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See back cover for an English translation of this cover.

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91586M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Pāngarau me te Tauanga (Te Tauanga), Kaupae 3, 2024

91586M Te whakahāngai tuari tūponotanga i te whakaoti rapanga

Ngā whiwhinga: E whā

Paetae	Kaiaka	Kairangi
Te whakahāngai tuari tūponotanga i te whakaoti rapanga.	Te whakahāngai tuari tūponotanga, mā roto i te whakaaro pānga, i te whakaoti rapanga.	Te whakahāngai tuari tūponotanga, mā roto i te whakaaro waitara e whānui ana, i te whakaoti rapanga.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te Puka mō ngā Ture Tātai me ngā Tūtohi L3–STATMF.

Whakaaturia ngā whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapa o ngā whārangi 2–27, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki tētahi wāhi e kitea ai te kauruku whakahāngai (⁘). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Ka taea te whakaahua te pito o te tangata hei "pito roto" (he pito whakaroto), hei "pito waho" (he pito kōture) rānei, e whakaahuatia ana i raro nei. He āhuatanga whakapaipai te āhua o te pito, ā, ehara i te tohu matua o te hauora, nō reira, kāore e rite tonu te kohia o ngā raraunga e pā ana ki te pito. Nā konā e uaua nei kia tika te whakatau tata i te hautanga o ngā pito waho i te taupori.

https://www.pampers.com/en-us/baby/newborn/article/outie-belly-button

I kohia ngā raraunga e pā ana ki te momo o te pito i te rangahau pito tuatahi o te ao. Mai i ēnei raraunga, i kitea he pito roto tō te 96% o ngā kaiurupare.

(i) Mā te whakamahi i tētahi tauira tuari tūponotanga e tika ana, tātaihia tētahi whakataunga tata mō te tūponotanga, i tētahi akomanga e 30 ngā ākonga, kāore he pito waho o tētahi ākonga.

Tuhia te tauira tuari tūponotanga me ngā tawhā i whakamahia rā hei wāhanga mō tō tuhinga.

(ii) Homai ngā take e RUA e tika ana te whakamahia o te tauira tuari tūponotanga i kōwhiria ai i te wāhanga (i).

Te take tuatahi:

Te take tuarua:

QUESTION ONE

(a) A person's belly button can be described as an "innie" (navel indented) or an "outie" (navel protruded), as pictured below. Belly-button shape is a cosmetic characteristic and not a significant indicator of health, so data about belly buttons is not routinely collected. It is therefore difficult to make a reliable estimate of the proportion of outie belly buttons in the population.

https://www.pampers.com/en-us/baby/newborn/article/outie-belly-button

Data on belly-button type was collected in the world's first belly button survey. From this data, it was found that 96% of respondents had an innie belly button.

(i) Using an appropriate probability distribution model, calculate an estimate for the probability that, in a class of 30 students, no students have an outie belly button.

State the probability distribution model and the parameters used as part of your answer.

(ii) Give TWO reasons why the use of the probability distribution model selected in part (i) is appropriate.

Reason one:

Reason two:

(iii) Ko te kerehunga o roto i te pito te whakawhāitinga o ngā kaka rānei, o ngā huru rānei, o ngā para rānei ka putu ki te pito o tētahi tangata. I kitea i tētahi rangahau pito, ko te 66% o ngā kaiurupare, he kerehunga i ō rātou pito.

Mā te whakamahi i tētahi tauira tuari tūponotanga e tika ana, tātaihia tētahi whakataunga tata mō te tūponotanga, i tētahi akomanga e 30 ngā ākonga, he pito waho tō tētahi ākonga kotahi, ā, he kerehunga kei te pito rā.

Me tuhi koe i te tauira tuari tūponotanga me ngā tawhā i whakamahia rā hei wāhanga mō tō tuhinga.

(iii) Belly button fluff is the accumulation of fibres, lint, or debris that collect in a person's belly button. A belly button survey found that 66% of all the respondents had fluff in their belly buttons.

Using an appropriate probability distribution model, calculate an estimate for the probability that one student in a class of 30 has an outie belly button, and it contains belly button fluff.

You should state the probability distribution model and the parameters used as part of your answer.

(b) He tino iti noa iho ngā mahi rangahau kua whakahaeretia e pā ana ki te rahi o ngā pito. E whakapaetia ana, ka taea te whakatauira te whitianga o ngā pito pakeke (te tawhiti i te wāhi whānui katoa) mā te tuari māori, e 2.05 cm te toharite, ā, e 0.9 cm te ine mahora.

(i) Tātaihia he whakataunga tata mō te tūponotanga ka 1.75 cm, ka whānui ake rānei i tērā, te whitianga o tētahi pito pakeke, i kōwhiri matapōkeretia ai, mēnā kei waenganui i te 1.5 cm me te 2.0 cm te whitianga.

(ii) Tuhia he kōrero mō te tika rānei o te whakamahi i ngā tawhā i hoatu ai i runga nei mō te whakatauira i te whānui o te pito pakeke.

(b) Very little research on the typical size of belly buttons has been undertaken. It is hypothesised that the diameter of adult belly buttons (the distance across at the widest point) could be modelled by a normal distribution with mean of 2.05 cm and standard deviation of 0.9 cm.

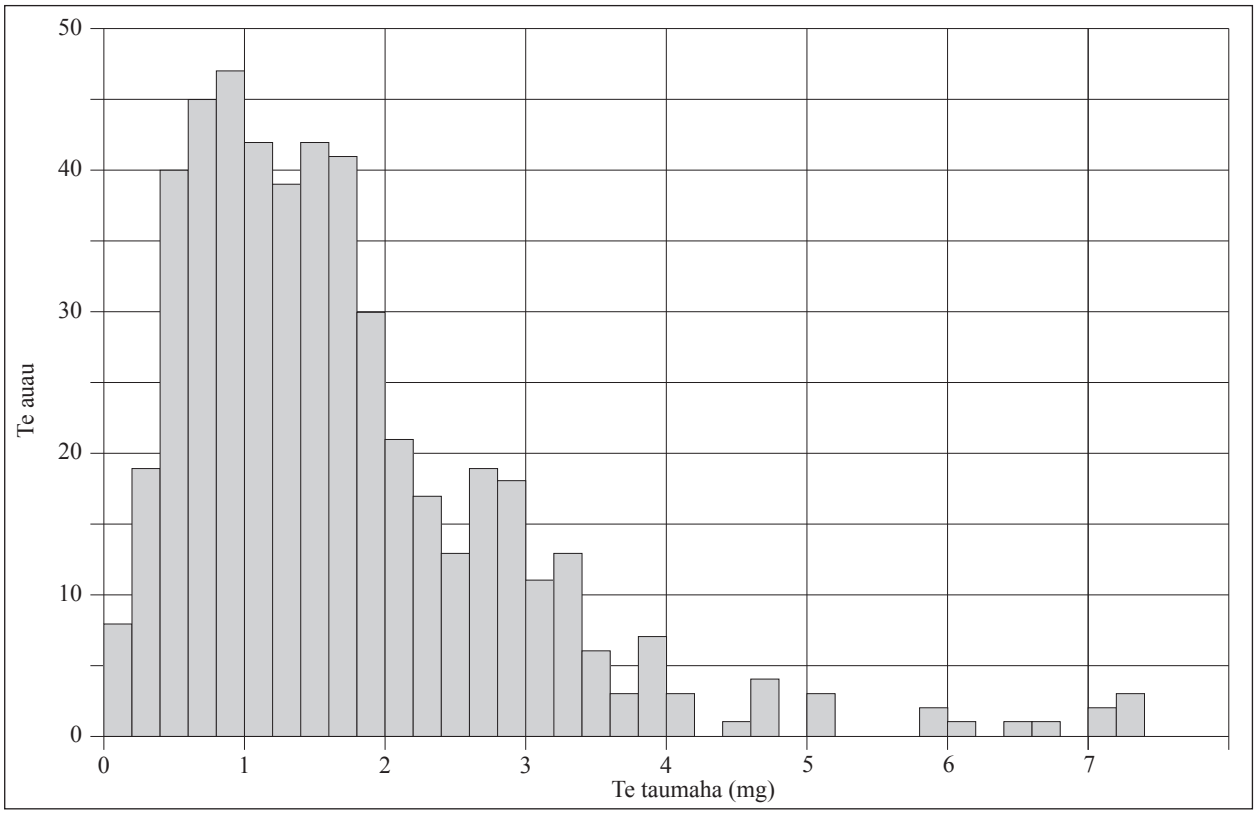
(i) Calculate an estimate for the probability that a randomly selected adult belly button has a diameter of at least 1.75 cm, given that its diameter is between 1.5 cm and 2.0 cm.

(ii) Comment on the suitability of using the parameters given above for modelling adult belly button width.

TE TŪMAHI TUARUA

Ka putu te kerehunga ki ngā pito o ētahi tāngata. I whakaputaina te mātaitanga o Te Āhua o te Kerehunga i ngā Pito (nā Steinhauser) i te tau 2009. I kohi te kairangahau o tēnei mātaitanga i ngā raraunga e pā ana ki te kerehunga i tōna ake pito. I ia rā, mō ngā rā e 502, i uwhiuwhi ia i te ata, ā, i horoi ia i tana pito. I te mutunga o ia rā, i huhuti ia i te kerehunga pito i putu ai i te roanga o te rā, ka ine ai i te taumaha.

(a) E whakaaturia ana i Te Ata 2(a) i raro nei te tuari taumaha o te nui o te huru pito (te kerehunga) i kohi rā ia i ia rā.



Te Ata 2(a): Te tuari taumaha o ngā kerehunga pito o ia rā e 502

E marohitia ana kia whakatauiratia te taumaha o ngā kerehunga pito o ia rā mā te whakamahi tuari tapatoru e whai tawhā ana:

te mōkito = 0 mg, te mōrahi = 7.4 mg, ā, ko te tau tānui = 1.0 mg

(i) Huahuatia tēnei tauira ki Te Ata 2(a).

(ii) Mā te whakamahi i ngā tawhā nei, me whakatau tata te tūponotanga i tētahi rangi kāore te taumata o te kerehunga pito e eke ki te 1.5 mg.

