SUPERVISOR'S USE ONLY

Tirohia te uhi o muri e kitea ai te whakapākehātanga o tēnei uhi

3

91578M

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Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2026

91578M Te whakahāngai i ngā tikanga whiriwhiri pārōnaki i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga whiriwhiri pārōnaki i te whakaoti rapanga.	Te whakahāngai i ngā tikanga whiriwhiri pārōnaki i te whakaoti rapanga, mā roto i te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pārōnaki i te whakaoti rapanga, mā roto i te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga ake o tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka o Ngā Ture Tātai me ngā Tūtohi L3–CALCFM.

Whakaaturia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–31 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki ngā paenga (////). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

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Kāore e whakaaetia ana kia tāruahia tētahi paku wāhanga o tēnei pukapuka ki te kore te Mana Tohu Mātauranga e mātua whakaae.

TE TŪMAHI TUATAHI

(a) Whiriwhiria te pārōnaki o $y = 5.25\,\text{cosec}\,(4x)$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) Whiriwhiria te rōnaki o te **rārangi hāngai** ki te kōpiko o $y = \dfrac{3}{(x^2+2)^3}$ i te pūwāhi o $\left(1, \dfrac{1}{9}\right)$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION ONE

(a) Differentiate $y = 5.25\,\text{cosec}\,(4x)$.

You do not need to simplify your answer.

(b) Find the gradient of the **normal** to the curve $y = \dfrac{3}{(x^2+2)^3}$ at the point $\left(1, \dfrac{1}{9}\right)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) E tohua ana te pānga ki te $f(x) = \frac{a}{x} + b\sqrt{x} - 6$, arā, he tau pūmau te a me te b.
E mōhiotia ana ko te whāruarua o $f(x)$ kei $(1, -3)$.

Whiriwhiria te uara o ngā tau pūmau o te a me te b.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) A function is given by $f(x) = \frac{a}{x} + b\sqrt{x} - 6$, where a and b are constants.
It is known that $f(x)$ has a local minimum at $(1, -3)$.

Find the value of the constants a and b.
You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Whiriwhiria ngā uara o x, i te wā e piki haere ana te pānga o $f(x) = \dfrac{16x^5 + 81k^5}{x^3}$.

Whakaaturia tō tuhinga nā runga i te k, arā, he tau pūmau **tōrunga** te k.

Me whakarūnā koe i tō tuhinga ki te mea rūnā katoa ka taea.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(d) Find the values of x for which the function $f(x) = \dfrac{16x^5 + 81k^5}{x^3}$ is increasing.

Give your answer in terms of k, where k is a **positive** constant.

Simplify your answer as far as possible.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Whakaaturia tā te pānga o $y = (a + bx)\mathrm{e}^{px}$ whakaea i te whārite pārōnaki

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2p\frac{\mathrm{d}y}{\mathrm{d}x} + p^2y = 0$, arā, he tau pūmau te a, te b, me te p.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(e) Show that the function $y = (a + bx)e^{px}$ satisfies the differential equation

$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + p^2y = 0$, where a, b, p are all constants.

You must use calculus and show any derivatives that you need to find when solving this problem.

TE TŪMAHI TUARUA

(a) Whiriwhiria te pārōnaki o $y = \dfrac{4x^3}{e^{4x}}$.

Kāore he take o te whakarūnā i tō tuhinga.

(b) Whiriwhiria te rōnaki o te pātapa ki te kōpiko o $y = \ln(3x^4 - 5x + 4)$ i te pūwāhi e pātahi ana te kōpiko ki te tuaka-y.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

QUESTION TWO

(a) Differentiate $y = \dfrac{4x^3}{e^{4x}}$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \ln(3x^4 - 5x + 4)$ at the point where the curve crosses the y-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) E haere tōtika ana tētahi mea. Ko tana peinga, $s(t)$, i tētahi pūwāhi pūmau, i te P, ā-mita nei, e tohua ana ki te $s(t) = 12\sqrt[3]{t} - 4\sqrt{t}$, arā, e inea ana te t ki ngā hēkona mai i te tīmatanga o te ine i te wā, me te $t > 1$.

