūwāhi e kitea ana e koe.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) Consider the function $f(x) = \frac{\ln x}{x}, x > 0.$

Find the coordinates of the point of inflection on the graph of the function.

You can assume that your point found is actually a point of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Ko te kauwhata o te pānga o te $y = \dfrac{xe^{3x}}{2x + k}$, arā, he tau pūmau, ehara i te kore, te k, ā, **kotahi anake te pūwāhi hurihanga** i te **Q**.

Whiriwhiria te taunga-x o te pūwāhi o te Q.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

He wāhi anō mō tō tuhinga mō tēnei tūmahi kei ngā whārangi e whai ake nei.

(e) The graph of the function $y = \dfrac{xe^{3x}}{2x + k}$, where k is a non-zero constant, **has a single turning point** at **Q**.

Find the x-coordinate of the point Q.

You must use calculus and show any derivatives that you need to find when solving this problem.

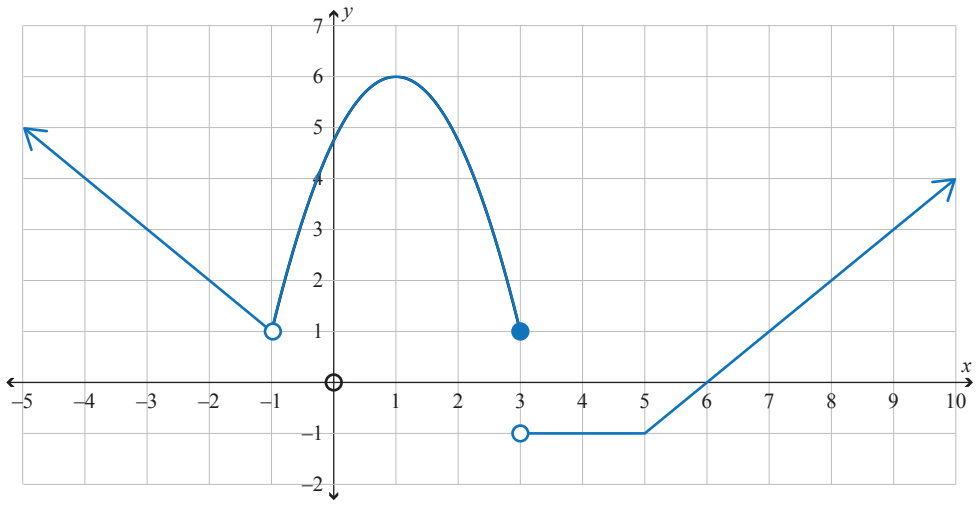
There is more space for your answer to this question on the following pages.

TE TŪMAHI TUATORU

(a) Whiriwhiria ngā pārōnaki o te $y = \sqrt{x}.\sec(6x)$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) E whakaatu ana te kauwhata i raro nei i te pānga o $y = f(x)$.



(i) Mō te pānga kei runga nei, whiriwhiria te/ngā uara o te x mehemea e motukore ana te $f(x)$, engari kāore e taea te whiriwhiri ōna pārōnaki.

(ii) Mō te pānga kei runga nei, whiriwhiria te/ngā uara o te x mehemea ko te $f'(x) = 0$.

(iii) Ko te aha te uara o te $\lim_{x \to -1} f(x)$?

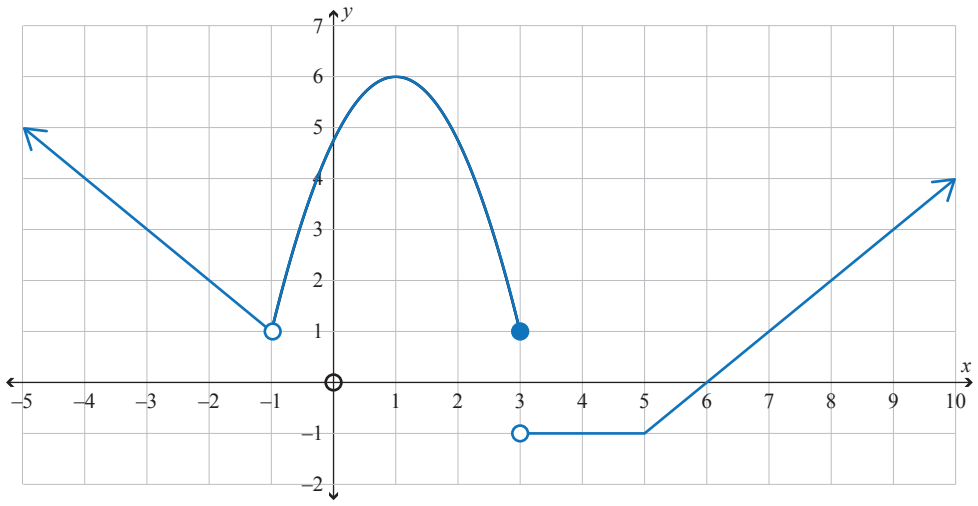
Me āta whakamārama mehemea karekau he uara.