QUESTION TWO

Fluff collects in the belly buttons (navels) of some people. The Nature of Navel Fluff study (Steinhauser) was published in 2009. The author of the study collected data about his own belly button fluff. Each day, for 502 days, he had a morning shower where he washed his belly button. At the end of each day, he removed and weighed the belly button fluff that had accumulated during the day.

(a) The weight distribution of the amount of belly button lint (fluff) he collected each day is shown in Figure 2(a) below.

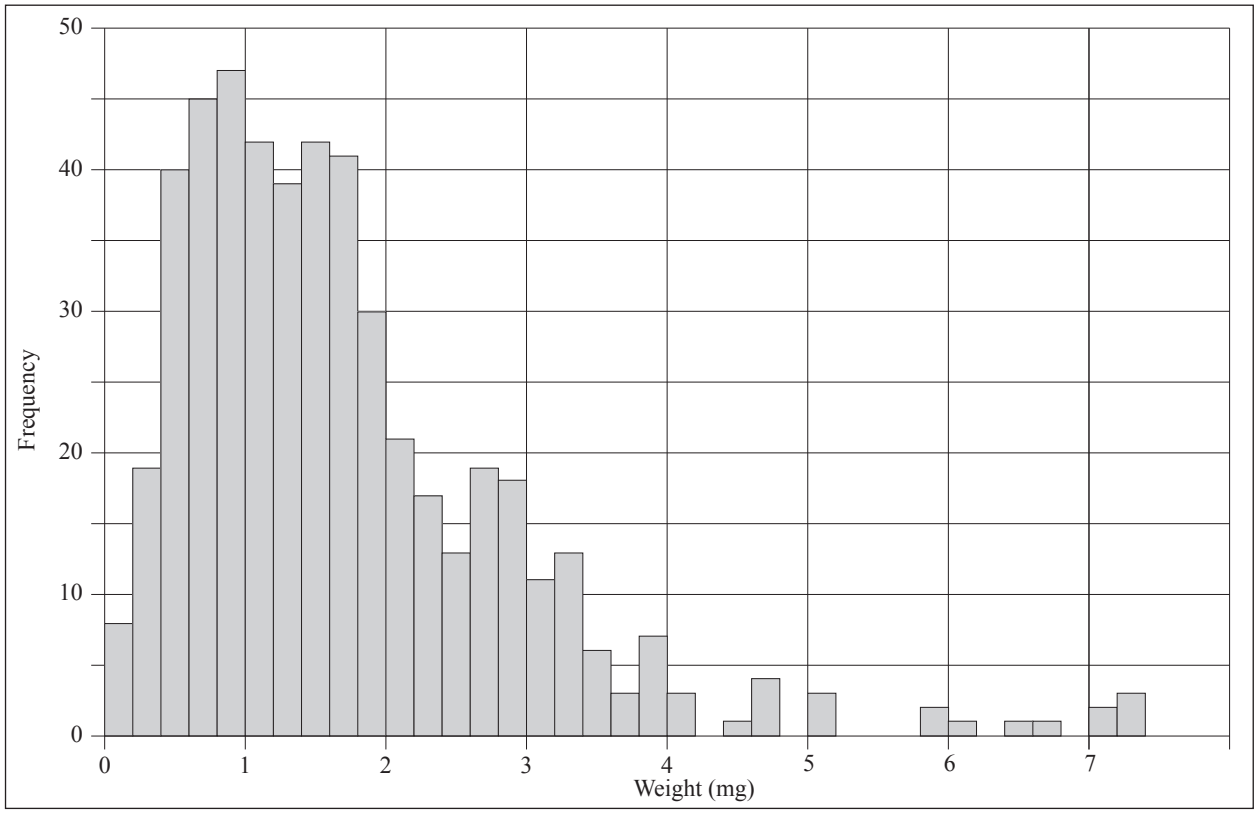


Figure 2(a): Weight distribution of 502 pieces of daily belly button fluff

It is proposed to model daily belly button fluff weight using a triangular distribution with parameters:

