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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2025

91578 Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in the margins (⋮⋮⋮). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Differentiate $f(x) = (5x^3 - 2x + 1)^5$.

You do not need to simplify your answer.

(b) The temperature of an oven is given by the formula $C = \frac{60}{\sqrt{t}} + 5\sqrt{t} + 15$, given that $0 < t \leq 12$,

where C is the temperature of the oven, in °C, and t is the time, in minutes, after the oven has been switched off.

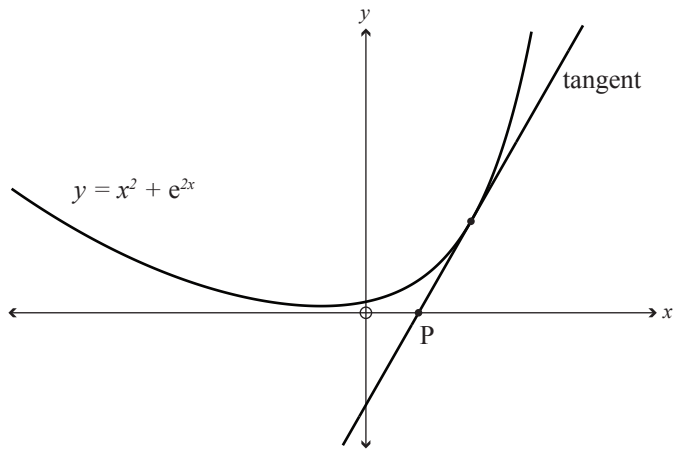
Find the rate of change of the temperature of the oven 4 minutes after the oven was switched off.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) What is the gradient of the **normal** to the curve when $y = \sin^3 x \cos x$ when $x = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) The graph below shows the function $f(x) = x^2 + e^{2x}$ and the tangent to the curve when $x = 1$.



Find the point P, the x-intercept of this tangent.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A curve is defined by the pair of parametric equations:

$x = 3t^2 + 6t + 1$ and $y = 2t^3$

Find the coordinates of any points of inflection on this curve that are also stationary points.

You may assume that any inflection point(s) found are actually inflection points.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION TWO

(a) Differentiate $f(x) = \frac{7}{x} - \ln(3x^3 - 2x^2 + 5)$

You do not need to simplify your answer.

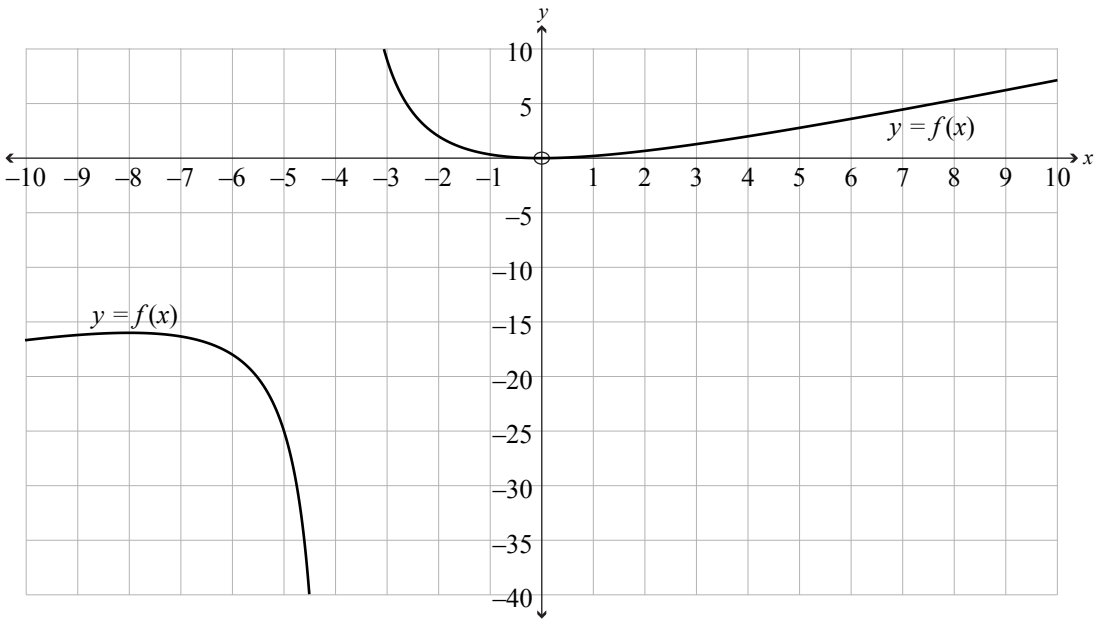
(b) The depth of water, in metres, in a particular harbour, is given by the formula

$$P = (30t - 5t^2)\mathrm{e}^{-t},$$

where t is measured in hours and $0 < t \leq 4$.

Show whether the depth of the water in the harbour is increasing or decreasing after 2 hours.
You must use calculus and show any derivatives that you need to find when solving this problem.

(c) There are two tangents to the curve $y = f(x) = \dfrac{x^2}{x+4}$ that have the equation $y = -3x + c$.



Find the coordinates of the two points of contact of these tangents with the curve $y = f(x)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Find the equation of the tangent to the graph defined by the pair of equations:

$x = 2\sec t$ and $y = 5\tan t$, at the point on the graph where $t = \frac{\pi}{6}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) The radius of curvature of a function is a measure of how much the curve is bending at a given point.