QUESTION THREE

(a) Differentiate $y = \sqrt{x}.\sec(6x)$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



(i) For the function above, find the value(s) of x where $f(x)$ is continuous but not differentiable.

(ii) For the function above, find the value(s) of x where $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to -1} f(x)$?

State clearly if the value does not exist.

(c) Whiriwhiria te/ngā uara-x o te/ngā pūwāhi tūnoa i te kauwhata o te pānga o $f(x)=\dfrac{x^2-5x+4}{x^2+5x+4}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

***Ehara i te mea** me whakatau rawa koe i te āhua o te/ngā pūwāhi tūnoa i whiriwhiria ai.*

(c) Find the x-value(s) of any stationary point(s) on the graph of the function $f(x)=\dfrac{x^2-5x+4}{x^2+5x+4}$.

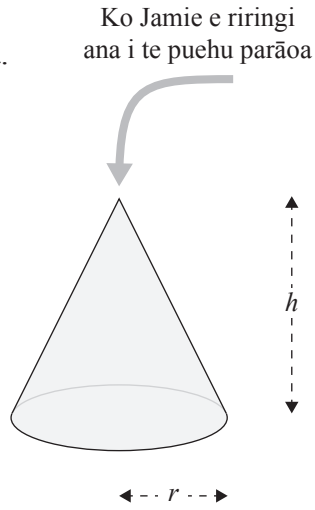
You must use calculus and show any derivatives that you need to find when solving this problem.
*You **do not need** to determine the nature of any stationary point(s) found.*

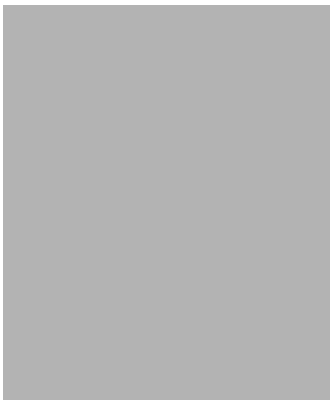
(d) Kei te tunu keke a Jamie, ā, e riringia ana te puehu parāoa kia puke koeko ai te āhua.

I ngā wā katoa, e ōrite ana te teitei o te puke ki te whitianga o te pūtake o te koeko.

Mēnā ka 3 cm^3 te tere pūmau o te pāpātanga o te āpitinga o te puehu parāoa i te hēkona, ko te aha te pāpātanga o te teitei haeretanga ka 4 cm ana te teitei o te puke?

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.





Te mātāpuna: https://www.istockphoto.com/nl/foto/man-gieten-bloem-uit-de-kom-van-de-maatregel-gm825182090-133804287

Kia mōhio noa, ko te rōrahi o tētahi koeko $= \frac{1}{3}\pi r^2 h$.

He wāhi anō mō tō tuhinga mō tēnei tūmahi kei ngā whārangi e whai ake nei.

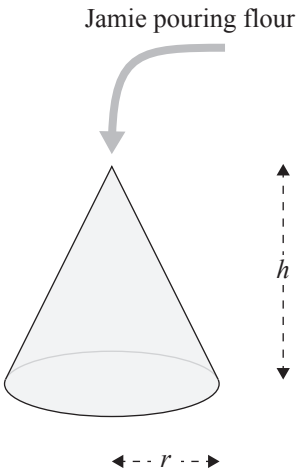
(d) Jamie is doing some baking and pouring the flour to form a conical pile.

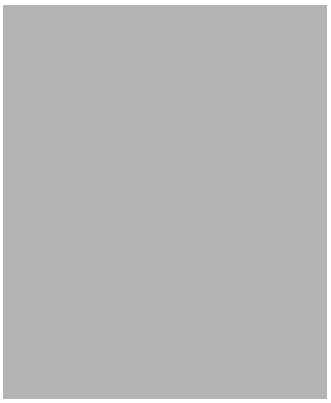
The height of the pile is always the same as the diameter of the base of the cone.

If the flour is being added at a constant rate of 3 cm^3 per second, at what rate is the height increasing when the pile is 4 cm in height?

You must use calculus and show any derivatives that you need to find when solving this problem.

Note that volume of a cone $= \frac{1}{3}\pi r^2 h$.





