Tirohia te uhi o muri e kitea ai te whakapākehātanga o tēnei uhi

3

L3–CALCMF

993208

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2026

TE PUKAPUKA MŌ NGĀ TURE TĀTAI ME NGĀ TŪTOHI mō te 91577M, mō te 91578M me te 91579M

Tirohia tēnei pukapuka hei whakaoti i ngā tūmahi o ngā Pukapuka Tūmahi me ngā Tuhinga.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

E ĀHEI ANA TŌ PUPURI I TĒNEI PUKAPUKA HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

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Kāore e whakaaetia ana kia tāruahia tētahi paku wāhanga o tēnei whakamātautau ki te kore te Mana Tohu Mātauranga e mātua whakaae.

TE PĀNGARAU – HE TURE TĀTAI WHAI HUA

TE TAURANGI

Te Whārite Pūrua

Mēnā ko te $ax^2 + bx + c = 0$

ko te $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ngā Pūkōaro

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b\left(xy\right) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Ngā tau tuatini

$z = x + \mathrm{i}y$
$\quad = r\,\mathrm{cis}\,\theta$
$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\overline{z} = x - \mathrm{i}y$
$\quad = r\,\mathrm{cis}\,(-\theta)$
$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = \left|z\right| = \sqrt{z\overline{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

arā, ko te $\cos\theta = \dfrac{x}{r}$

ā, ko te $\sin\theta = \dfrac{y}{r}$

Te Ture a De Moivre

Mēnā he tau tōpū te n, ko te

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}\,(n\theta)$

KO TE ĀHUAHANGA TAUNGA

Te Rārangi Tōtika

Te Whārite $y - y_1 = m(x - x_1)$

TE TUANAKI

Te Pārōnaki

$y = f(x)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\,x$	$-\mathrm{cosec}\,x\,\cot x$
$\cot x$	$-\mathrm{cosec}^2 x$

Te Pāwhaitua

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\left\|x\right\| + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\left\|f(x)\right\| + c$

Te Pānga Tawhā

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

Te Ture Otinga

$(f.g)' = g.f' + f.g'$ mēnā rānei ko te $y = uv$, ko te $\frac{\mathrm{d}y}{\mathrm{d}x} = v\frac{\mathrm{d}u}{\mathrm{d}x} + u\frac{\mathrm{d}v}{\mathrm{d}x}$

Te Ture Huawehe

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$ mēnā rānei ko te $y = \frac{u}{v}$, ko te $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Ngā Pānga Hiato, ngā Ture Mekameka rānei

$\left(f\left(g\right)\right)' = f'\left(g\right).g'$

mēnā rānei ko te $y = f(u)$, ā, ko te $u = g(x)$, kāti, ko te $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}.\frac{\mathrm{d}u}{\mathrm{d}x}$

NGĀ TIKANGA TAU

Te Ture Taparara

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

arā, ko te $h = \frac{b-a}{n}$, ā, ko te $y_r = f(x_r)$

Te Ture a Simpson

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

arā, ko te $h = \frac{b-a}{n}$, ā, ko te $y_r = f(x_r)$, ka mutu, he taurua te n.

MATHEMATICS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b (xy) = \log_b x + \log_b y$

$\log_b \left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b (x^n) = n \log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex numbers

$z = x + \mathrm{i}y$
$\quad = r \operatorname{cis}\theta$
$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\overline{z} = x - \mathrm{i}y$
$\quad = r \operatorname{cis}(-\theta)$
$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\overline{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r \operatorname{cis}\theta)^n = r^n \operatorname{cis}(n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

Parametric Function

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x}$

Product Rule

$(f.g)' = g.f' + f.g'$ or if $y = uv$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = v\dfrac{\mathrm{d}u}{\mathrm{d}x} + u\dfrac{\mathrm{d}v}{\mathrm{d}x}$

Quotient Rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Composite Function or Chain Rule

$\big(f(g)\big)' = f'(g).g'$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u}.\dfrac{\mathrm{d}u}{\mathrm{d}x}$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\Big[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\Big]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\Big[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\Big]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$ and n is even.

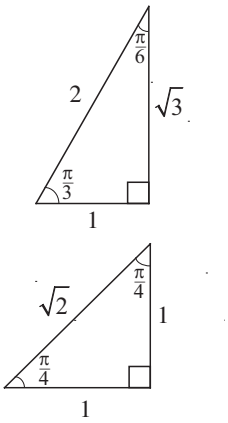
TE PĀKOKI

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$



Te Ture Aho

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Te Ture Whenu

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ngā Tuakiri

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

Ngā Otinga Whānui

Mēnā ko te $\sin\theta = \sin\alpha$, ko te $\theta = n\pi + (-1)^n\alpha$

Mēnā ko te $\cos\theta = \cos\alpha$, ko te $\theta = 2n\pi \pm \alpha$

Mēnā ko te $\tan\theta = \tan\alpha$, ko te $\theta = n\pi + \alpha$

arā, he tau tōpū te n

Ngā Koki Pūhui

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Ngā Koki Rearua

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Ngā Otinga

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Ngā Tapeke

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

TE INENGA

Te Tapatoru

Te horahanga $= \frac{1}{2}ab\sin C$

Te Taparara

Te horahanga $= \frac{1}{2}(a+b)h$

Te Pewanga

Te horahanga $= \frac{1}{2}r^2\theta$

Te roa o te pewa $= r\theta$

Te Rango

Te rōrahi $= \pi r^2 h$

Te horahanga mata kōpiko $= 2\pi rh$

Te Koeko

Te rōrahi $= \frac{1}{3}\pi r^2 h$

Te horahanga mata kōpiko $= \pi rl$

arā, ko $l =$ te teitei o te mata rōnaki

Te Poi

Te rōrahi $= \frac{4}{3}\pi r^3$

Te horahanga mata $= 4\pi r^2$

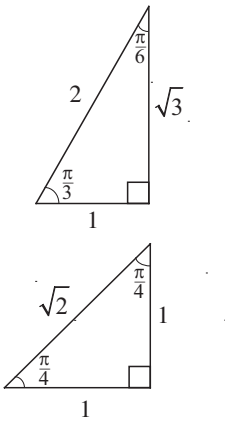
TRIGONOMETRY

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$



Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

MEASUREMENT

Triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Trapezium

$$\text{Area} = \frac{1}{2}(a+b)h$$

Sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Arc length $= r\theta$

Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

Cone

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Curved surface area $= \pi rl$ where l = slant height

Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Surface area $= 4\pi r^2$

English translation of the wording on the front cover

L3–CALCMF

Level 3 Calculus 2026

FORMULAE AND TABLES BOOKLET
for 91577M, 91578M, and 91579M

Refer to this booklet to answer the questions in your Question and Answer Booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

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