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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2026

91578 Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in the margins (////). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

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QUESTION ONE

(a) Differentiate $y = 5.25\,\text{cosec}\,(4x)$.

You do not need to simplify your answer.

(b) Find the gradient of the **normal** to the curve $y = \dfrac{3}{(x^2+2)^3}$ at the point $\left(1, \dfrac{1}{9}\right)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A function is given by $f(x) = \frac{a}{x} + b\sqrt{x} - 6$, where a and b are constants.
It is known that $f(x)$ has a local minimum at $(1, -3)$.

Find the value of the constants a and b.
You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Find the values of x for which the function $f(x) = \dfrac{16x^5 + 81k^5}{x^3}$ is increasing.

Give your answer in terms of k, where k is a **positive** constant.

Simplify your answer as far as possible.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Show that the function $y = (a + bx)e^{px}$ satisfies the differential equation

$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + p^2y = 0$, where a, b, p are all constants.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION TWO

(a) Differentiate $y = \dfrac{4x^3}{e^{4x}}$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \ln(3x^4 - 5x + 4)$ at the point where the curve crosses the y-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) An object is travelling in a straight line. Its displacement, $s(t)$, from a fixed point P, in metres, is given by $s(t) = 12\sqrt[3]{t} - 4\sqrt{t}$, where t is measured in seconds since the timing began, and $t > 1$.

Calculate when the object comes to rest.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) For a given function $f(x)$, it is known that its gradient function $f'(x) = (x^3 - 3x^2)(5x - 9)$.

Find the ***x*-value(s)** of any stationary points on the graph of this function, and **determine their nature**.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that any possible inflection point(s) found are actually inflection points.

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The assessment continues on the following page.

(e) Jaime is training for his surf club competition.

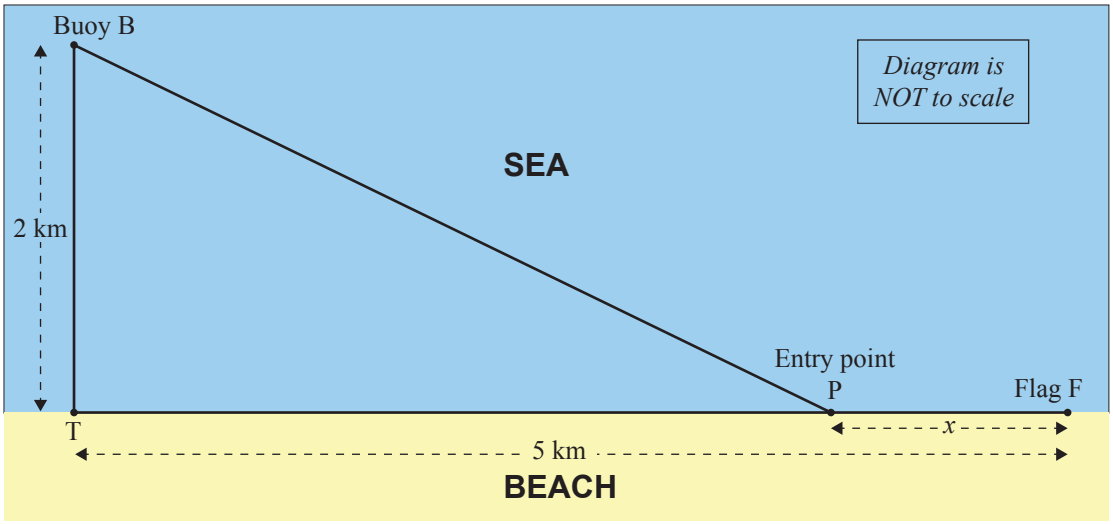
He starts from flag F, on the beach at the edge of the sea.

He runs straight along the beach and then enters the water at point P.

He then swims in a straight line to the buoy at B.

He knows that he is able to run twice as fast on the sand as he can swim in the sea.

The diagram below illustrates his race.