Tātaihia te wā e tau ai te mea.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) An object is travelling in a straight line. Its displacement, $s(t)$, from a fixed point P, in metres, is given by $s(t) = 12\sqrt[3]{t} - 4\sqrt{t}$, where t is measured in seconds since the timing began, and $t > 1$.

Calculate when the object comes to rest.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Mō tētahi pānga kua tohua ki te $f(x)$, e mōhiotia ana e pēnei ana tōna pānga rōnaki:
$f'(x) = (x^3 - 3x^2)(5x - 9)$.

Whiriwhiria te/ngā **uara-*x*** o te/ngā pūwāhi tūnoa o te kauwhata o tēnei pānga, ā, **whakatauria hoki te āhua o aua pūwāhi**.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

Ehara i te mea me hāpono rawa e koe, ko te/ngā pūwāhi tūpā tērā pea ka kitea he pūwāhi tūpā motuhenga.

(d) For a given function $f(x)$, it is known that its gradient function $f'(x) = (x^3 - 3x^2)(5x - 9)$.

Find the ***x*-value(s)** of any stationary points on the graph of this function, and **determine their nature**.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that any possible inflection point(s) found are actually inflection points.

(e) E whakangungu ana a Jaime mō tētahi whakataetae i tana karapu kauhauora.

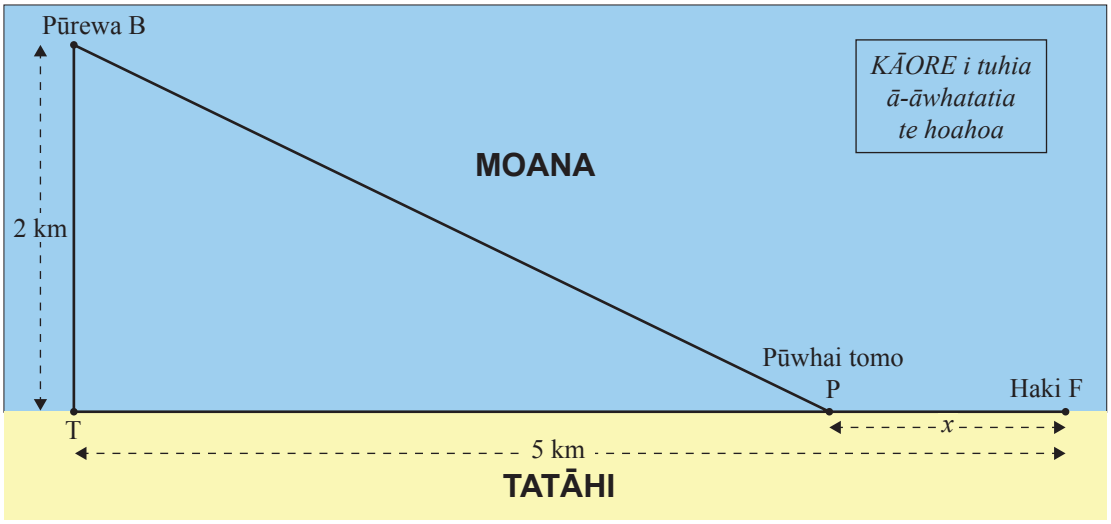
Ka tīmata ia i te haki F, i tātahi, i te tapa o te moana.

Ka oma tōtika ia i tātahi, ka kuhu ai ki te wai ki te pūwāhi o P.

Kātahi ka tōtika tana kauhoe ki te pūrewa kei B.

E mōhio ana ia ka rearua te tere ake o tana oma i te onepū, tēnā i tana kauhoe i te moana.

E whakaahuatia ana tana reihi ki te hoahoa kei raro nei.



Me mātua whakatau a Jaime i te tawhiti me oma e ia i te onepū, arā, ko te tawhiti o FP, x, i mua i tana kuhu ki te moana e poto katoa ai te roa o tana reihi.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

Ehara i te mea me whakaatu rawa e koe, ko te uara i whiriwhirihia ai e koe, ko te mōkito tonu.

(e) Jaime is training for his surf club competition.

