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91579M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2024

91579M Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā roto i te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Whakaaturia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (⁄⁄⁄). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

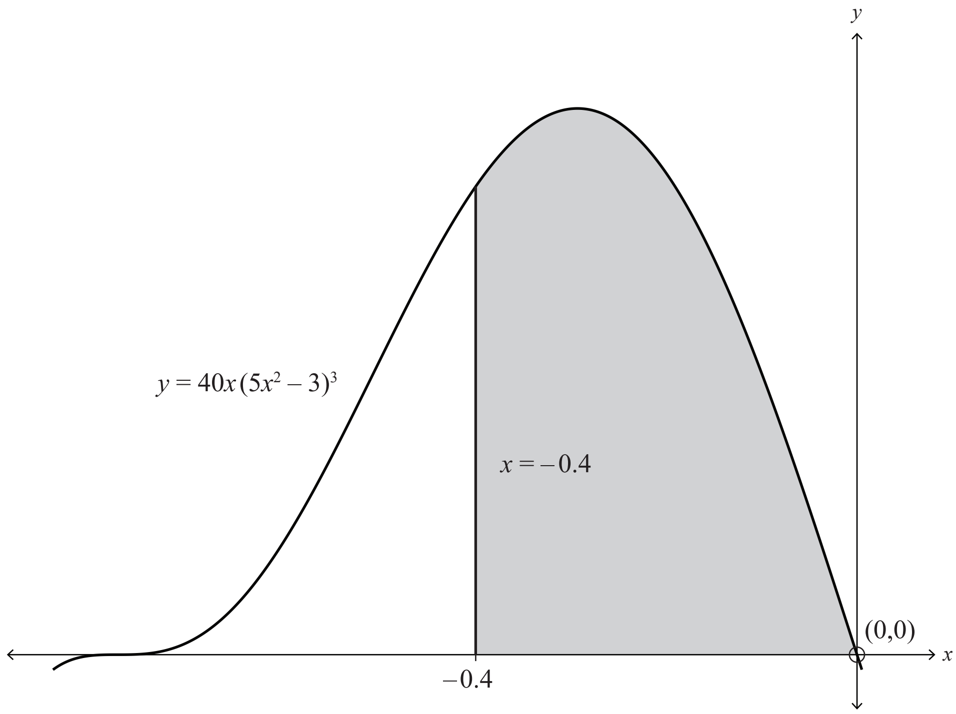
TE TŪMAHI TUATAHI

(a) Whiriwhiria te $\int 6\sec(2x)\tan(2x)\,dx$.

(b) E whakaatuhia ana i te kauwhata i raro nei te pānga o te $y = 40x(5x^2 - 3)^3$.

Whiriwhiria te horahanga o te wāhi kua kaurukutia.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.