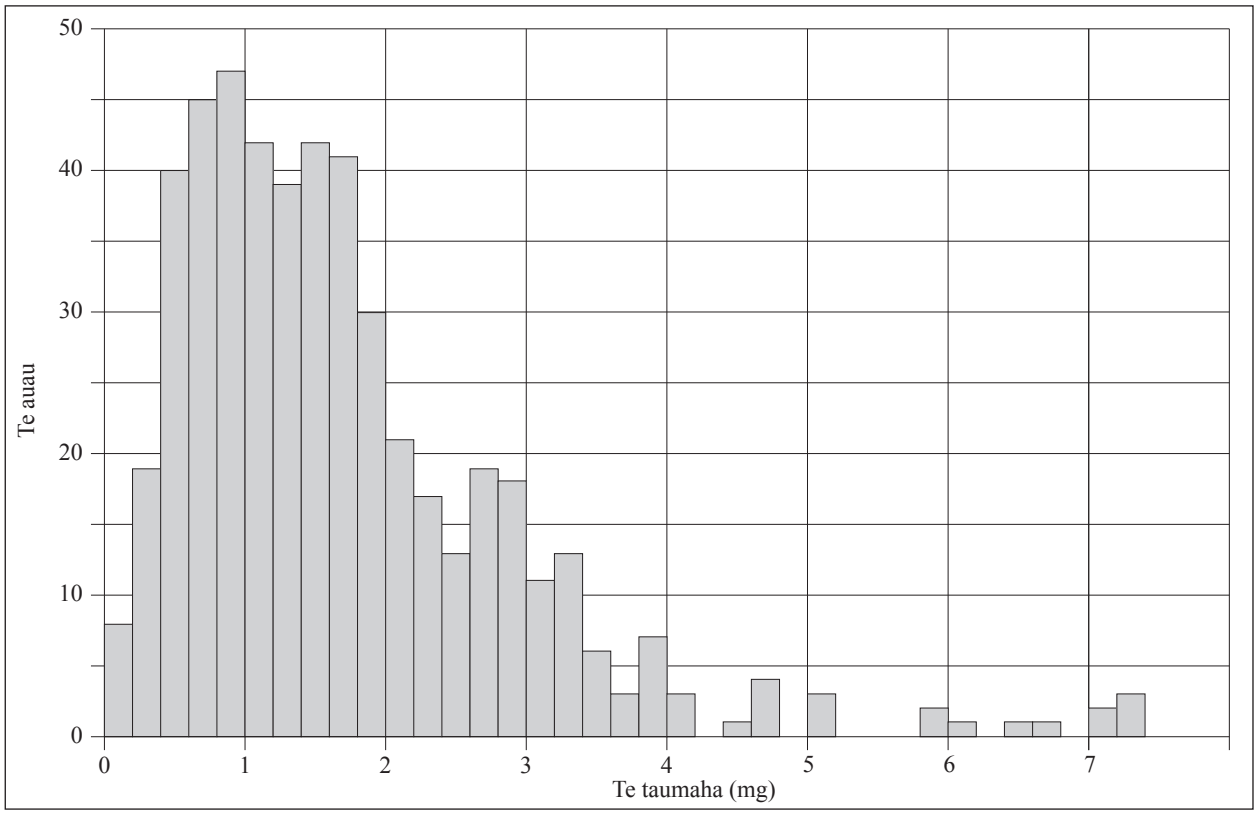
minimum = 0 mg, maximum = 7.4 mg, and mode = 1.0 mg

(i) Sketch this model on Figure 2(a).

(ii) Using the given parameters, estimate the probability that on any given day the belly button fluff weight is less than 1.5 mg.

(iii) Tautapahia ētahi atu tawhā mō tētahi tuari tapatoru e whai tikanga ake ana ki ēnei raraunga, ā, huahuatia tō tauira hou ki Te Ata 2(b) i raro nei.

Hei wāhanga mō tō tuhinga, parahautia tō kōwhiri i ēnei tawhā rerekē.



Te Ata 2(b): Te tuari taumaha o ngā kerehunga pito o ia rā e 502

(iii) Suggest alternative parameters for a triangular distribution that will give a better fit to this data and sketch your new model onto Figure 2(b) below.

As part of your answer, justify your choice of these alternative parameters.