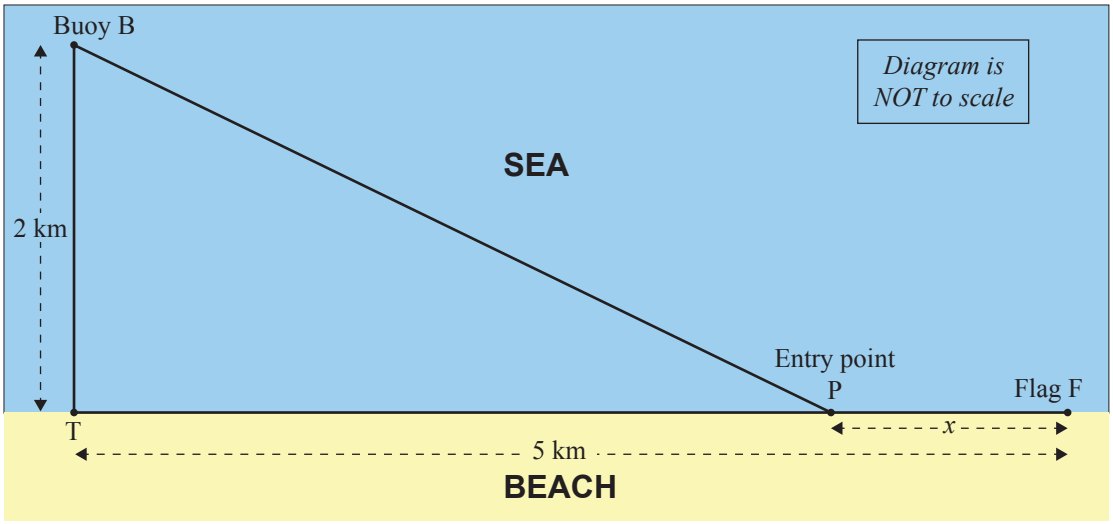
He starts from flag F, on the beach at the edge of the sea.

He runs straight along the beach and then enters the water at point P.

He then swims in a straight line to the buoy at B.

He knows that he is able to run twice as fast on the sand as he can swim in the sea.

The diagram below illustrates his race.



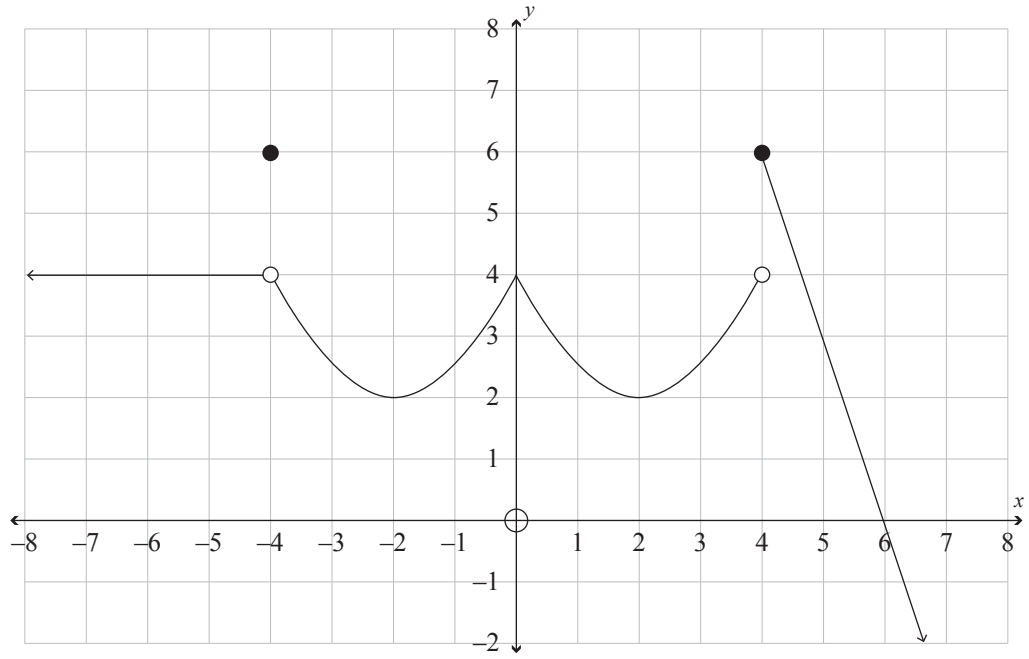
Jaime needs to decide how far he should run along the sand, distance FP, x, before entering the sea to achieve his minimum race time.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to show that the value that you find is actually the minimum.

