SUPERVISOR'S USE ONLY

Tirohia te uhi o muri e kitea ai te whakapākehātanga o tēnei uhi

3

91578M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2025

91578M Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe Te Pukapuka mō Ngā Ture Tātai me ngā Tūtohi L3–CALCMF.

Whakaaturia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki ngā paenga (✂✂✂). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria ngā pārōnaki o te $f(x) = (5x^3 - 2x + 1)^5$.

Ehara i te mea me whakarūnā koe i tō tuhinga.

(b) E tohua ana te paemahana o tētahi umu ki te ture tātai o te $C = \frac{60}{\sqrt{t}} + 5\sqrt{t} + 15$, mēnā ko te $0 < t \leq 12$,

arā, ko C te paemahana, ā-tohurau nei (°C), o te umu, ā, ko t te wā, ā-meneti nei, i muri i te whakawetonga o te umu.

Whiriwhiria te pāpātanga o te whiti o te paemahana o te umu e whā meneti i muri mai i te whakawetonga o te umu.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) Ko te aha te rōnaki o te **rārangi hāngai** ki te kōpiko i te wā ko te $y = \sin^3 x \cos x$, arā, ko te $x = \frac{\pi}{4}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION ONE

(a) Differentiate $f(x) = (5x^3 - 2x + 1)^5$.

You do not need to simplify your answer.

(b) The temperature of an oven is given by the formula $C = \frac{60}{\sqrt{t}} + 5\sqrt{t} + 15$, given that $0 < t \leq 12$,

where C is the temperature of the oven, in °C, and t is the time, in minutes, after the oven has been switched off.

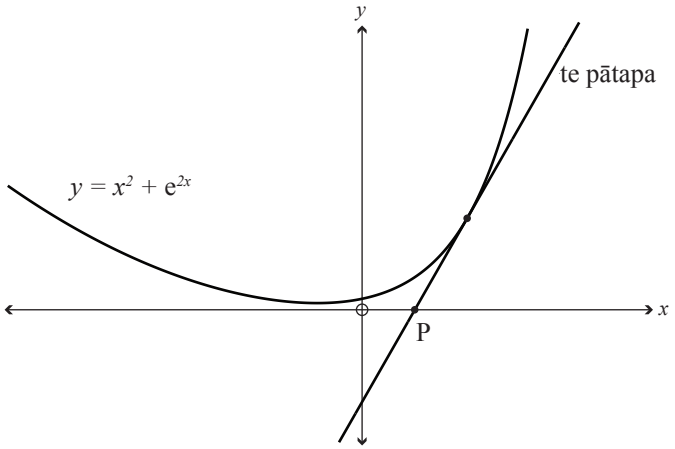
Find the rate of change of the temperature of the oven 4 minutes after the oven was switched off.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) What is the gradient of the **normal** to the curve when $y = \sin^3 x \cos x$ when $x = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

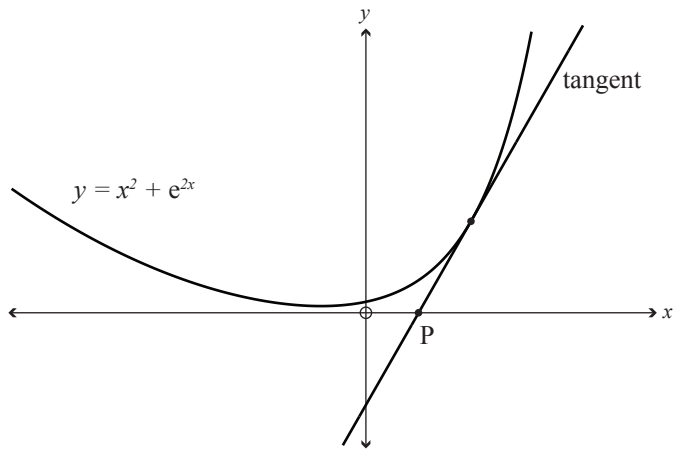
(d) E whakaatuhia ana i te kauwhata o raro nei te pānga o te $f(x) = x^2 + e^{2x}$ me te pātapa ki te kōpiko i te wā ko te $x = 1$.