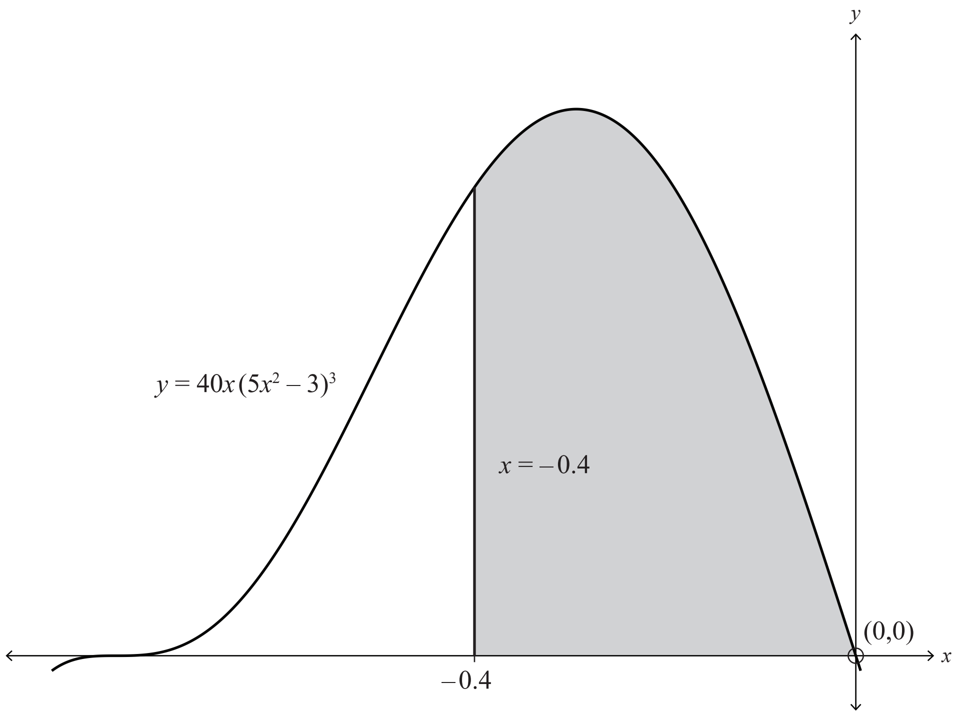
QUESTION ONE

(a) Find $\int 6\sec(2x)\tan(2x)\,\mathrm{d}x$.

(b) The graph below shows the function $y = 40x(5x^2 - 3)^3$.

Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.



(c) Ka taea te tere o tētahi mea te whakatauira ki te whārite o te $v(t) = 26.4\sqrt[3]{t}$,

arā, ko te v te tere o te mea hei m s^{-1},
ā, ko te t te wā ā-hēkona mai i te tīmatanga o te inenga o te wā.

I te tīmatanga, i 360 mita te tawhiti o te mea i tētahi pūwāhi P.

Tātaihia te tawhiti o te haere a tēnei mea mai i te pūwāhi P i te wā ka eke ki te 264 m s^{-1} te tere.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) An object's velocity can be modelled by the equation $v(t) = 26.4\sqrt[3]{t}$,

where v is the velocity of the object in m s^{-1},
and t is the time in seconds since the start of timing.

Initially, the object was 360 metres from a point P.

Calculate the distance that this object has travelled from the point P when it has reached a velocity of 264 m s^{-1} .

You must use calculus and show the results of any integration needed to solve the problem.

(d) Whakaarohia te whārite pārōnaki o te $\frac{dy}{dx} = 24\cos(3x)\sin(x)$.

Mehemea ko te $y = 6$ i te wā ko te $x = \frac{\pi}{3}$, whiriwhiria te/ngā uara o te y i te wā ko te $x = \frac{\pi}{2}$.

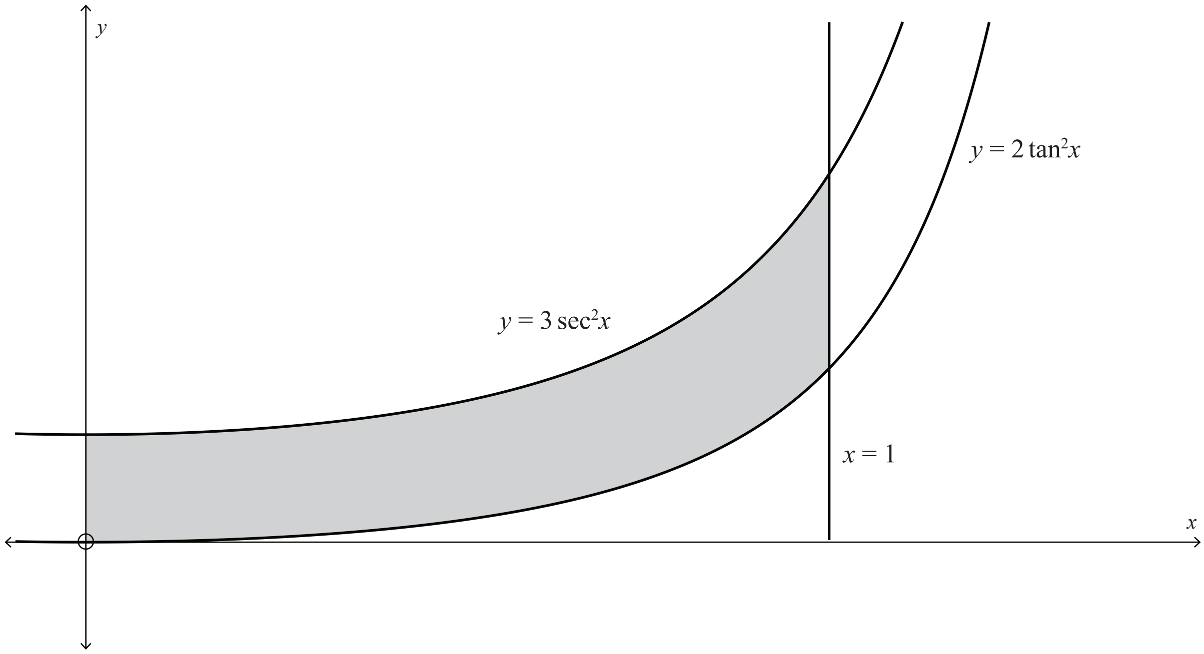
Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) Consider the differential equation $\frac{dy}{dx} = 24\cos(3x)\sin(x)$.

