SUPERVISOR'S USE ONLY

Tirohia te uhi o muri e kitea ai te whakapākehātanga o tēnei uhi

3

91579M

915795

Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2025

91579M Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka mō Ngā Ture Tātai me ngā Tūtohi L3–CALCMF.

Whakaaturia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki ngā paenga (⁄⁄⁄). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TŪMAHI TUATAHI

(a) Whiriwhiria te $\int 6\sec(3x)\tan(3x)\,\mathrm{d}x$.

(b) Whakaotihia te whārite pārōnaki o te $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sqrt{x} + \frac{2}{\sqrt{x}}$, mehemea ko te $y = 10$ i te wā ko te $x = 4$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION ONE

(a) Find $\int 6\sec(3x)\tan(3x)\,\mathrm{d}x$.

(b) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sqrt{x} + \frac{2}{\sqrt{x}}$, given that $y = 10$ when $x = 4$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Ko te $\int_1^2 \left(a - \frac{6k}{x^2}\right) \mathrm{d}x = 3$, ā, ko te $\int_1^k 6x \, \mathrm{d}x = a$, ka mutu, he pūmau te a me te k.

Whakatauria te/ngā momo uara o k e tika ana.

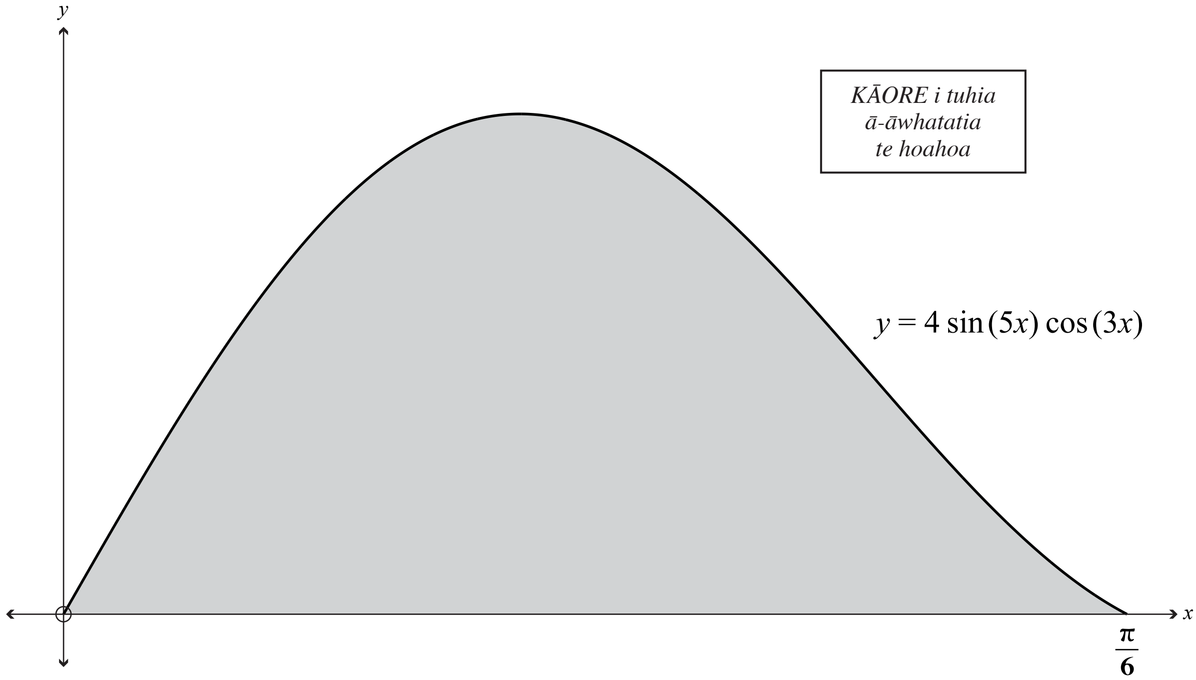
Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) It is given that $\int_1^2 \left(a - \frac{6k}{x^2}\right) \mathrm{d}x = 3$ and $\int_1^k 6x \, \mathrm{d}x = a$, where a and k are constants.

Determine the possible value(s) of k.

You must use calculus and show the results of any integration needed to solve the problem.