Whiriwhiria te pūwāhi P, te haukotinga-*x* o tēnei pātapa.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) The graph below shows the function $f(x) = x^2 + e^{2x}$ and the tangent to the curve when $x = 1$.



Find the point P, the x-intercept of this tangent.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) E tautuhia ana tētahi kōpiko ki ngā whārite tawhā e rua:

Ko te $x = 3t^2 + 6t + 1$ me te $y = 2t^3$

Whiriwhiria ngā taunga o ngā pūwāhi tūpā i tēnei kōpiko he pūwāhi tūnoa hoki.

Me whakapono noa koe he pūwāhi tūpā mārika te/ngā pūwāhi tūpā e kitea ana e koe.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(e) A curve is defined by the pair of parametric equations:

$x = 3t^2 + 6t + 1$ and $y = 2t^3$

Find the coordinates of any points of inflection on this curve that are also stationary points.

You may assume that any inflection point(s) found are actually inflection points.

You must use calculus and show any derivatives that you need to find when solving this problem.

TE TŪMAHI TUARUA

(a) Whiriwhiria ngā pārōnaki o te $f(x) = \frac{7}{x} - \ln(3x^3 - 2x^2 + 5)$

Ehara i te mea me whakarūnā koe i tō tuhinga.

(b) Ko te hōhonu o te wai, ā-mita nei, i tētahi whanga, e tohua ana ki te ture tātai o te

$$P = (30t - 5t^2)\mathrm{e}^{-t},$$

arā, ka inea ā-hāoratia te t, ā, ko te $0 < t \leq 4$.

Whakaaturia te hōhonu haere, te pāpaku haere rānei o te wai i te whanga i muri i te 2 hāora.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION TWO

(a) Differentiate $f(x) = \frac{7}{x} - \ln(3x^3 - 2x^2 + 5)$

You do not need to simplify your answer.

(b) The depth of water, in metres, in a particular harbour, is given by the formula

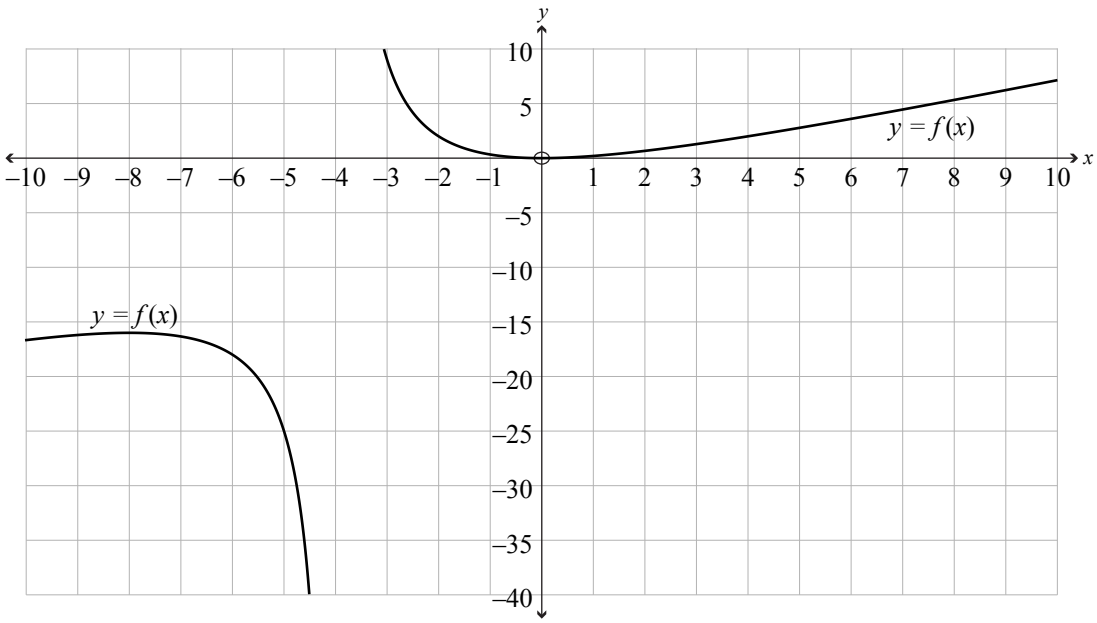
$$P = (30t - 5t^2)\mathrm{e}^{-t},$$

where t is measured in hours and $0 < t \leq 4$.