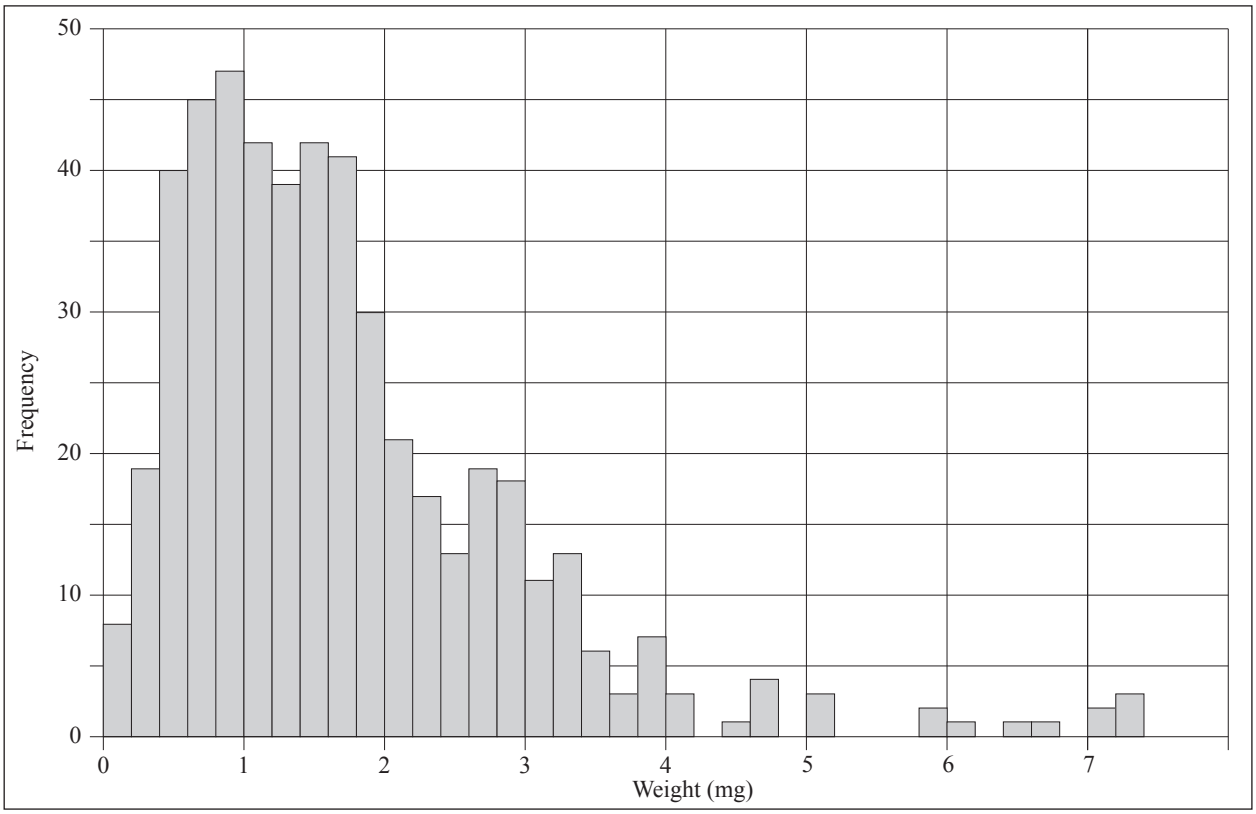


Figure 2(b): Weight distribution of 502 pieces of daily belly button fluff

(b) E whakaatu ana te tūtohi o raro nei i te tuari tūponotanga o te taurangi matapōkere, o te T, arā, ko te nui o ngā tāora i ia tangata ka whakamahia mō ngā kainoho o tētahi tāone, puta noa i tētahi rangi.

t	0	1	2	3	4	5	6
P(T = t)	0.129	0.271	0.264	0.185	0.095	0.039	0.017

(i) E hia te toharite o ngā tāora i whakamahia e tētahi kainoho o taua tāone i te rangi kotahi?

(ii) Me kī, ko te taurangi matapōkere, ko te C, te nui o ngā huinga kākahu i mauria mō ngā kainoho o taua tāone i taua rangi kotahi. Ko te 3 te nui mōrahi o ngā panonitanga kākahu e ngā kainoho o taua tāone i taua rangi.

Ko te SD(C) o te taurangi matapōkere o te C = ko te 0.5.

Whakaaturia he iti iho te ine mahora o te C, tēnā i tō te T, ā, homai tētahi take e pēnei ana pea.

(iii) Ko te SD(T + C) = 1.754

He motuhake rānei te T me te C?

Tuhia he kōrero mō tā tēnei e tohu nei e pā ana ki te nui o ngā tāora i whakamahia rā me te nui o ngā panonitanga kākahu mō ngā kainoho o taua tāone.

Taunakitia tō tuhinga ki te/ngā tauākī ā-tauanga me te/ngā tātainga ā-tauanga.

(b) The table below shows the probability distribution of the random variable, T, the number of towels used per person for residents from one city over one day.

t	0	1	2	3	4	5	6
P(T = t)	0.129	0.271	0.264	0.185	0.095	0.039	0.017

(i) How many towels, on average, were used by a resident from this city over the one day?

(ii) Let the random variable, C, be the number of sets of clothing worn for residents from this city on this one day. The maximum number of clothing changes by residents of this city on this day is 3.

The random variable C has SD(C) = 0.5.

Show that C has a lower standard deviation than T, and give one reason why this might be the case.

(iii) SD(T + C) = 1.754

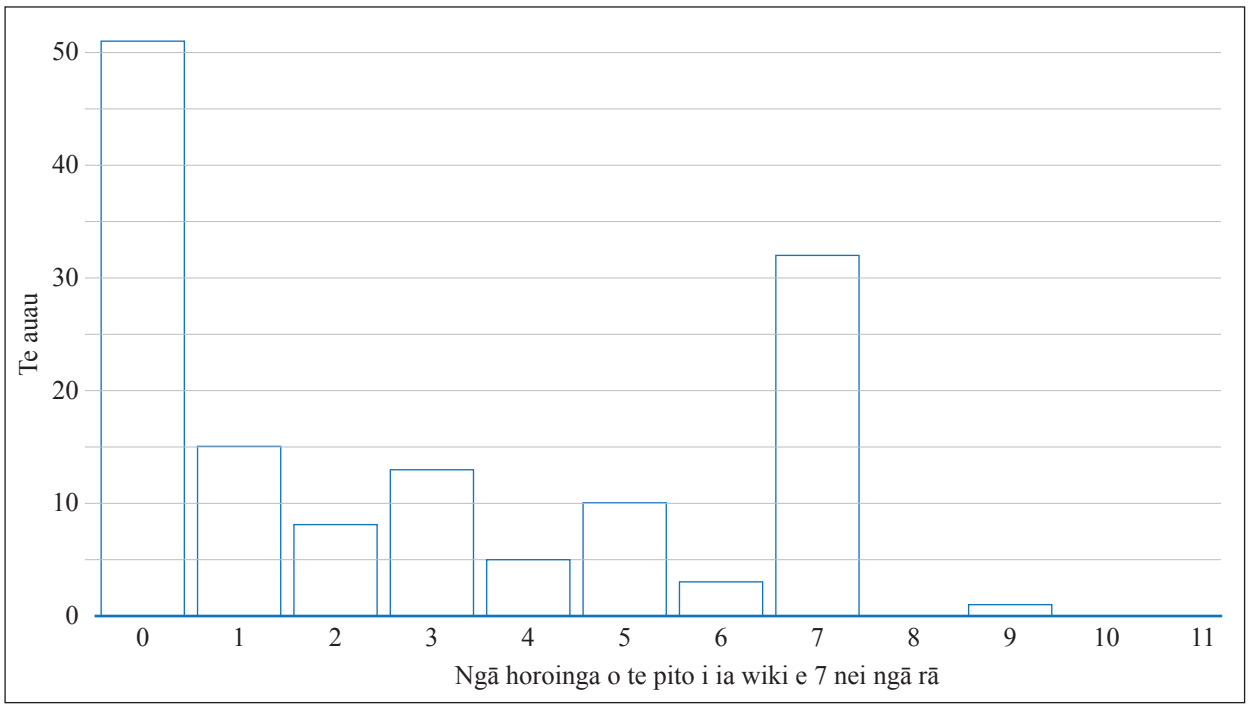
Are T and C independent?

Comment on what this suggests about the number of towels used and number of clothing changes for residents of this city.

Support your answer with statistical statement(s) and calculation(s).

TE TŪMAHI TUATORU

I kohia i tētahi mātaitanga, i whakaputaina ai i te tau 2012, ngā pārongo e pā ana ki ngā ritenga mō ngā pito, e hāngai ana ki ngā raraunga i kohia rā i ngā tāngata 138 i whai wāhi atu ki tētahi kaupapa pūtaiao ā-motu o ngā kiriraraau o Amerika, i te tau 2011. E whakaaturia ana i Te Ata 3(a) te tuari o te auau o ngā horoinga (ko te tapeke o ngā horoinga o te pito i ia wiki e 7 nei ngā rā).



Te Ata 3(a): Te nui o ngā wā i horoi ai ngā kaiurupare o te mātaitanga i ō rātou pito i te wiki

(a) E whai ake nei ko te upoko kōrero o tētahi pānui pāpāho e pūrongorongo ana i ngā hua o te mātaitanga:

"TE KINO HOKI – he iti iho i te hauwhā o ngā kiriraraau o Amerika ka horoi i ō rātou pito i ia rā".

Tuhia he kōrero mō te tika rānei o tēnei upoko kōrero.

Taunakitia tō tuhinga ki ngā tātainga me ngā whakaaro ā-tauanga.

QUESTION THREE

A study published in 2012, based on data collected from 138 people participating in a nationwide American citizen science project in 2011, collected information about belly button habits. The distribution of washing frequency (defined as the number of belly button washes per 7-day week), is shown in Figure 3(a).

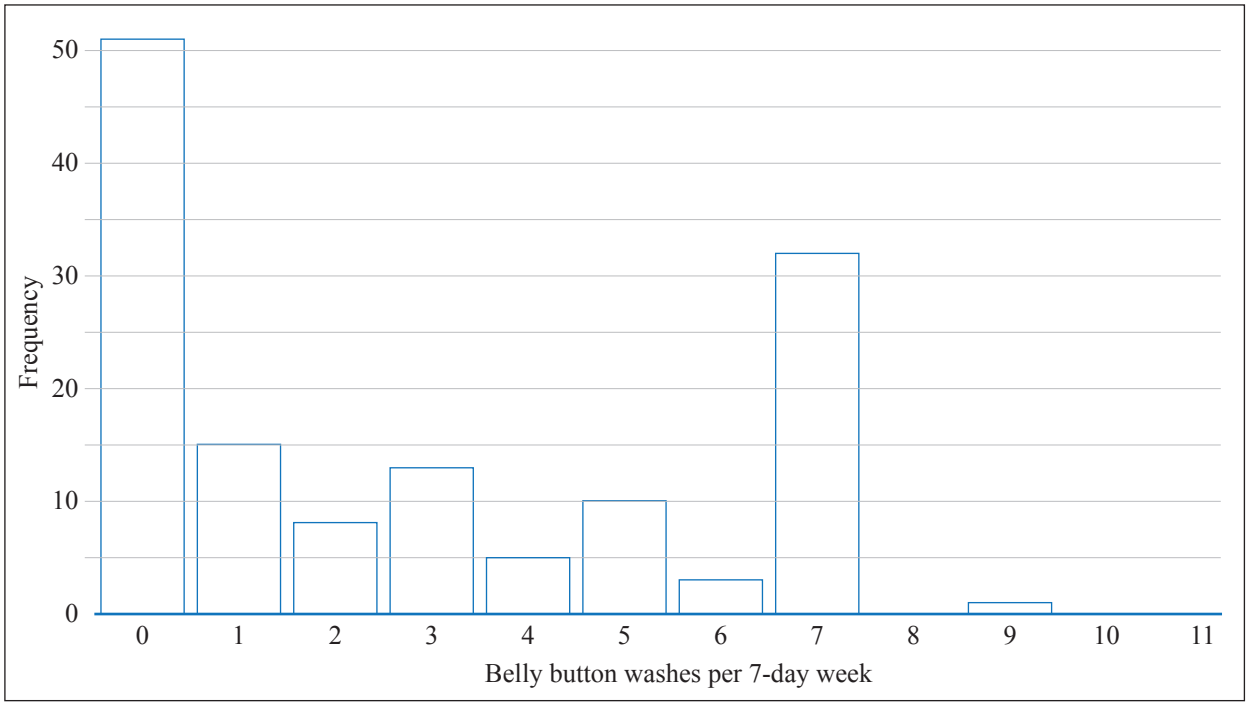


Figure 3(a): The number of reported times survey respondents washed their belly buttons per week

(a) A media release reporting on the study's results had the following headline:
"SHOCKING – less than a quarter of US citizens wash their belly button daily".

