SUPERVISOR'S USE ONLY

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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Mathematics and Statistics (Statistics) 2024

91585 Apply probability concepts in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Educators use writing detectors like Turnitin to detect if students have used AI (Artificial Intelligence) to write their assignments. When designing writing detectors, scientists focus on accuracy; if they say AI writing is present in a piece of work, they want to be pretty certain that the work is AI-generated, to ensure that students are not falsely accused of misconduct. This means that the detector may not always detect all AI writing; some may be missed.

A false positive in AI writing detection refers to incorrectly identifying fully human-written text as AI-generated. Suppose that a particular AI writing detector has a 1% false positive rate but that 74% of the time it fails to detect AI-generated writing.

Based on a confidential survey of students in one teacher's course, it is thought that 22% of pieces of student writing contain content that is generated by AI.

(i) If 120 pieces of writing are screened by this AI detector, approximately how many would be detected as potentially being generated by AI?

(ii) A piece of writing is detected as potentially being generated by AI.

Comment on whether a teacher should be concerned that the student could be unfairly accused of cheating.

Support your answer with statistical reasoning.

(b) Exam supervisors are sometimes concerned that students who complete tests quickly might have cheated. Based on data collected from one school, for a particular NCEA standard with an allocated time of 60 minutes, it is known that:

- 1% of students cheat on the assessment for this standard
- 20% of students complete the assessment in less than 25% of the allocated time
- 80% of students who cheat on the assessment complete it in less than 25% of the allocated time.

(i) Comment on whether the events, 'student cheats' and 'student completes the assessment in less than 25% of the allocated time', are independent of each other.

Use statistical reasoning to support your answer.

AND interpret your answer in terms of the relationship between the events 'student cheats' and 'student completes the assessment in less than 25% of the allocated time'.

(ii) Estimate the proportion of students across New Zealand who cheat and complete the assessment for this standard in less than 25% of the allocated time.

(iii) Give TWO reasons why care should be taken when using this data to estimate the proportion of students being assessed for NCEA who will cheat and complete the assessment in less than 25% of the allocated time.

Reason one:

Reason two:

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The assessment continues on the following page.

QUESTION TWO

(a) Data science is commonly thought of as the intersection of three main skill sets: coding, mathematical and statistical knowledge, and domain (or subject-specific) understanding, e.g. finance, biology, health.

From 35 applications for a data science role, where all three skill sets were desired, the following information is known:

- 7 applicants had none of the three skill sets
- 21 applicants had coding skills
- 16 applicants had mathematical and statistical knowledge
- 6 applicants had the necessary subject-specific understanding
- 9 applicants had exactly two of the three main skill sets
- 7 applicants were good candidates for the role except that they didn't have any subject-specific knowledge
- 1 applicant had only subject-specific understanding
- out of those with only one main skill set, $\frac{3}{8}$ had only mathematical and statistical knowledge.

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(i) A shortlist of people with all three skill sets was created.

How many applicants were selected for the shortlist?

(ii) Calculate the proportion of applicants who had only one of the required skills.

(iii) It is claimed that, for those applicants with the required mathematical and statistical knowledge, applicants are twice as likely to have subject-specific understanding compared to coding skills.

Does this data support the claim?

Use calculations and statistical reasoning to support your answer.

(b) ‘In 2022, 82.4% of all 18-year-olds attained the equivalent of NCEA Level 2 or above. Of those who turned 18 in 2022, 78.3% of them attained at least NCEA Level 2 or above in school, and 4.1% of them attained at least NCEA Level 2 or above post-school in a tertiary or vocational setting.’

Figure 1: Most 18-year-olds with NCEA Level 2 or above attained their qualification in school

Source: www.educationcounts.govt.nz/statistics/18-year-olds-with-level-2-or-equivalent

(i) Based on the data, is the suggestion that the total proportion of 18-year-olds with NCEA Level 2 has increased between 2012 and 2022 correct?

Support your answer with statistical reasoning.

(ii) Comment on the contribution of post-school qualifications to the total proportion of 18-year-olds with NCEA Level 2 or above between 2012 and 2022.

Support your answer with statistical reasoning.

QUESTION THREE

(a) A player plays two different games, A and B, by rolling a pair of dice.