Given that $y = 6$ when $x = \frac{\pi}{3}$, find the value(s) of y when $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

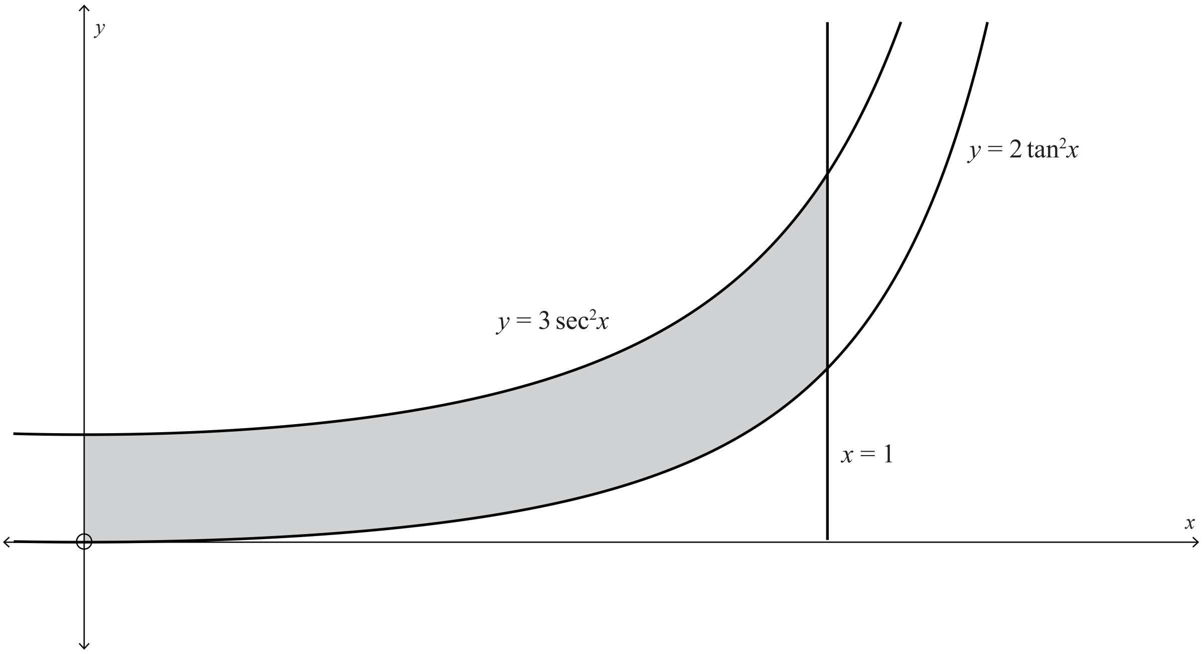
(e) E whakaatuhia ana i te kauwhata i raro nei ngā kōpiko o te $y = 3\sec^2 x$ me te $y = 2\tan^2 x$.



Whiriwhiria te horahanga o te wāhi kua kaurukutia e rohea ana e ngā kōpiko e rua, e te $x = 1$, e te tuaka-y hoki.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) The graph below shows the curves $y = 3\sec^2 x$ and $y = 2\tan^2 x$.



Find the area of the shaded region enclosed by the two curves, $x = 1$, and the y-axis.

You must use calculus and show the results of any integration needed to solve the problem.

TE TŪMAHI TUARUA

(a) Whiriwhiria te $\int \left(3x^4 + 4\right)^2 \mathrm{d}x$.

(b) Whiriwhiria te uara o te k, mehemea ko te $\int_k^{16} 3\sqrt{x}\,\mathrm{d}x = 112$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION TWO

(a) Find $\int\left(3x^4+4\right)^2\,\mathrm{d}x$.

(b) Find the value of k, given that $\int_k^{16} 3\sqrt{x}\,\mathrm{d}x = 112$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Whakaarohia te whārite pārōnaki o te $\frac{dy}{dx} = 12y^2e^{3x}$.

Mehemea ko te $y = 0.5$ i te wā ko te $x = 0$, whiriwhiria te uara o te y i te wā ko te $x = \frac{1}{3}$.