Comment on whether this headline is correct.
Support your answer with statistical calculations and reasoning.

(b) Me kī, ka whakatauiratia te nui o ngā horoinga o te pito i te wiki mō te taupori whānui mā te tuari Poisson, ā, ko te $\lambda = 2.8$.

(i) Whakamahia tēnei tauira Poisson hei tātai i tētahi whakataunga tata o te tūponotanga ka kotahi, ka neke atu rānei, te horoinga a tētahi tangata nō te taupori whānui, i kōwhiri matapōkeretia ai, i tōna pito.

(ii) E whakahāngaitia ai te tuari i whakamahia rā i te wāhanga (b)(i), me kotahi, me neke atu rānei, te whakapae.

Tautuhia TĒTAHI o ngā whakapae kāore pea e whai mana, ā, matapakina te take e pēnei ana.

Ka rere tonu te Tūmahi Tuatoru i te whārangi e whai ake nei.

(b) Suppose the number of belly button washes per week for the general population is modelled by a Poisson distribution with $\lambda = 2.8$.

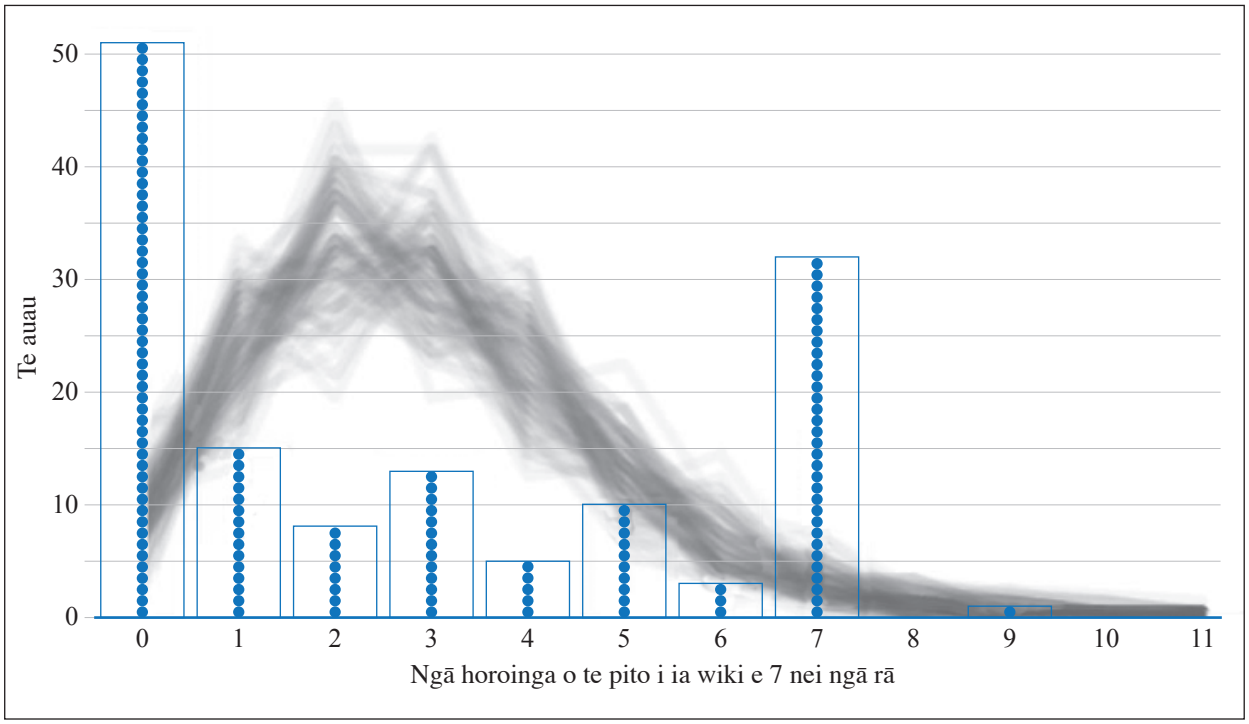
(i) Use this Poisson model to calculate an estimate for the probability that a randomly selected person from the general population washes their belly button at least once in a day.

(ii) To apply the distribution used in part (b)(i), at least one assumption must be made.

Identify ONE assumption made that may be invalid and discuss why this is the case.

Question Three continues on the next page.

E whakaaturia ana i Te Ata 3(a) te 1000 kuhunga o ngā tīpakonga raraunga ki tētahi tauira whaihanga, ā, e whakapaetia ana ka whakamahia tētahi tuari Poisson, ka mutu, ko te $\lambda = 2.8$, hei whakatauira i te nui o ngā horoinga o te pito i te wiki, e 7 nei ngā rā, mō te taupori whānui. E whakaaturia ana i Te Ata 3(b) ngā hua o te tauira whaihanga me ngā raraunga taketake i tirohia rā (ngā ira kahurangi).