TE TŪMAHI TUATORU

(a)



Mō te pānga kei runga nei:

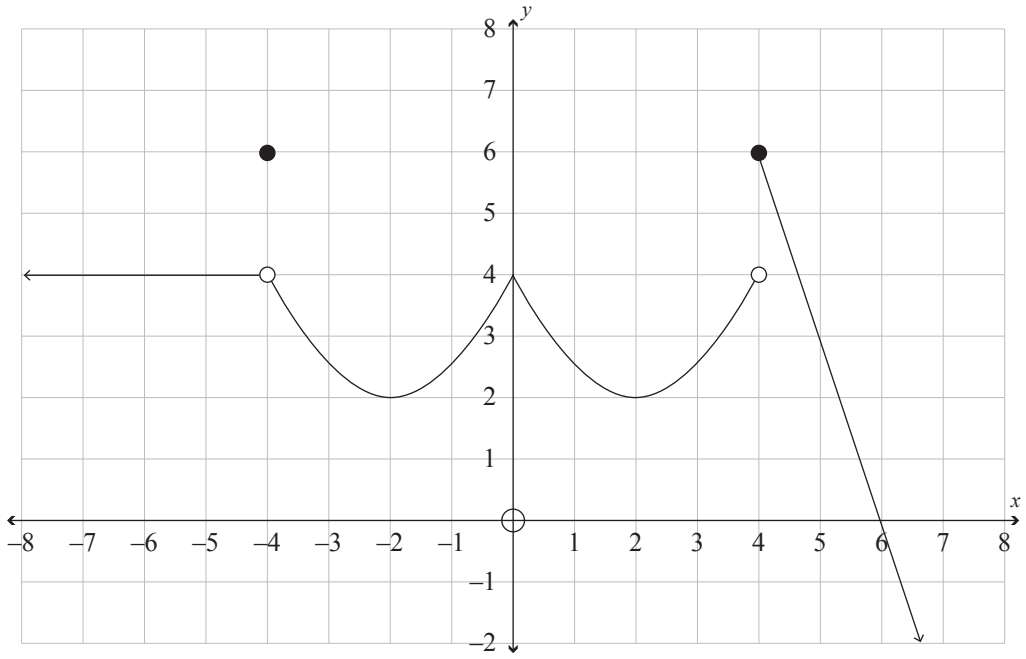
(i) Whiriwhiria te/ngā uara o x , arā, kāore e taea ana ngā pārōnaki o $f(x)$ te whiriwhiri.

(ii) Whiriwhiria te/ngā uara o x , arā, ko te $f'(x) = 0$.

(iii) Ko te aha te uara o te $\lim_{x \to 4} f(x)$?
Me āta whakamārama mehemea karekau he uara.

QUESTION THREE

(a)



For the function above:

(i) Find the value(s) of x where $f(x)$ is not differentiable.

(ii) Find the value(s) of x where $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to 4} f(x)$?
State clearly if the value does not exist.

(b) Whiriwhiria te/ngā taunga-x o te/ngā pūwāhi tūnoa i te kōpiko o $f(x) = e^{2x} - 4x$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(c) Whiriwhiria te whārite o te pātapa ki te kōpiko e tohua ana ki ngā whārite tawhā e rua nei: $x = 2\sqrt{2}\cos(3t)$ me $y = 3\sin(4t)$ i te pūwāhi o te kōpiko, arā, ko te $t = \frac{\pi}{4}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