Show whether the depth of the water in the harbour is increasing or decreasing after 2 hours.

You must use calculus and show any derivatives that you need to find when solving this problem.

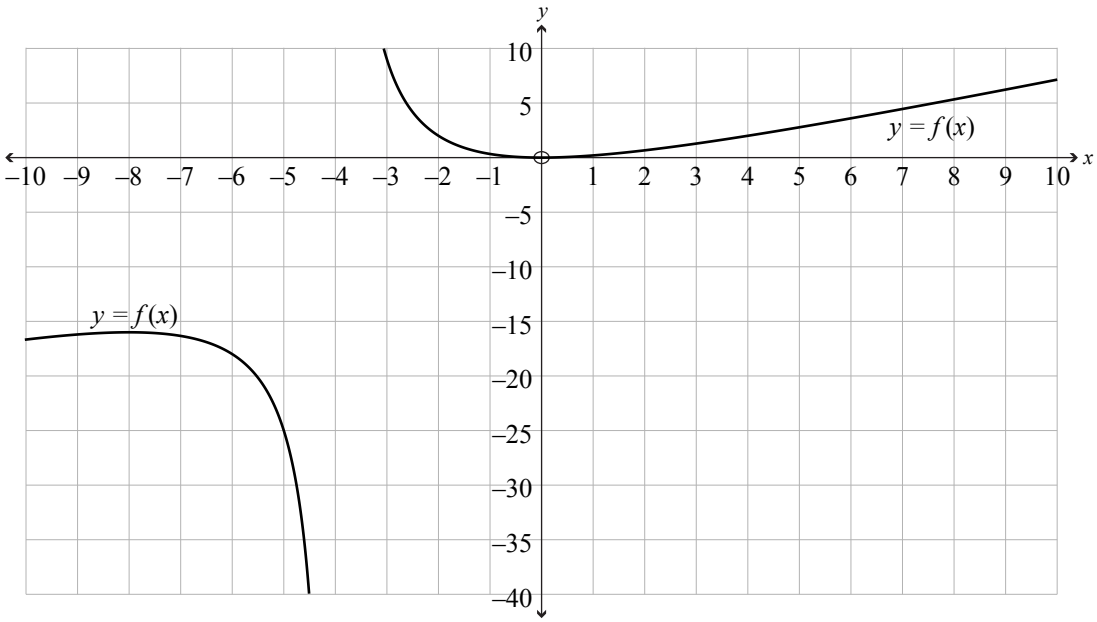
(c) E rua ngā pātapa ki te kōpiko $y = f(x) = \dfrac{x^2}{x+4}$ e whai ana i te whārite o te $y = -3x + c$.



Whiriwhiria ngā taunga o ngā pūwāhi pā e rua o ēnei pātapa e whai ana i te kōpiko $y = f(x)$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) There are two tangents to the curve $y = f(x) = \dfrac{x^2}{x+4}$ that have the equation $y = -3x + c$.



Find the coordinates of the two points of contact of these tangents with the curve $y = f(x)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Whiriwhiria te whārite o te pātapa ki te kauwhata e tautuhia ana ki ngā whārite e rua:
ki te $x = 2\sec t$, me te $y = 5\tan t$, i te pūwāhi o te kauwhata ko te $t = \frac{\pi}{6}$.
Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) Find the equation of the tangent to the graph defined by the pair of equations:
$x = 2\sec t$ and $y = 5\tan t$, at the point on the graph where $t = \frac{\pi}{6}$.
You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Ko te pūtoro o te kōpiko o tētahi pānga, he inenga nō te nui o te piko o te kōpiko i tētahi pūwāhi.

E tohua ana te ture hei tātai i te pūtoro o te kōpiko ki te $p = \dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$

Whiriwhiria te pūtoro o te kōpiko mō te kōpiko $y = \cos^2(2x)$ i te wā ko te $x = \dfrac{\pi}{3}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(e) The radius of curvature of a function is a measure of how much the curve is bending at a given point.