Te Ata 3(b): Ngā hua o te tauira whaihanga me ngā raraunga taketake i tirohia rā

(iii) Whakamāramahia tā ngā rārangi kiwikiwi e whakaatu nei i Te Ata 3(b).

(iv) E ai ki ngā hua o te tauira whaihanga me ngā raraunga taketake i tirohia rā (i Te Ata 3(b)), matapakina mēnā rānei e tika ana te tauira tuari Poisson (e whakaaturia ana i te whārangi o mua), hei whakatauira i te nui o ngā horoinga o te pito i te wiki e 7 nei ngā rā.

Taunakitia tō tuhinga ki ngā whakaaro ā-tauanga.

(v) Marohitia, ka parahau ai i te whakamahinga o tētahi tauira atu anō ka tika pea hei whakatauira i te nui o ngā horoinga o te pito i te wiki e 7 nei ngā rā.

Me tuhi koe i te tauira tuari tūponotanga me ngā tawhā i whakamahia ai hei wāhanga mō tō tuhinga.

The sample data shown in Figure 3(a) is run through a simulation model 1000 times, assuming that a Poisson distribution with $\lambda = 2.8$ is used to model the number of belly button washes per 7-day week for the general population. Figure 3(b) shows the results of the simulation model and the original observed data (blue dots).

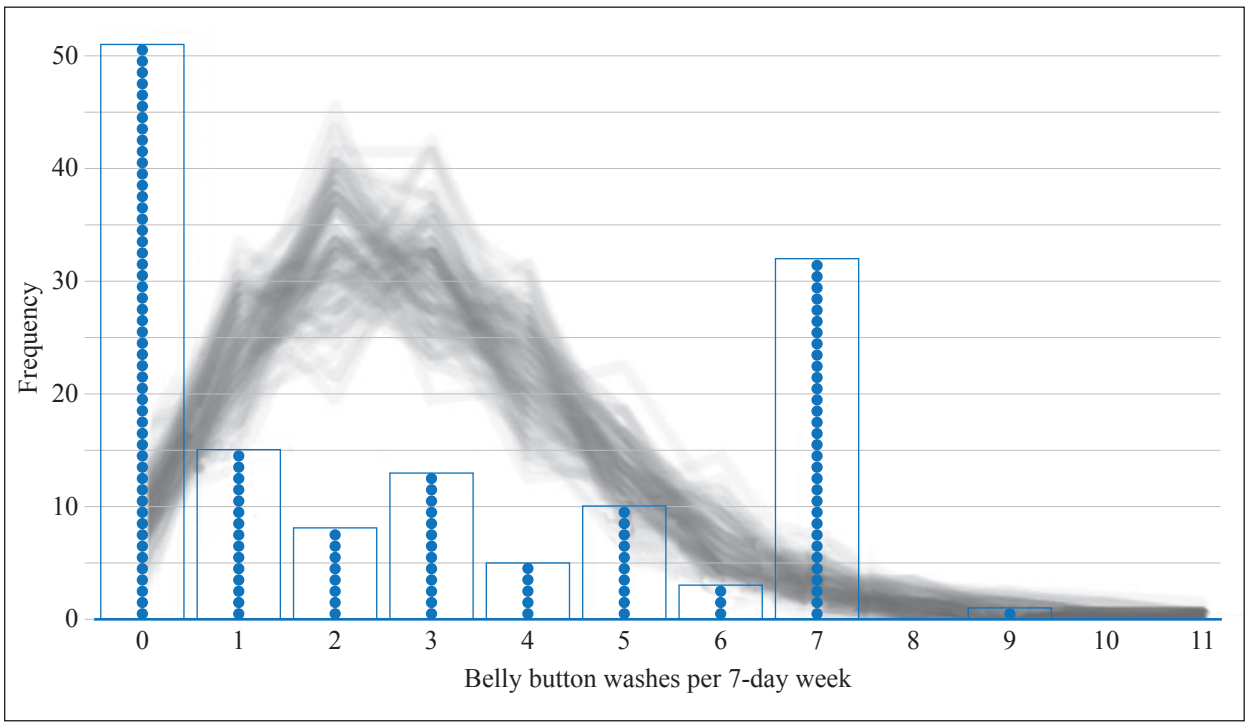


Figure 3(b): Results of the simulation model and original observed data

(iii) Explain what the grey band is showing in Figure 3(b).

(iv) Based on the results of the simulation model and the original observed data (Figure 3(b)), discuss whether the Poisson distribution model (presented above) appears to be appropriate for modelling the number of belly button washes per 7-day week.

Support your answer with statistical reasoning.

(v) Propose and justify the use of an alternative model that could be appropriate for modelling the number of belly button washes per 7-day week.

You should state the probability distribution model and the parameters used as part of your answer.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Mathematics and Statistics (Statistics) 2024

91586M Apply probability distributions in solving problems

Credits: Four

91586M

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATFM.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⁄⁄⁄). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.