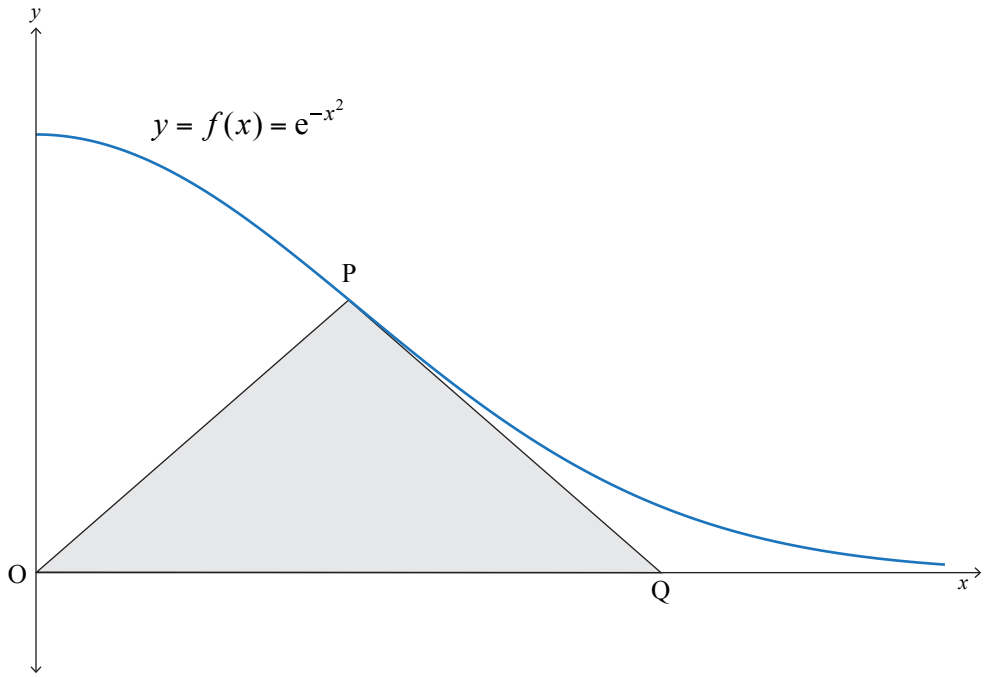
Source: https://www.istockphoto.com/nl/foto/man-gieten-bloem-uit-de-kom-van-de-maatregel-gm825182090-133804287

There is more space for your answer to this question on the following pages.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

Question Three continues on the next page.

(e) E whakaatuhia ana i te hoahoa i raro nei tētahi wāhanga o te kauwhata o te pānga o te $f(x) = e^{-x^2}$, ā, ko te $x \geq 0$.



Ka noho te pūwāhi o te P ki te kōpiko, ā, ka noho te pūwāhi o te Q ki te tuaka-x kia OP = PQ, ā, ko te O te pū.

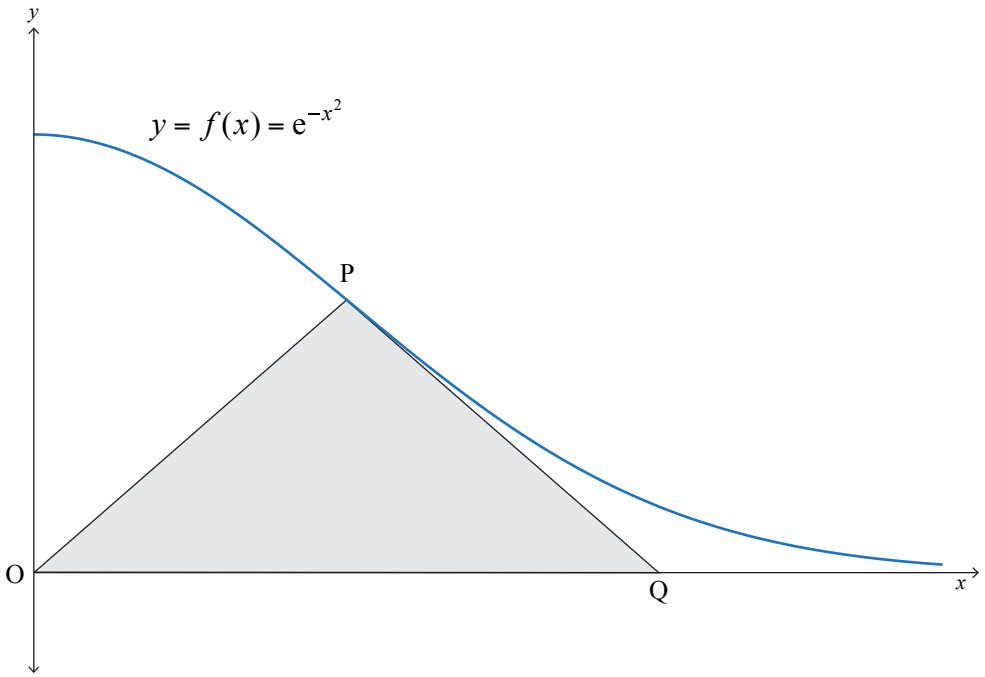
Whakaaturia ko te horahanga nui katoa e taea ana o te tapatoru o OPQ, ko te $\frac{1}{\sqrt{2e}}$.

Ehara i te mea me whakaatu rawa koe, ko te horahanga i whiriwhiria ai e koe, he mōrahi.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

He wāhi anō mō tō tuhinga mō tēnei tūmahi kei ngā whārangi e whai ake nei.

(e) The diagram below shows part of the graph of the function $f(x) = e^{-x^2}$, where $x \geq 0$.



The point P lies on the curve and the point Q lies on the x-axis so that OP = PQ, where O is the origin.

Prove that the largest possible area of the triangle OPQ is $\frac{1}{\sqrt{2e}}$.

You do not need to show that the area you have found is a maximum.

You must use calculus and show any derivatives that you need to find when solving this problem.

There is more space for your answer to this question on the following pages.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2024

91578M Apply differentiation methods in solving problems

Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–31 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⁄⁄⁄). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.