

91579



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if you have NOT written in this booklet

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Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2025

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in the margins (✂✂✂). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

- (a) Find $\int 6 \sec(3x) \tan(3x) dx$.

- (b) Solve the differential equation $\frac{dy}{dx} = 3\sqrt{x} + \frac{2}{\sqrt{x}}$, given that $y = 10$ when $x = 4$.

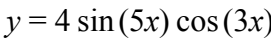
You must use calculus and show the results of any integration needed to solve the problem.

- (c) It is given that $\int_1^2 \left(a - \frac{6k}{x^2}\right) dx = 3$ and $\int_1^k 6x \, dx = a$, where a and k are constants.

Determine the possible value(s) of k .

You must use calculus and show the results of any integration needed to solve the problem.

- Diagram is NOT to scale*



You must use calculus and show the results of any integration needed to solve the problem.

- (e) Consider the differential equation $y - xy - (1 + x)\frac{dy}{dx} = 0$.

Given that $y = 3$ when $x = 0$, find the value(s) of y when $x = 2$.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION TWO

(a) Find $\int \frac{10}{(2x+1)^6} dx$.

(b) The rate of change of a particular function, p , is given by $\frac{dp}{dt} = 5 \cos(4t)$.

Find the function, given that $p = 8$ when $t = \frac{\pi}{24}$.

You must use calculus and show the results of any integration needed to solve the problem.

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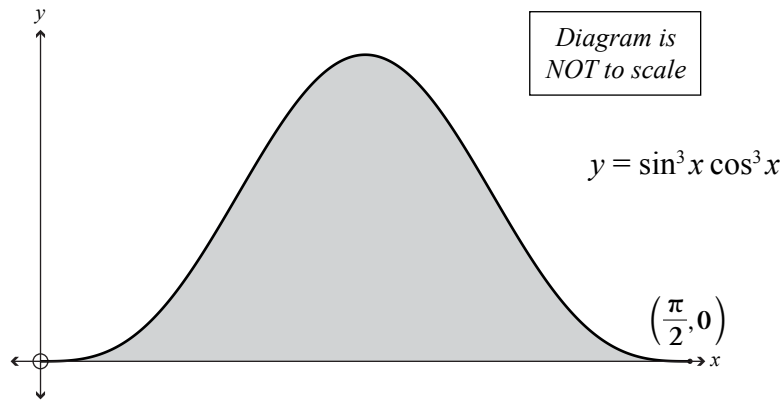
- (c) Find the value of the constant k , given that $\int_0^k \frac{1}{\sqrt{4x+1}} \, dx = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

- Initially, at a fixed point P, the particle had a velocity of 6 m s^{-1} .

You must use calculus and show the results of any integration needed to solve the problem.

- (e) The graph below shows the function $y = \sin^3 x \cos^3 x$.

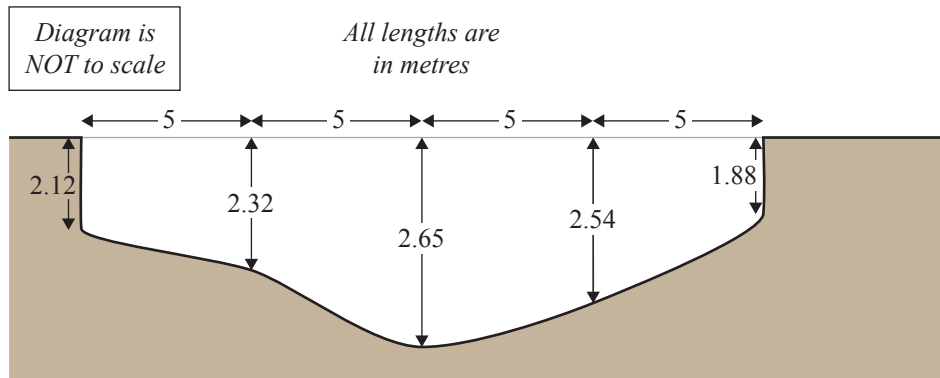


Find the shaded area under the curve between the values of $x = 0$ and $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

- (a) The diagram below shows the cross-section of a hole dug in the ground.
The depth of the hole is measured every 5 metres across the top of the hole.



Using the trapezium rule, find an estimate for the area of the cross-section of the hole.

- (b) Find $\int_1^k \frac{10}{2x-1} dx$, giving your answer in terms of k , where k is a constant and $k > 1$.

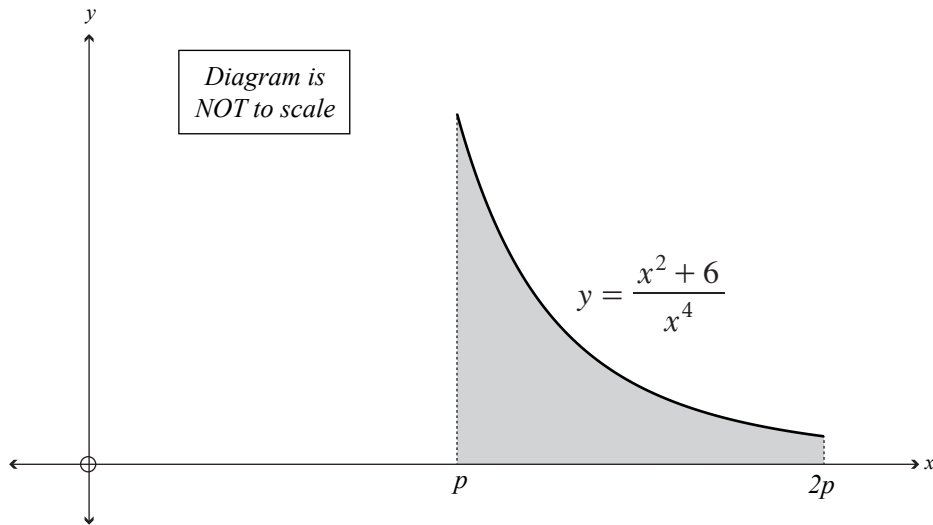
You must use calculus and show the results of any integration needed to solve the problem.

- (c) Consider the differential equation $\frac{dy}{dx} = \frac{\sqrt{4y+1}}{x^2}$.

Given that $y = 2$ when $x = \frac{2}{3}$, find the value(s) of y when $x = \frac{4}{5}$.

You must use calculus and show the results of any integration needed to solve the problem.

- (d) The graph below shows part of the curve $y = \frac{x+6}{x^4}$, where $x > 0$.



The area of the shaded region is $\frac{9}{4}$ units².

Prove that $9p^3 - 2p^2 - 7 = 0$.

You must use calculus and show the results of any integration needed to solve the problem.

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Murray has researched a way to make the picture balance by using the following formula

where $f(x)$ is the equation of the curved edge of the piece of art.

Use this formula to find the x -value of the hanging point.

**Extra space if required.
Write the question number(s) if applicable.**

QUESTION
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