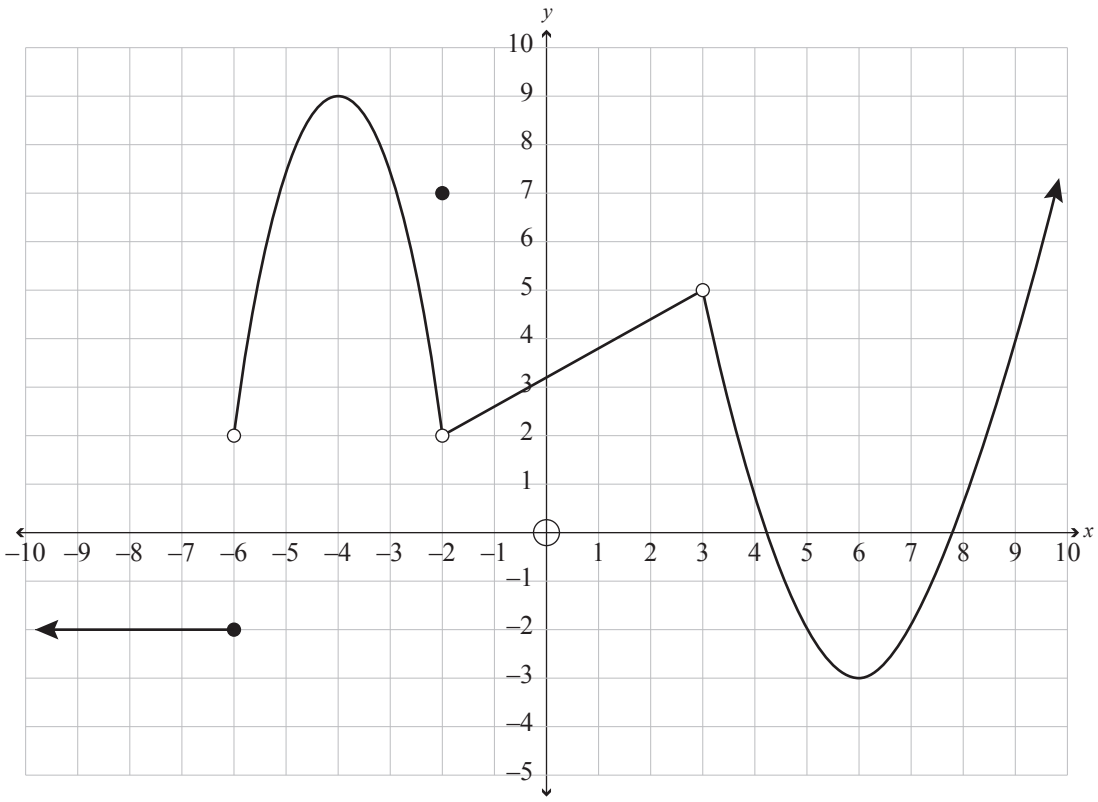
The formula to calculate the radius of curvature is given by $p = \dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$

Find the radius of curvature for the curve $y = \cos^2(2x)$ when $x = \dfrac{\pi}{3}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREE

(a) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x where $f(x)$ is not differentiable.

(ii) Find the value(s) of x where $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to -2} f(x)$?
State clearly if the value does not exist.

(b) A curve has equation $y = \dfrac{\ln x}{2x}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A solid spherical lump of ice is melting while maintaining its spherical shape.

At the instant when the radius is 6 cm, the radius of the ball of ice is decreasing by 0.05 cm s^{-1}.

Find the rate at which the volume of the spherical ice ball is decreasing at the instant when the radius is 6 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) The equation of a curve is given by the pair of parametric equations:

$$x = \frac{5}{e^{2t}} \text{ and } y = 5e^{2t}.$$

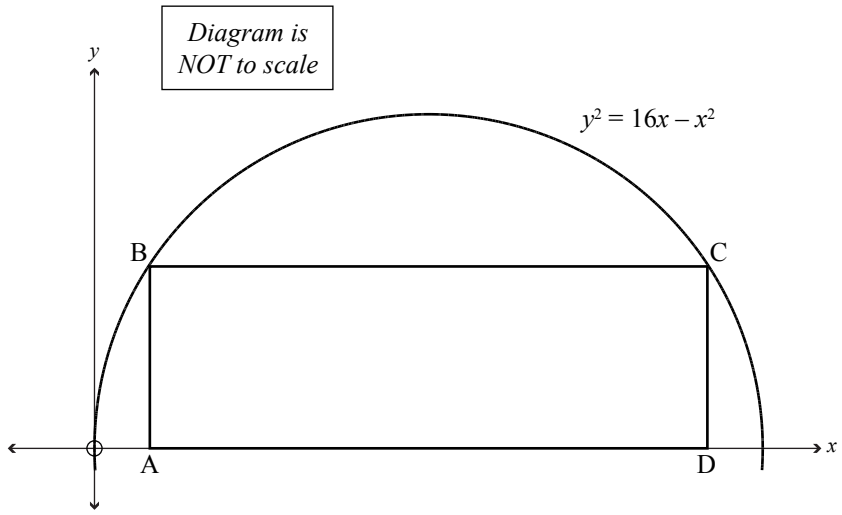
Find the **coordinates** of the point(s) on the curve where the gradient is –2.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the next page.

(e) The diagram below shows part of the symmetrical graph $y^2 = 16x - x^2$.

A rectangle ABCD is drawn inside the curve with its vertices B and C lying on the curve.



Find the **length AD** so that the rectangle has its maximum area.

You can assume that the value found is a maximum.

You must use calculus and show any derivatives that you need to find when solving this problem.

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