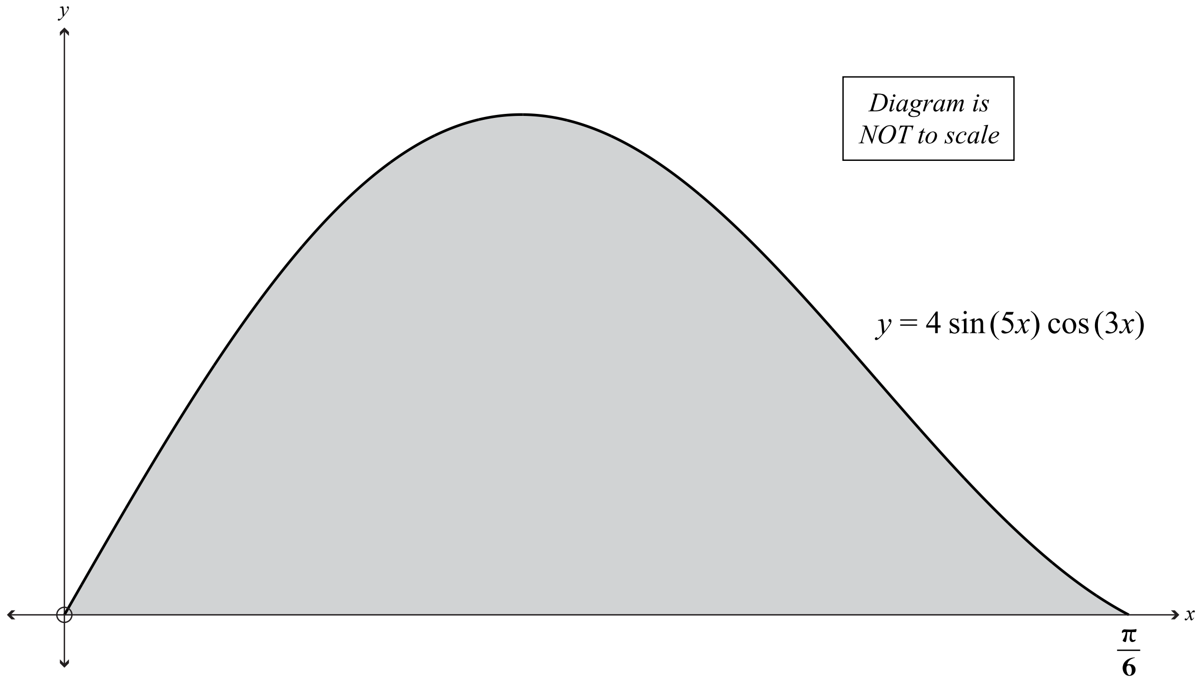
(d) E whakaatuhia ana i te kauwhata i raro nei tētahi wāhanga o te kauwhata o te pānga o te $y = 4\sin(5x)\cos(3x)$.



Whiriwhiria te horahanga o te wāhi kua kaurukutia i raro i te kōpiko i waenga i te $x = 0$ me te $x = \frac{\pi}{6}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) The graph below shows part of the graph of the function $y = 4\sin(5x)\cos(3x)$.



Find the shaded area under the curve between $x = 0$ and $x = \frac{\pi}{6}$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Whakaarohia te whārite pārōnaki o te $y - xy - (1 + x)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

Mehemea ko te $y = 3$ i te wā ko te $x = 0$, whiriwhiria te/ngā uara o te y i te wā ko te $x = 2$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) Consider the differential equation $y - xy - (1 + x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$.

Given that $y = 3$ when $x = 0$, find the value(s) of y when $x = 2$.

You must use calculus and show the results of any integration needed to solve the problem.

TE TŪMAHI TUARUA

(a) Whiriwhiria te $\int \frac{10}{(2x+1)^6} \mathrm{d}x$.

(b) Ko te pāpātanga o te whiti o tētahi pānga, o te p, e tohua ana ki te $\frac{\mathrm{d}p}{\mathrm{d}t} = 5\cos(4t)$.

Whiriwhiria te pānga, mehemea ko te $p = 8$ i te wā ko te $t = \frac{\pi}{24}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION TWO

(a) Find $\int \frac{10}{(2x+1)^6}\,\mathrm{d}x$.

(b) The rate of change of a particular function, p, is given by $\frac{\mathrm{d}p}{\mathrm{d}t} = 5\cos(4t)$.

Find the function, given that $p = 8$ when $t = \frac{\pi}{24}$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Whiriwhiria te uara o te pūmau o k, mehemea ko te $\int_0^k \frac{1}{\sqrt{4x+1}}\,\mathrm{d}x = 1$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) Find the value of the constant k, given that $\int_0^k \frac{1}{\sqrt{4x+1}}\,\mathrm{d}x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Ka taea te whakaterenga o tētahi korakora te whakatauira ki te whārite o te $a(t) = 2.4e^{-0.3t} - 1$,

arā, ko te $a(t)$ te whakaterenga o te korakora, hei m s^{-2}

ā, ko te t te wā ā-hēkona mai i te tīmatanga o te inenga o te wā.

I te tīmatanga, i te pūwāhi pūmau o P, ko te 6 m s^{-1} te tere whaiahu o te korakora.

E pēhea ana te tawhiti o te korakora i te pūwāhi o P e 3 hēkona i muri i te tīmatanga o te inenga o te wā?

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) A particle's acceleration can be modelled by the equation $a(t) = 2.4e^{-0.3t} - 1$,

where $a(t)$ is the acceleration of the particle, in m s^{-2}

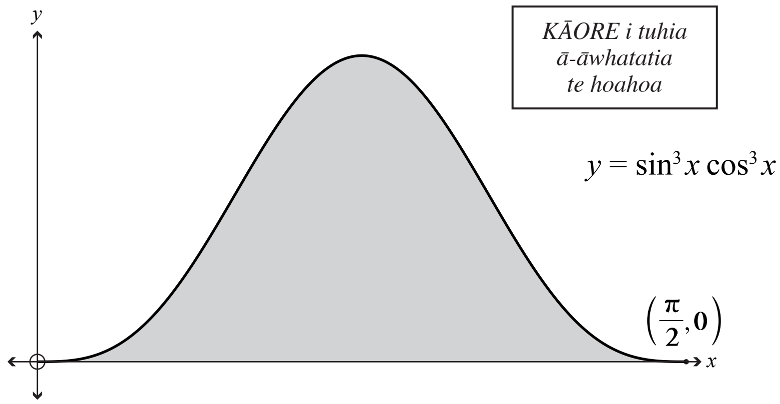
and t is the time, in seconds, from the start of timing.

Initially, at a fixed point P, the particle had a velocity of 6 m s^{-1}.

How far from the point P is the particle 3 seconds after timing started?

You must use calculus and show the results of any integration needed to solve the problem.

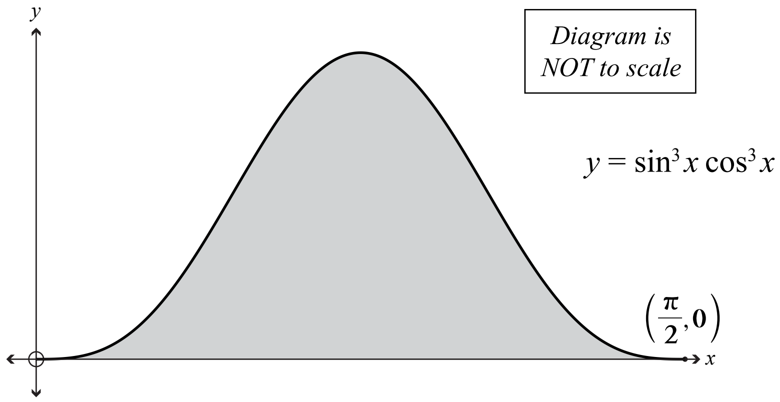
(e) E whakaatuhia ana i te kauwhata i raro nei te pānga o te $y = \sin^3 x \cos^3 x$.



Whiriwhiria te horahanga o te wāhi kua kaurukutia i raro i te kōpiko i waenga i te $x = 0$ me te $x = \frac{\pi}{2}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(e) The graph below shows the function $y = \sin^3 x \cos^3 x$.



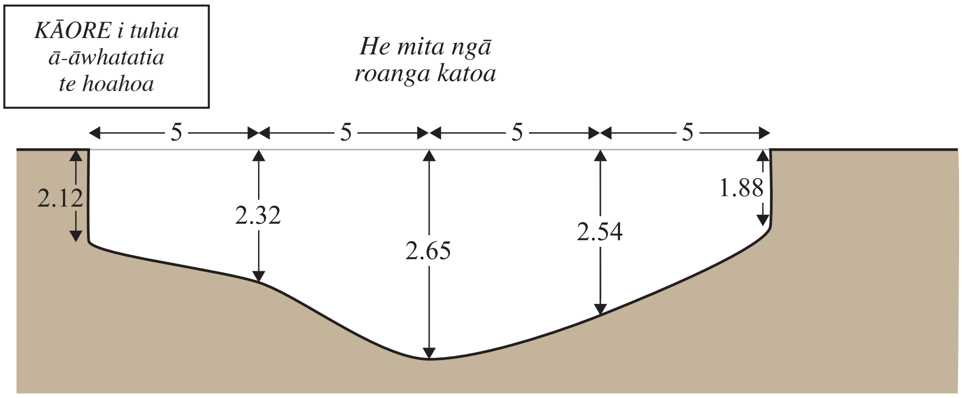
Find the shaded area under the curve between the values of $x = 0$ and $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

TE TŪMAHI TUATORU

(a) E whakaatuhia ana i te hoahoa i raro nei te motuhanga o tētahi rua kua keria i te whenua.

Kua inea te hōhonu o te rua i ia 5 mita mai i tētahi pito o te rua ki tērā atu pito.



Mā te whakamahi i te ture taparara, whiriwhiria tētahi whakataunga tata mō te horahanga o te motuhanga o te rua.

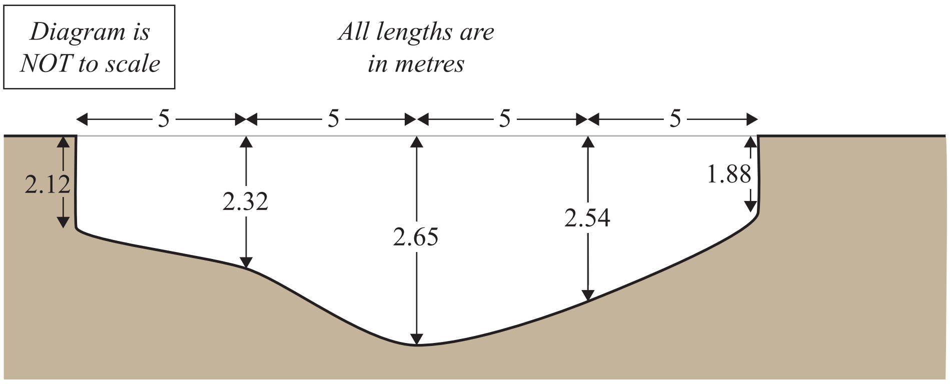
(b) Whiriwhiria te $\int_1^k \frac{10}{2x-1}\,dx$, mā te whakaatu i tō otinga e whai wāhi ai te k, otirā, he pūmau te k, ā, ko te $k > 1$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

QUESTION THREE

(a) The diagram below shows the cross-section of a hole dug in the ground.

The depth of the hole is measured every 5 metres across the top of the hole.



Using the trapezium rule, find an estimate for the area of the cross-section of the hole.

(b) Find $\int_{1}^{k} \frac{10}{2x-1}\,dx$, giving your answer in terms of k, where k is a constant and $k > 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Whakaarohia te whārite pārōnaki o te $\frac{dy}{dx} = \frac{\sqrt{4y+1}}{x^2}$.

Mehemea ko te $y = 2$ i te wā ko te $x = \frac{2}{3}$, whiriwhiria te/ngā uara o te y i te wā ko te $x = \frac{4}{5}$.

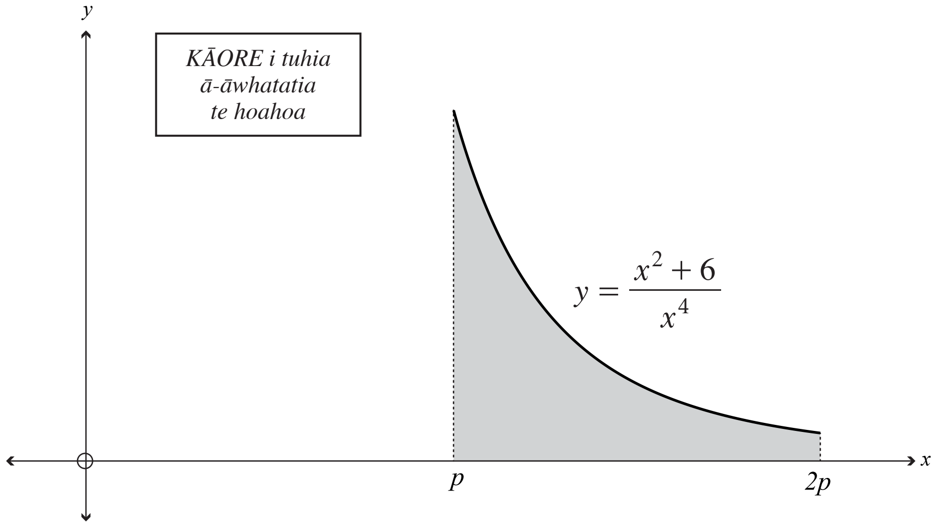
Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(c) Consider the differential equation $\frac{dy}{dx} = \frac{\sqrt{4y+1}}{x^2}$.

Given that $y = 2$ when $x = \frac{2}{3}$, find the value(s) of y when $x = \frac{4}{5}$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaatuhia ana i te kauwhata i raro nei tētahi wāhanga o te kōpiko o te $y = \dfrac{x^2 + 6}{x^4}$, mehemea ko te $x > 0$.

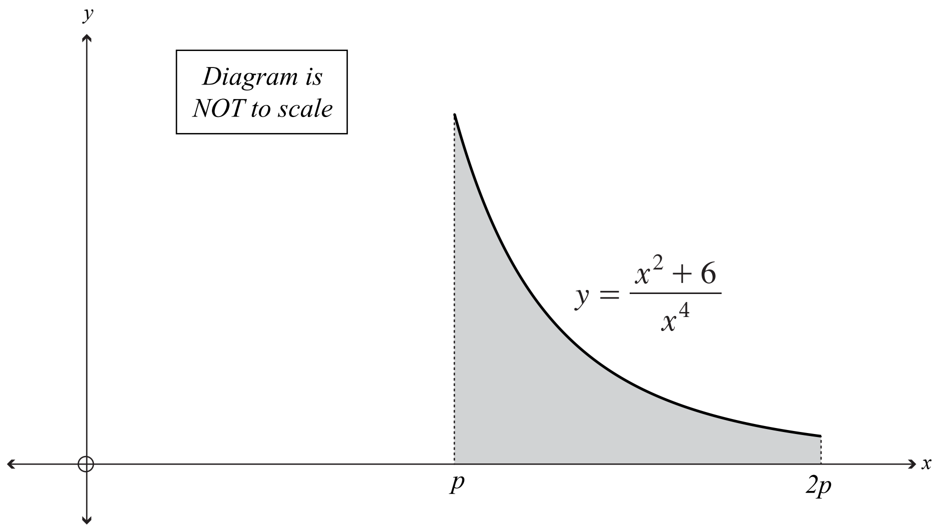


Ko te $\dfrac{9}{4}$ waeine² te horahanga o te wāhi kua kaurukutia.

Whakaaturia te $9p^3 - 2p^2 - 7 = 0$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

(d) The graph below shows part of the curve $y = \dfrac{x^2+6}{x^4}$, where $x > 0$.

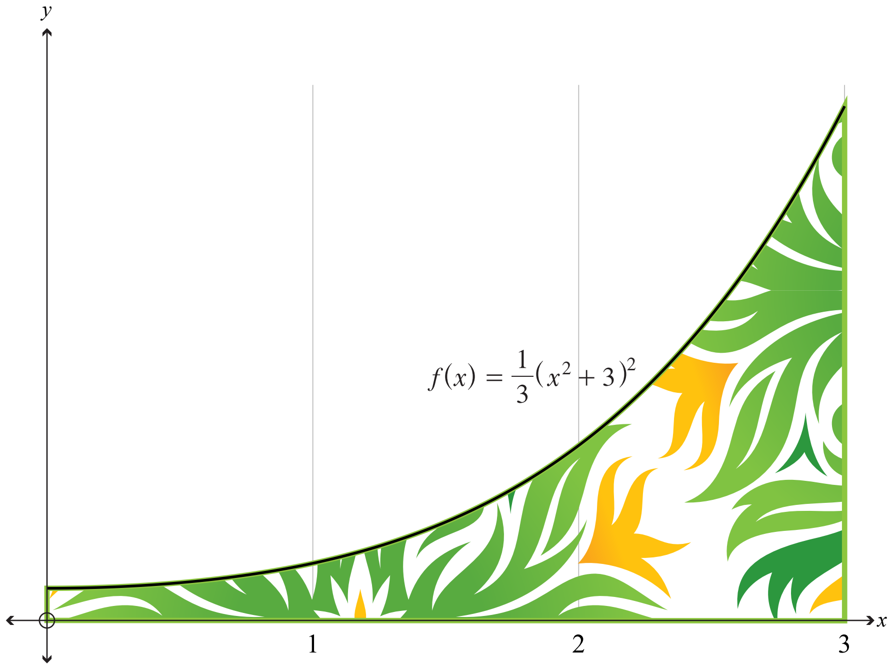


The area of the shaded region is $\dfrac{9}{4}$ units².

Prove that $9p^3 - 2p^2 - 7 = 0$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) E whakarite ana a Murray ki te whakairi i tētahi o ana mahi toi ki tētahi pakitara. E whakaatuhia ana tēnei i te hoahoa i raro nei.



Ko te whārite o te tapa kōpiko o te mahi toi, ko te $f(x) = \frac{1}{3}(x^2 + 3)^2$.

Kua rangahau a Murray i tētahi huarahi e taurite ai te whakairinga o te pikitia mā te whakamahi i te ture tātai e whai ake nei hei whiriwhiri i te uara-x o te pūwāhi whakairi: $\dfrac{\int_0^3 x\, f(x)\, \mathrm{d}x}{\int_0^3 f(x)\, \mathrm{d}x}$,

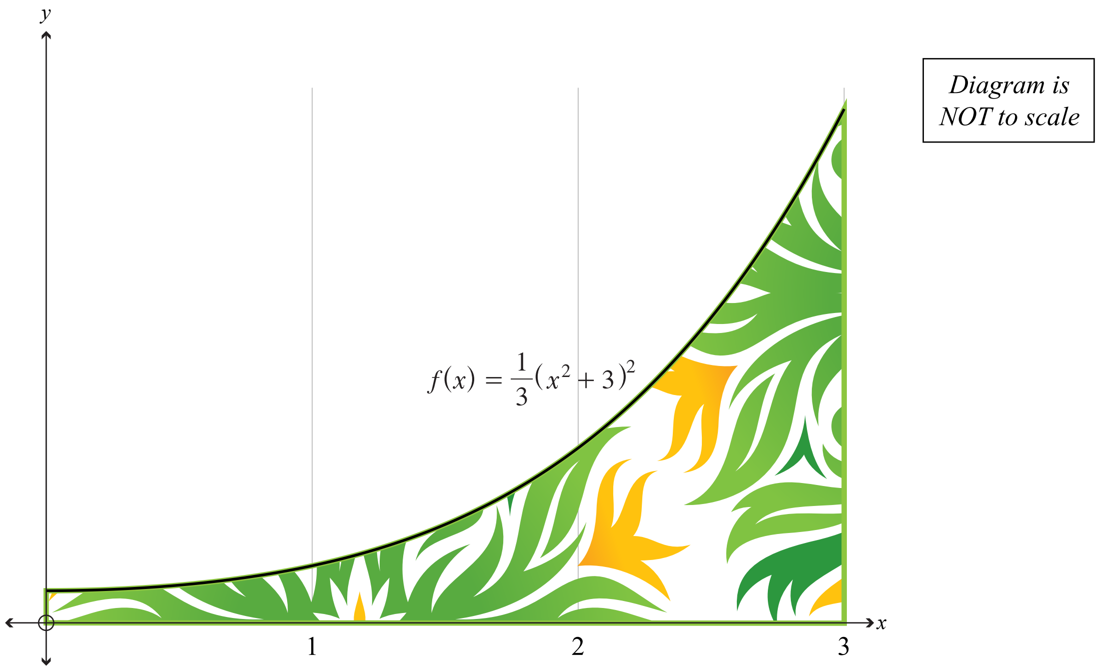
arā, ko te $f(x)$ te whārite o te tapa kōpiko o te mahi toi.

Whakamahia tēnei ture tātai hei whiriwhiri i te uara-x o te pūwāhi whakairi.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai e oti ai te rapanga.

E rere tonu ana te Tūmahi Tuatoru i te whārangi e whai ake ana.

(e) Murray is planning to hang a piece of his art on a wall. This is shown in the diagram below.



The equation of the curved edge of the piece of art is $f(x) = \frac{1}{3}(x^2 + 3)^2$.

Murray has researched a way to make the picture balance by using the following formula

to find the x-value of the hanging position: $\dfrac{\int_0^3 x\, f(x)\, \mathrm{d}x}{\int_0^3 f(x)\, \mathrm{d}x}$,

where $f(x)$ is the equation of the curved edge of the piece of art.

Use this formula to find the x-value of the hanging point.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the next page.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2025

91579M Apply integration methods in solving problems

Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in the margins ( ). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.