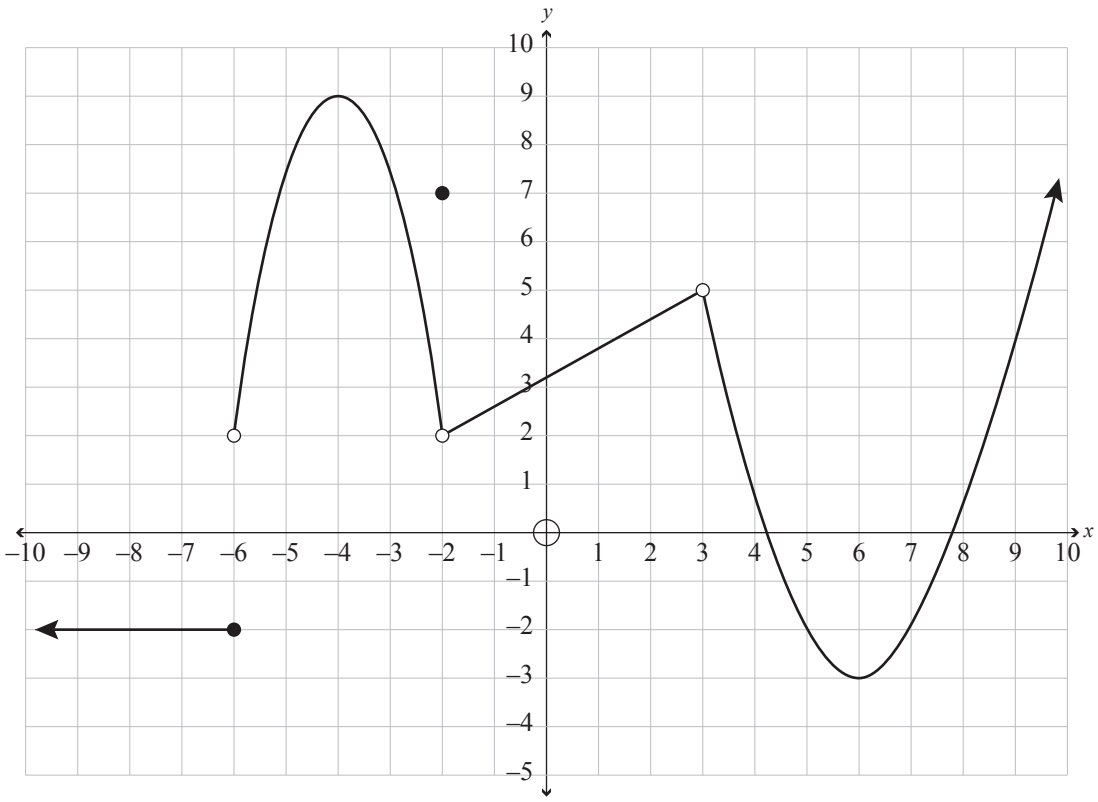
The formula to calculate the radius of curvature is given by $p = \dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$

Find the radius of curvature for the curve $y = \cos^2(2x)$ when $x = \dfrac{\pi}{3}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

TE TŪMAHI TUATORU

(a) E whakaatuhia ana i te kauwhata i raro nei te pānga o $y = f(x)$.



Mō te pānga kei runga nei:

(i) Whiriwhiria te/ngā uara o te x mēnā kāore e taea te whiriwhiri ngā pārōnaki o te $f(x)$.