(i) For game A, if the total of two dice rolls is between 5 and 10 (inclusive), then the player wins.

Calculate the probability that a player wins at least once when playing 3 times.

(ii) For game B, a version of the game called Hazard is played. The rules of this game are as follows:

- When a player rolls the dice for the first time, any combination of the two dice that adds up to 7 is a winner.
- On the first roll, any dice total that equals 2, 3, 11, or 12 is an immediate loser.
- If the first roll has not produced a winner or a loser, the total of the dice becomes known as the point.
- For all successive rolls, the player will win a game if the point is rolled again. However, if a 7 is rolled before the point is rolled, the player loses.

Calculate the probability that the game is lost before the player has to roll the dice for a third time.

Question Three continues on the next page.

(b) The player is concerned that one of their dice is biased. The outcomes from 1000 rolls of this die are summarised in the table below.

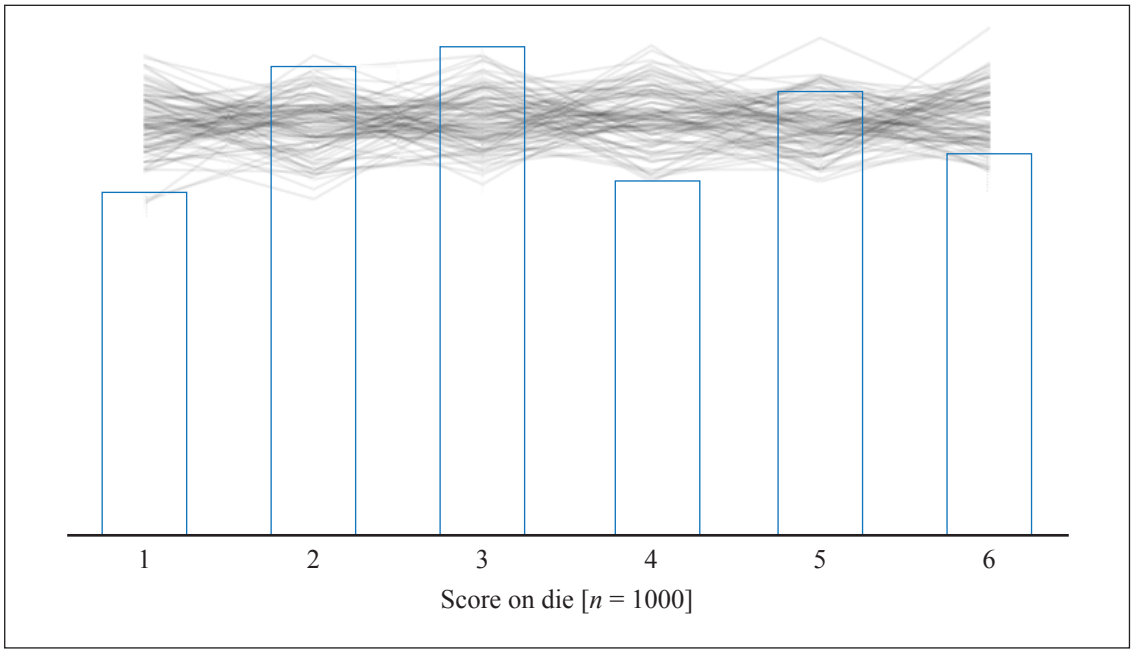
Outcome	1	2	3	4	5	6
Totals	138	189	197	143	179	154

(i) For these 1000 rolls, which is more likely? Rolling 3 or less OR rolling 4 or more?

Support your answer with statistical calculations.

The diagram below shows the results of 1000 trials of a simulation model. The simulation assumed that each outcome on the die was equally likely to occur.

The height of the blue vertical bars shows the relative frequencies of each observed digit outcome on the die, as shown in the table above. The grey band shows the variation expected for each outcome, based on simulating 1000 throws of a fair die.



(ii) Should the player be concerned that one of their dice is biased?

Use the results of the simulation model shown in the diagram on the previous page and the outcomes of the 1000 rolls given in the table, and refer to experimental, theoretical, and true probability as part of your answer.

(iii) Explain what this result means for the chance of throwing a total of 2 using this particular die as one of the pair of dice in game B.

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