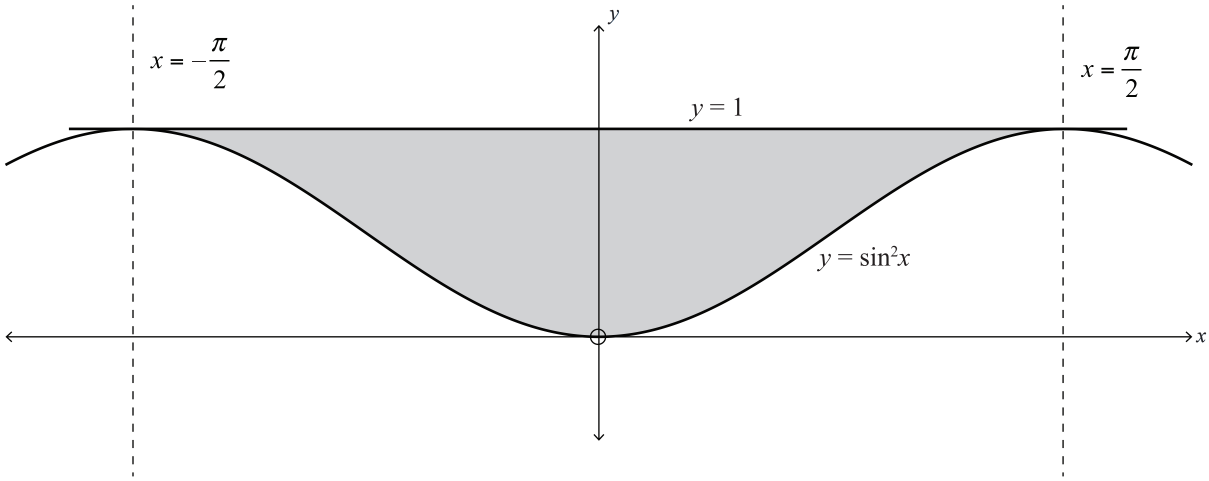
Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 12y^2\mathrm{e}^{3x}$.

Given that $y = 0.5$ when $x = 0$, find the value of y when $x = \frac{1}{3}$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaatuhia ana i te kauwhata i raro nei tētahi wāhanga o te kauwhata o te pānga o te $y = \sin^2 x$.

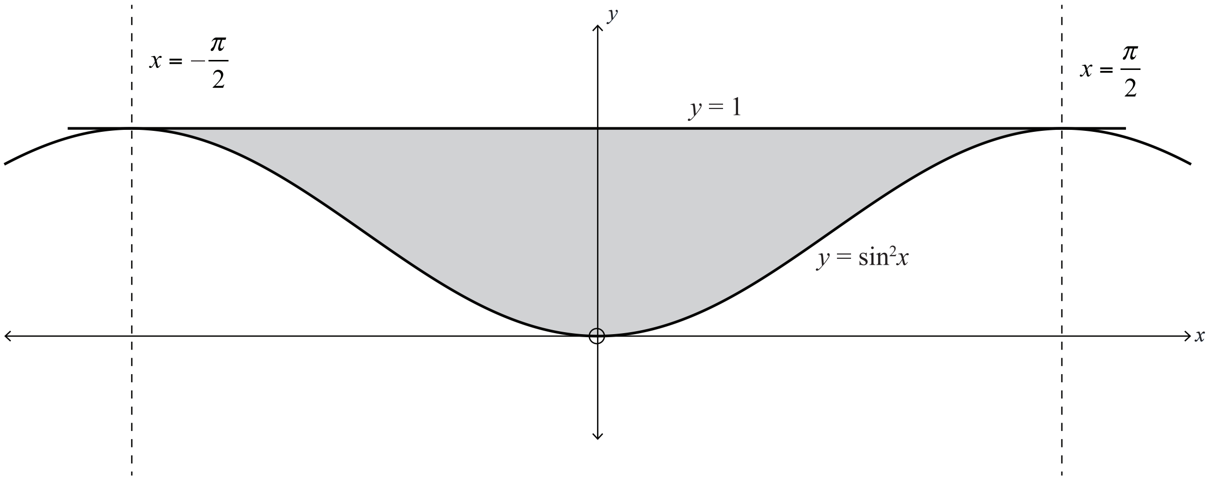


Whiriwhiria te horahanga o te wāhi kua kaurukutia e rohea ana e ngā rārangi

$y = \sin^2 x,\ \ y = 1,\ \ x = -\dfrac{\pi}{2},\ \ x = \dfrac{\pi}{2}.$

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) The graph below shows part of the graph of the function $y = \sin^2 x$.



Find the shaded area enclosed between the lines $y = \sin^2 x$, $y = 1$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Ka taea te papatipu, te M, o tētahi mea he poi te āhua, me te pūtoro o p, te āwhiwhi ki te

$$M = \int_0^p 4\pi r^2 \frac{a}{\left(1 + br^3\right)} \mathrm{d}r$$

arā, he tau pūmau tōrunga te a, te b, me te p.

Mā te whakamahi i tēnei ture, whiriwhiria he kīanga mō te papatipu, mō te M, o tētahi mea he poi te āhua, mā te whakaatu i tō whakautu e ai ki te a, ki te b, ki te p, me te π.

(e) The mass, M, of a spherical object, with radius p, can be approximated by

$$M = \int_0^p 4\pi r^2 \frac{a}{\left(1 + br^3\right)} \mathrm{d}r$$

where a, b, and p are all positive constants.

Using this formula, find an expression for the mass, M, of a spherical object, giving your answer in terms of a, b, p, and π.

TE TŪMAHI TUATORU

(a) Whiriwhiria te $\int\left(e^{2x}+\frac{3}{e^{4x}}\right)dx$.

(b) Whakaotihia te whārite pārōnaki o te $\frac{dy}{dx}=\frac{5}{4x-3}$, mehemea ko te $y = 10$ i te wā ko te $x = 1$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION THREE

(a) Find $\int \left(e^{2x} + \frac{3}{e^{4x}} \right) dx$.

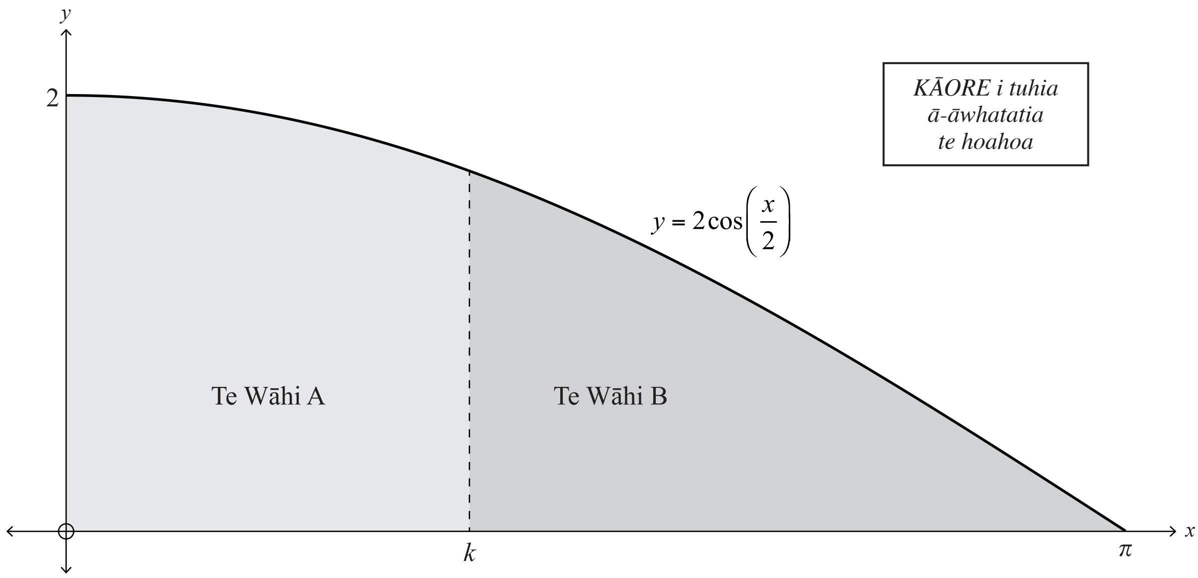
(b) Solve the differential equation $\frac{dy}{dx} = \frac{5}{4x - 3}$, given that $y = 10$ when $x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Whiriwhiria te uara o te m, mehemea ko te $\int_{-1}^{m} \left(\frac{4x+5}{2x+3} \right) dx = 2m$.

(c) Find the value of m, given that $\int_{-1}^{m} \left(\frac{4x+5}{2x+3} \right) \mathrm{d}x = 2m$.