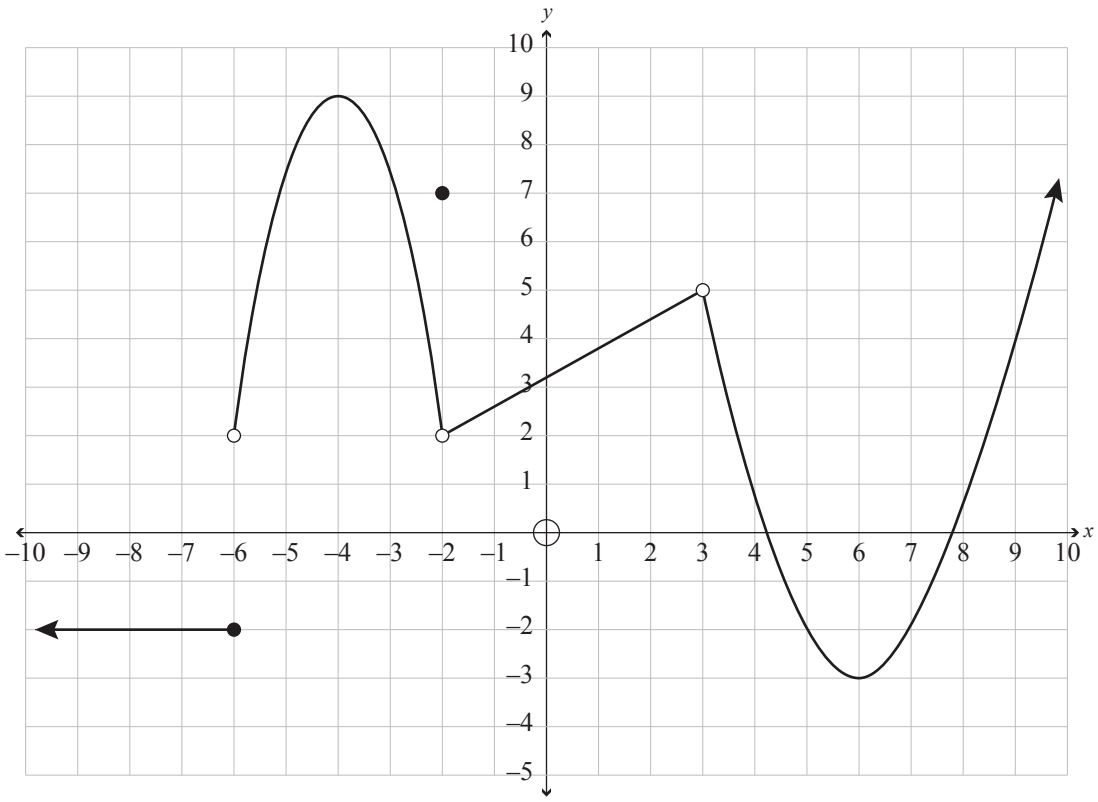
(ii) Whiriwhiria te/ngā uara o te x mēnā ko te $f'(x) = 0$.

(iii) Ko te aha te uara o te $\lim_{x \to -2} f(x)$?

Me āta whakatakoto koe i te kōrero mēnā karekau he uara.

QUESTION THREE

(a) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x where $f(x)$ is not differentiable.

(ii) Find the value(s) of x where $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to -2} f(x)$?
State clearly if the value does not exist.

(b) Ko te whārite o te kōpiko, ko te $y = \dfrac{\ln x}{2x}$.

Whiriwhiria te/ngā taunga-x o te/ngā pūwāhi tūnoa i te kōpiko.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) E rewa ana tētahi ponguru tio mārō, he poi te hanga, engari kei te mau tonu tōna hanga ā-poi.

I te wā ko te 6 cm te pūtoro, ko te 0.05 cm s^{-1} te hekenga haeretanga o te pūtoro o te poi tio.

Whiriwhiria te pāpātanga o te hekenga haeretanga o te rōrahi o te poi tio ka 6 cm ana te pūtoro.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(b) A curve has equation $y = \dfrac{\ln x}{2x}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A solid spherical lump of ice is melting while maintaining its spherical shape.

At the instant when the radius is 6 cm, the radius of the ball of ice is decreasing by 0.05 cm s^{-1}.

Find the rate at which the volume of the spherical ice ball is decreasing at the instant when the radius is 6 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) E tohua ana te whārite o tētahi kōpiko ki ngā whārite tawhā e rua:

Ko te $x = \dfrac{5}{e^{2t}}$, ā, ko te $y = 5e^{2t}$.

