

Tirohia te uhi o muri e kitea ai te
whakapākehātanga o tēnei uhi

3

L3-CALCMF



Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2025

TE PUKAPUKA MŌ NGĀ TURE TĀTAI ME NGĀ TŪTOHI
mō te 91577M, mō te 91578M, me te 91579M

Tirohia tēnei pukapuka hei whakaoti i ngā tūmahi o ngā Pukapuka Tūmahi me ngā Tuhiinga.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu,
kāore tētahi o aua whārangi i te takoto kau.

E ĀHEI ANA TŌ PUPURI KI TĒNEI PUKAPUKA HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE PĀNGARAU – HE TURE TĀTAI WHAI HUA

TE TAURANGI

Te Whārite Pūrua

Mēnā ko te $ax^2 + bx + c = 0$

$$\text{ko te } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ngā Pūkōaro

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Ngā tau tuatini

$$z = x + iy$$

$$= r \operatorname{cis} \theta$$

$$= r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy$$

$$= r \operatorname{cis}(-\theta)$$

$$= r(\cos \theta - i \sin \theta)$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2}$$

$$\theta = \arg z$$

$$\text{arā, ko te } \cos \theta = \frac{x}{r}$$

$$\text{ā, ko te } \sin \theta = \frac{y}{r}$$

Te Ture a De Moivre

Mēnā he tau tōpū te n , ko te

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$$

KO TE ĀHUAHANGA TAUNGA

Te Rārangi Tōtika

Te Whārite $y - y_1 = m(x - x_1)$

TE TUANAKI

Te Pārōnaki

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Te Pāwhaitua

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1} + c$ ($n \neq -1$)
$\frac{1}{x}$	$\ln x + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$

Te Pānga Tawhā

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

MATHEMATICS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Logarithms

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Complex numbers

$$z = x + iy$$

$$= r \operatorname{cis} \theta$$

$$= r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy$$

$$= r \operatorname{cis}(-\theta)$$

$$= r(\cos \theta - i \sin \theta)$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$$

$$\theta = \arg z$$

$$\text{where } \cos \theta = \frac{x}{r}$$

$$\text{and } \sin \theta = \frac{y}{r}$$

De Moivre's Theorem

If n is any integer, then

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$$

COORDINATE GEOMETRY

Straight Line

$$\text{Equation } y - y_1 = m(x - x_1)$$

CALCULUS

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1} + c$ ($n \neq -1$)
$\frac{1}{x}$	$\ln x + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$

Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

Te Ture Otinga

$$(f \cdot g)' = g \cdot f' + f \cdot g' \text{ mēnā rānei ko te } y = uv, \text{ ko te } \frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Te Ture Huawehe

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \text{ mēnā rānei ko te } y = \frac{u}{v}, \text{ ko te } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Ngā Pānga Hiato, ngā Ture Mekameka rānei

$$(f(g))' = f'(g) \cdot g'$$

$$\text{mēnā rānei ko te } y = f(u), \text{ ā, ko te } u = g(x), \text{ kāti, ko te } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

NGĀ TIKANGA TAU**Te Ture Taparara**

$$\int_a^b f(x) \, dx \approx \frac{1}{2} h [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{arā, ko te } h = \frac{b-a}{n}, \text{ ā, ko te } y_r = f(x_r)$$

Te Ture a Simpson

$$\int_a^b f(x) \, dx \approx \frac{1}{3} h [y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{arā, ko te } h = \frac{b-a}{n}, \text{ ā, ko te } y_r = f(x_r), \text{ ka mutu, he taurua te } n.$$

Product Rule

$$(f.g)' = g.f' + f.g' \text{ or if } y = uv \text{ then } \frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2} \text{ or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Composite Function or Chain Rule

$$(f(g))' = f'(g).g'$$

$$\text{or if } y = f(u) \text{ and } u = g(x) \text{ then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

NUMERICAL METHODS**Trapezium Rule**

$$\int_a^b f(x) \, dx \approx \frac{1}{2}h[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r)$$

Simpson's Rule

$$\int_a^b f(x) \, dx \approx \frac{1}{3}h[y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r) \text{ and } n \text{ is even.}$$

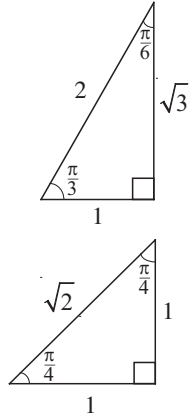
TE PĀKOKI

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Te Ture Aho**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Te Ture Whenu

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Ngā Tuakiri

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

Ngā Otinga Whānui

Mēnā ko te $\sin \theta = \sin \alpha$, ko te $\theta = n\pi + (-1)^n \alpha$

Mēnā ko te $\cos \theta = \cos \alpha$, ko te $\theta = 2n\pi \pm \alpha$

Mēnā ko te $\tan \theta = \tan \alpha$, ko te $\theta = n\pi + \alpha$

arā, he tau tōpū te n

Ngā Koki Pūhui

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Ngā Koki Rearua

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

Ngā Otinga

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Ngā Tapeke

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

TE INENGA**Te Tapatoru**

$$\text{Te horahanga} = \frac{1}{2} ab \sin C$$

Te Taparara

$$\text{Te horahanga} = \frac{1}{2} (a+b)h$$

Te Pewanga

$$\text{Te horahanga} = \frac{1}{2} r^2 \theta$$

$$\text{Te roa o te pewa} = r\theta$$

Te Rango

$$\text{Te rōrahi} = \pi r^2 h$$

$$\text{Te horahanga mata kōpiko} = 2\pi rh$$

Te Koeko

$$\text{Te rōrahi} = \frac{1}{3} \pi r^2 h$$

$$\text{Te horahanga mata kōpiko} = \pi rl$$

arā, ko l = te teitei o te mata ahu-3

Te Poi

$$\text{Te rōrahi} = \frac{4}{3} \pi r^3$$

$$\text{Te horahanga mata} = 4\pi r^2$$

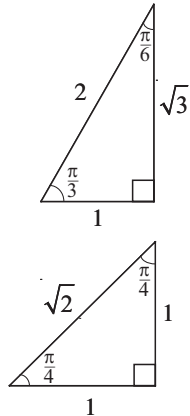
TRIGONOMETRY

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Sine Rule**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

General Solutions

$$\text{If } \sin \theta = \sin \alpha \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$\text{If } \cos \theta = \cos \alpha \text{ then } \theta = 2n\pi \pm \alpha$$

$$\text{If } \tan \theta = \tan \alpha \text{ then } \theta = n\pi + \alpha$$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Products

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

MEASUREMENT**Triangle**

$$\text{Area} = \frac{1}{2} ab \sin C$$

Trapezium

$$\text{Area} = \frac{1}{2} (a+b)h$$

Sector

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$\text{Arc length} = r\theta$$

Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi rh$$

Cone

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi rl \text{ where } l = \text{slant height}$$

Sphere

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$

English translation of the wording on the front cover



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New Zealand Qualifications Authority

Level 3 Calculus 2025

FORMULAE AND TABLES BOOKLET for 91577M, 91578M, and 91579M

Refer to this booklet to answer the questions in your Question and Answer Booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.