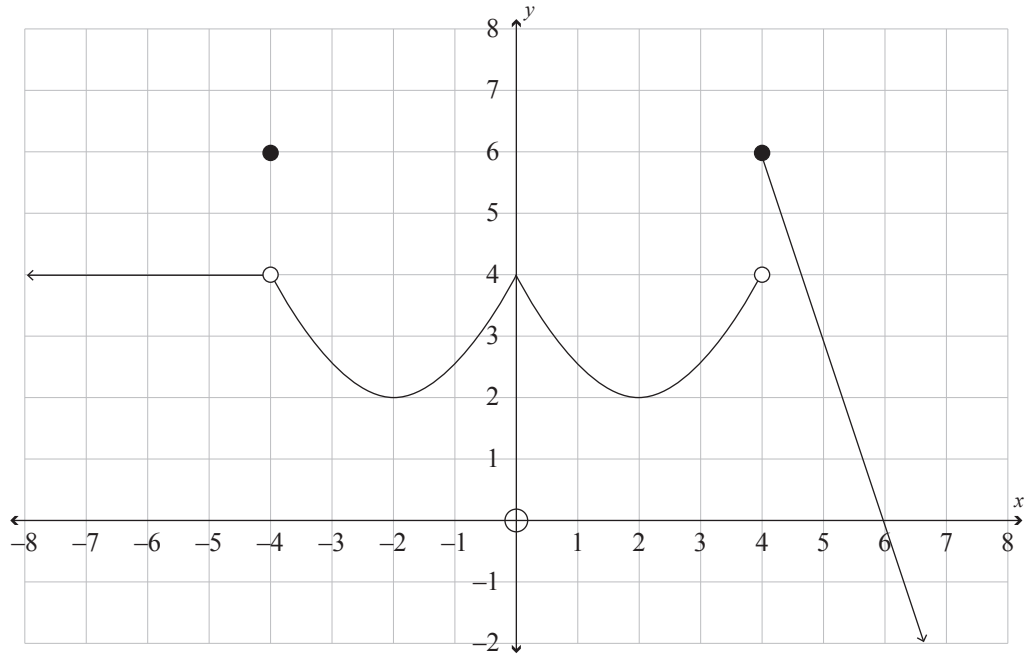
Jaime needs to decide how far he should run along the sand, distance FP, x, before entering the sea to achieve his minimum race time.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to show that the value that you find is actually the minimum.

QUESTION THREE

(a)



For the function above:

(i) Find the value(s) of x where $f(x)$ is not differentiable.

(ii) Find the value(s) of x where $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to 4} f(x)$?
State clearly if the value does not exist.

(b) Find the x-coordinate(s) of any stationary point(s) on the curve $f(x) = e^{2x} - 4x$.

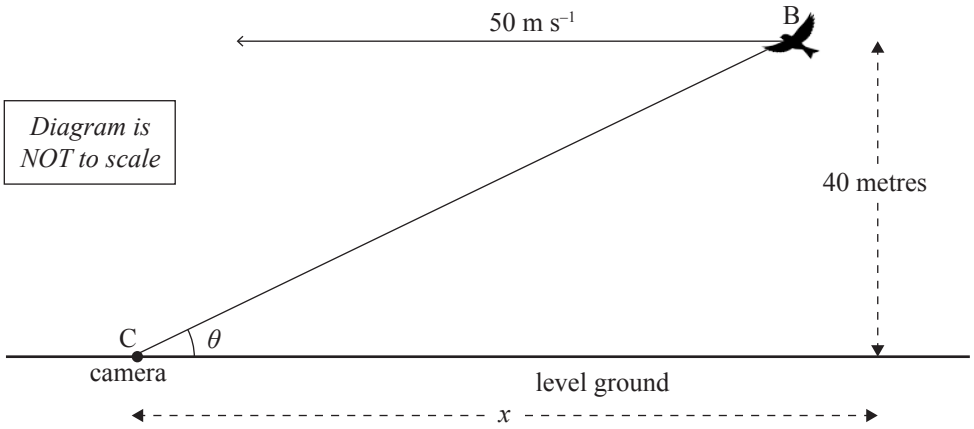
You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the equation of the tangent to the curve defined by the pair of parametric equations $x = 2\sqrt{2}\cos(3t)$ and $y = 3\sin(4t)$ at the point on the curve where $t = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the following page.

(d) A camera fixed on flat ground at point C is filming a bird at B flying horizontally, 40 metres above the ground, towards the camera. The bird is flying at a constant speed of 50 m s^{-1}.



Find the rate of increase of the angle of elevation, θ, between the camera and the bird when the angle of elevation is $\frac{\pi}{6}$ radians.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Show that the equation of the tangent to the curve $y = \ln(k - \sqrt{x})$ at its point of inflection is $y = -\frac{2}{k^2}x + \frac{1}{2} + \ln\left(\frac{k}{2}\right)$ where $\boldsymbol{k}$ is a positive constant.

You must use calculus and show any derivatives that you need to find when solving this problem.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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