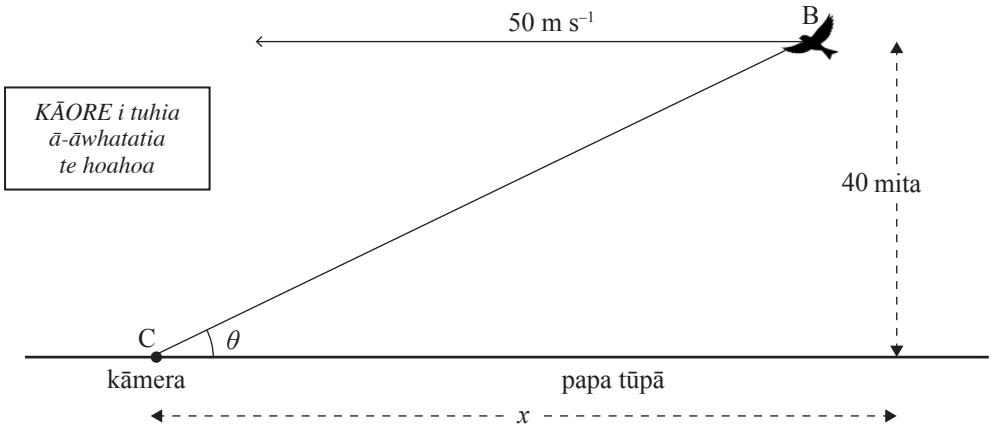
(b) Find the x-coordinate(s) of any stationary point(s) on the curve $f(x) = e^{2x} - 4x$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the equation of the tangent to the curve defined by the pair of parametric equations $x = 2\sqrt{2}\cos(3t)$ and $y = 3\sin(4t)$ at the point on the curve where $t = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Kua mau tētahi kāmera ki te papa paparahi i te pūwāhi o C e whakaahua ana i tētahi manu e rere huapae ana i te pūwāhi o B, e 40 mita i runga ake o te papa, ki te kāmera. Ko te tere pūmau o te rere a te manu, ko te 50 m s^{-1}.

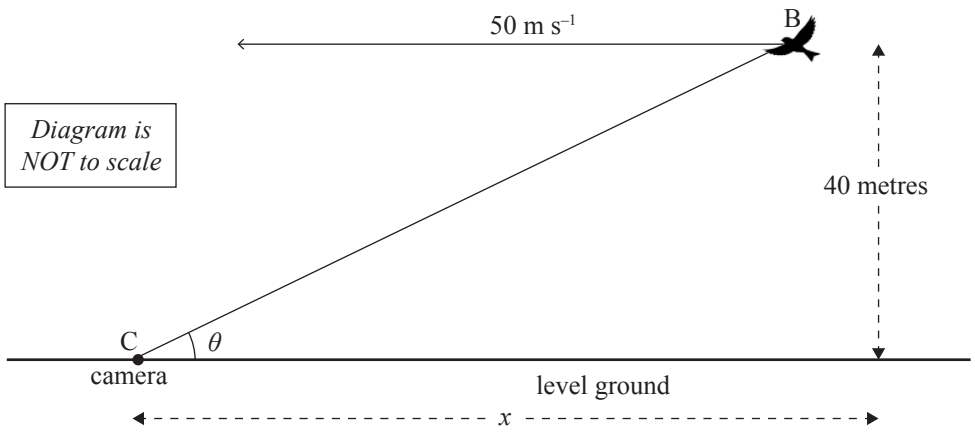


Whiriwhiria te pāpātanga o te whakapikitanga o te koki rewa, θ, i waenga i te kāmera me te manu, i te wā ko te $\frac{\pi}{6}$ tātoro te koki rewa.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

Ka rere tonu te Tūmahi Tuatoru i te whārangi e whai ake nei.

(d) A camera fixed on flat ground at point C is filming a bird at B flying horizontally, 40 metres above the ground, towards the camera. The bird is flying at a constant speed of 50 m s^{-1}.



Find the rate of increase of the angle of elevation, θ, between the camera and the bird when the angle of elevation is $\frac{\pi}{6}$ radians.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the following page.

(e) Whakaaturia ko te whārite o te pātapa ki te kōpiko o $y = \ln(k - \sqrt{x})$ i te pūwāhi tūpā, ko $y = -\frac{2}{k^2}x + \frac{1}{2} + \ln\left(\frac{k}{2}\right)$, arā, he tau pūmau tōrunga te $\boldsymbol{k}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu rawa hoki i ngā pārōnaki me mātua whiriwhiri i te wā e whakaotihia ana tēnei rapanga.

(e) Show that the equation of the tangent to the curve $y = \ln(k - \sqrt{x})$ at its point of inflection is $y = -\frac{2}{k^2}x + \frac{1}{2} + \ln\left(\frac{k}{2}\right)$ where $\boldsymbol{k}$ is a positive constant.

You must use calculus and show any derivatives that you need to find when solving this problem.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2026

91578M Apply differentiation methods in solving problems

Credits: Six

91578M

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–31 in the correct order and that none of these pages is blank.

Do not write in the margins (////). This area will be cut off when the booklet is marked.

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