(d) E whakaatuhia ana i te kauwhata i raro nei te pānga o te $y = 2\cos\left(\frac{x}{2}\right)$.

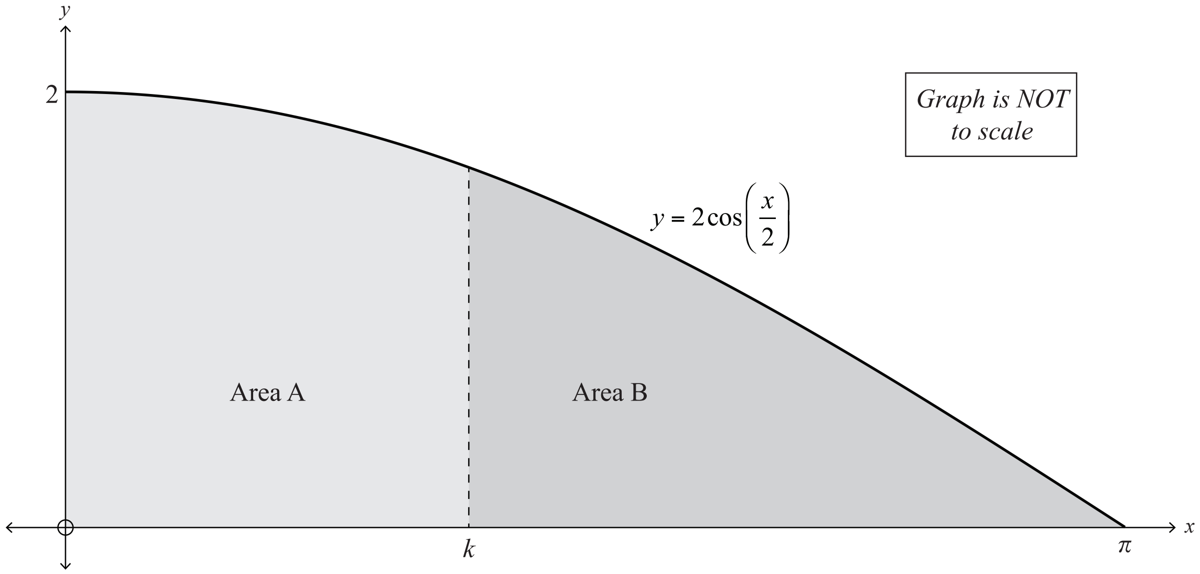


Whiriwhiria te uara o te *k* e rite ai Te Wāhi A kua kaurukutia, ki Te Wāhi B kua kaurukutia.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(d) The graph below shows the function $y = 2\cos\left(\frac{x}{2}\right)$.



Find the value of k so that the shaded Area A will be equal to the shaded Area B.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the next page.

(e) Ka mahi kapu kawhe tētahi kaiako i te tīmatanga o te paramanawa. Ka waiho te kaiako i te kapu kawhe ki te rūma kaimahi, ā, ko te 18 °C te paemahana o te rūma.

Ā muri i te 30 meneti, ko te 50 °C te paemahana o te kapu kawhe, engari e whakapono ana te kaiako kei te wera rawa tonu te kawhe hei inu māna.

Ka hoki atu ana anō te kaiako, ā muri i tētahi anō haora, kua heke te paemahana o te kapu kawhe ki te 30 °C.

E riterite ana te pāpātanga e panoni ai te paemahana o te kapu kawhe, ahakoa te wā, ki te rerekētanga i waenga i te paemahana o te kapu kawhe, o te N, me te paemahana o te rūma.

Tuhia mai he whārite pārōnaki e whakatauira ana i tēnei tūāhua, ka whakaotihia ai hei tātai i te paemahana o te kapu kawhe i te wā i mahia mai ai.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) A teacher makes a cup of coffee at the start of interval. The teacher leaves the cup of coffee in the staff room where the temperature is 18 °C.

After 30 minutes, the temperature of the cup of coffee is 50 °C, but the teacher believes that the coffee is still too hot to drink.

When the teacher returns again, after a further one hour, the cup of coffee has cooled down to a temperature of 30 °C.

The rate at which the temperature of the cup of coffee changes at any instant is proportional to the difference between the temperature of the cup of coffee, N, and the temperature of the room.

Write a differential equation that models this situation, and then solve it to calculate the temperature of the cup of coffee when it was made.

You must use calculus and show the results of any integration needed to solve the problem.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2024

91579M Apply integration methods in solving problems

Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.