Whiriwhiria ngā **taunga** o te/ngā pūwāhi i te kōpiko mēnā ko te –2 te rōnaki.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(d) The equation of a curve is given by the pair of parametric equations:

$$x = \frac{5}{e^{2t}} \text{ and } y = 5e^{2t}.$$

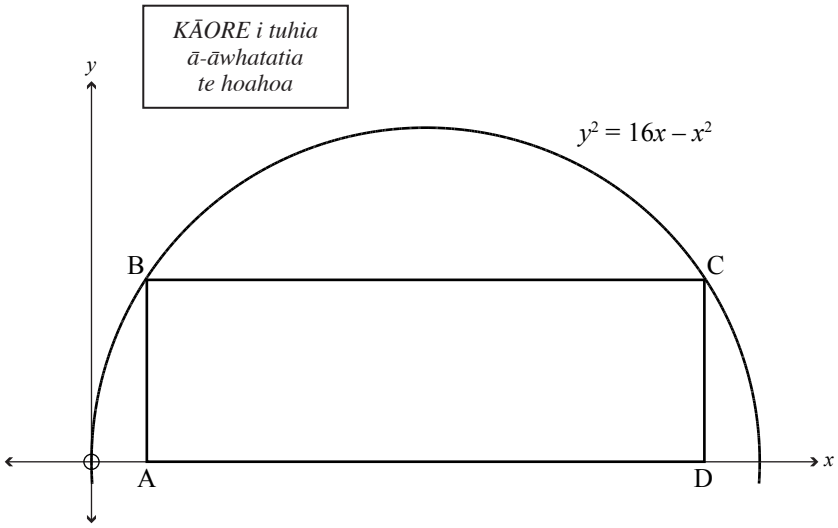
Find the **coordinates** of the point(s) on the curve where the gradient is –2.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the next page.

(e) E whakaatuhia ana i te hoahoa i raro nei tētahi wāhanga o te kauwhata hangarite $y^2 = 16x - x^2$.

Kua tuhia tētahi tapawhā hāngai o ABCD ki roto i te kōpiko, ā, ko ōna kokonga o B me C e takoto ana i runga i te kōpiko.



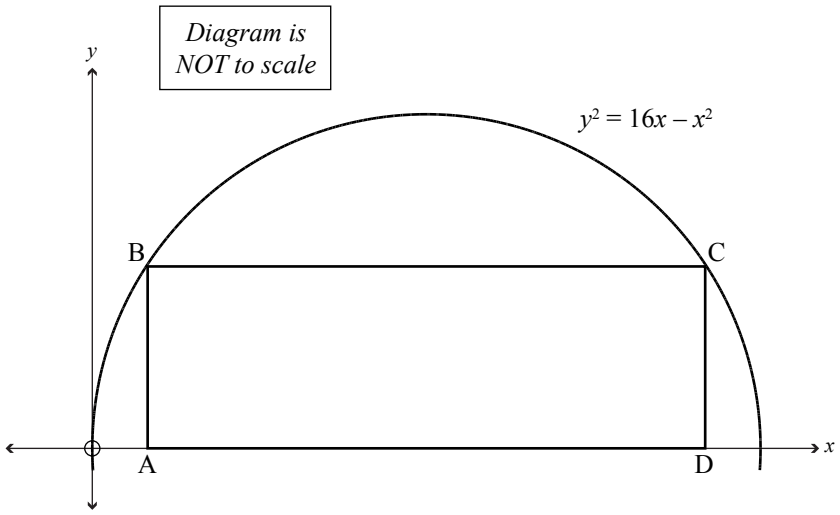
Whiriwhiria te **roa o AD** e whai ai te tapawhā hāngai i tōna horahanga mōrahi.

Me kī, he mōrahi te uara e kitea ana.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(e) The diagram below shows part of the symmetrical graph $y^2 = 16x - x^2$.

A rectangle ABCD is drawn inside the curve with its vertices B and C lying on the curve.



Find the **length AD** so that the rectangle has its maximum area.

You can assume that the value found is a maximum.

You must use calculus and show any derivatives that you need to find when solving this problem.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2025

91578M Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

91578M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in the margins (⁝⁝⁝). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.