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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2025

91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in the margins (⋮⋮⋮). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) When the polynomial $3x^4 + px^3 - 4x + 5$ is divided by $x - 2$, the remainder is 21.

Find the value of p.

(b) Solve the equation $x^2 - 6kx = k^2$ for x.

Simplify your answer as far as possible, giving your answer in terms of k.

(c) Prove that the equation $(kx)^2 = 3 - \frac{x}{k}$ has real roots for all real values of k, where $k \neq 0$.

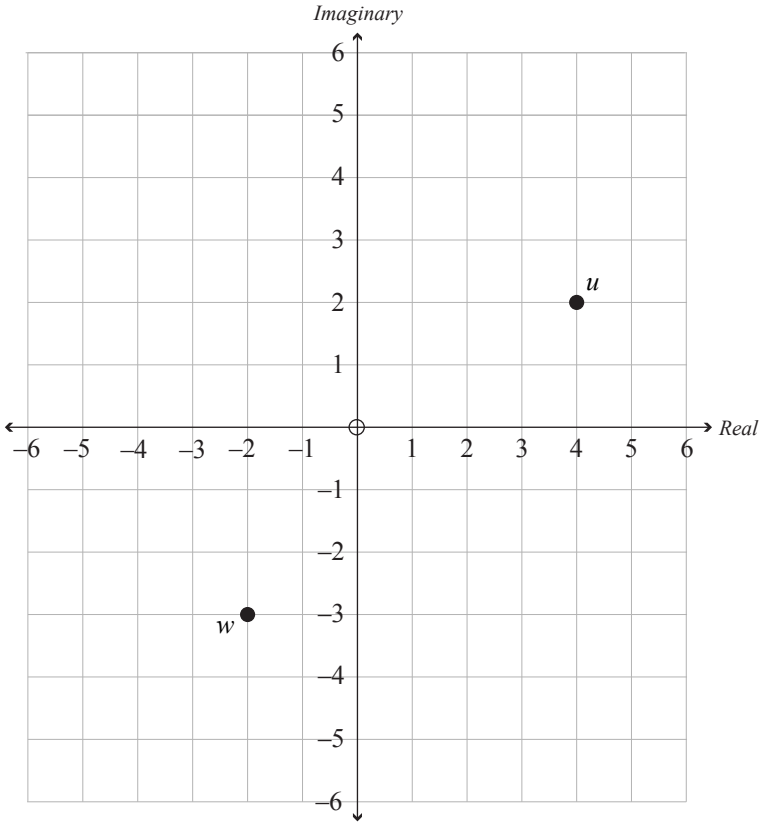
(d) Solve the equation $z^3 + 8m^{27}i = 0$, where m is a positive real constant.
Write your solution(s) in polar form, in terms of m.

(e) The locus of a complex number z is described by $|z - 5i| - |z + 5i| = 4$.

Find the Cartesian equation of the locus of z, giving your answer in the form $ay^2 - bx^2 = k$, where a, b, k are constants.

QUESTION TWO

(a) The complex numbers u and w are represented on the Argand diagram below.



If $z = 2u + 3w$, find z, and clearly show it on the Argand diagram above.

(b) If $u = m\,\text{cis}\left(\frac{3\pi}{10}\right)$, write u^5 **in the form** $\boldsymbol{a + bi}$, where a and b are both real numbers, giving your answer in terms of m.

(c) Solve the following equation for m, where m is a real number.

$$|5 - mi| = \sqrt{5m^2}$$

(d) If $(g + 2i)(3 + hi) = (10 - 4i)(3 - i)$, then find all possible values of g and h.

(e) Given that $\arg\left(\dfrac{d+6\mathrm{i}}{1-d\mathrm{i}}\right)=\dfrac{\pi}{4}$, find the possible value(s) of d, where d is a real constant.

QUESTION THREE

(a) One solution of a quadratic equation, with real coefficients, is $x = 2 + \sqrt{p}\,\mathrm{i}$.

Find the quadratic equation, in terms of p, giving your answer in the form $ax^2 + bx + c = 0$.

(b) Expand and simplify $(\sqrt{3a} - \sqrt{12a}\,\mathrm{i})^2$, giving your answer in terms of a, where a is a real number.

(c) Solve the following equation for x.

$$(1 + 2\sqrt{x})(3 + 2\sqrt{x}) = 5 + 6\sqrt{x}$$

(d) One solution of the equation $3z^3 + pz^2 + qz - 8 = 0$ is $z = 1 + i$.

If p and q are both real, find the other two solutions of the equation, and the value of **both** p and q.

Question Three continues on the next page.

(e) Given that u and v are both complex numbers, solve the following pair of simultaneous equations, giving solutions in the form $a + b\text{i}$.

$$u\text{i} + 2v = 3$$
$$u + (1 - \text{i})v = 4$$

Show your working clearly.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER