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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2026

91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in the margins (////). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

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QUESTION ONE

(a) If $u = k^{12}\,\text{cis}\left(\frac{2\pi}{3}\right)$ and $v = k^{3}\,\text{cis}\left(\frac{3\pi}{5}\right)$, write $\frac{u}{v}$ in the form $r\,\text{cis}\,\theta$.

(b) Given that $(x - k)$ is a factor of $f(x) = x^3 + x^2 - x + k$, find the value(s) of the **non-zero** constant k.

(c) The complex number z is defined by $z = \dfrac{p + 2\mathrm{i}}{p - \mathrm{i}}$, where p is a **positive**, real constant.

If the real part of z is $\dfrac{1}{2}$, then find the value(s) of $\boldsymbol{p}$.

(d) One solution of the equation $3z^3 + pz^2 + qz - 26 = 0$ is $z = 2 - 3\text{i}$.

Given that p and q are both real, find the other two solutions of the equation, and the values of **both** p and q.

(e) Given that u and v are two complex numbers, prove that $|u-v|^2 = |u|^2 + |v|^2 - 2Re(u\overline{v})$.

QUESTION TWO

(a) Rationalise the denominator of $\dfrac{k+4\sqrt{3}}{5-3\sqrt{3}}$, simplifying your answer as far as possible, giving your answer in terms of k.

(b) Given that p and q are real constants and $(3+\mathrm{i})(p-5\mathrm{i}) = 17-q\mathrm{i}$, find the values of p and q.

(c) Solve the following equation for x, giving your answer in terms of p and q:

$$p\sqrt{9x+4} - q\sqrt{4x-1} = 0$$

(d) Given that $\arg(u) = \frac{\pi}{10}$ and $\arg(v) = \frac{\pi}{5}$, prove that $u^{20}v^{25}$ is purely real.

(e) The equations of two loci are given by:

$|z-3| = |z-7-2\mathrm{i}|$ and $\arg(z-3-2\mathrm{i}) = \frac{\pi}{4}$.

Find the complex number z which lies on **both** loci.

QUESTION THREE

(a) Determine the nature of the roots of the equation $x^2 + (2k + 3)x + k^2 + 3k + 1 = 0$, where k is a positive real constant.

(b) Given that $z = 16 + k\text{i}$ and $w = 1 + 2k\text{i}$, where k is a **positive** real number, find the value(s) of k if zw is purely imaginary.

Question Three continues on the following page.

(c) Solve the equation $2z^3 - 128m^6\mathrm{i} = 0$, where m is a positive real constant.

Write your solution(s) in polar form, in terms of m.

(d) Find the Cartesian equation of the locus described by $z\overline{z} + 3(z + \overline{z}) = 16$, giving your answer in the form $(x + p)^2 + (y + q)^2 = r$

(e) Solve the equation for z, giving your answer in the form $z = a + b\mathrm{i}$.

$$\frac{2\overline{z}(1-2\mathrm{i})}{5z} + \frac{\mathrm{i}}{1+2\mathrm{i}} = \frac{2-3\mathrm{i}}{z}$$

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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