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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2026

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in the margins (////). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

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QUESTION ONE

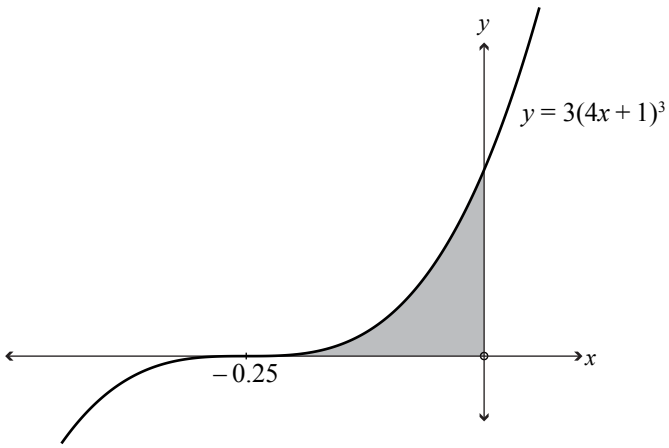
(a) Find $\int \left(5e^{3x} - \frac{5}{x^3} \right) dx$.

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(b) The graph below shows the function $y = 3(4x + 1)^3$.

Find the shaded area between the values of $x = -0.25$ and $x = 0$.

You must use calculus and show the results of any integration needed to solve the problem.



(c) Consider the differential equation $\frac{dy}{dx} = (3y - 2)\cos(3x)$.

Given that $y = 1$ when $x = \frac{\pi}{6}$, find the value of y when $x = \frac{5\pi}{18}$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) An object's acceleration can be modelled by the equation $a(t) = 32\cos(5t)\sin(3t)$, where $\boldsymbol{a}$ is the acceleration of the object in m s^{-2} and $\boldsymbol{t}$ is the time in seconds since the start of timing.

The initial velocity of the object was 10 m s^{-1}.

Calculate the velocity of this object $\frac{\pi}{8}$ seconds after the start of timing.

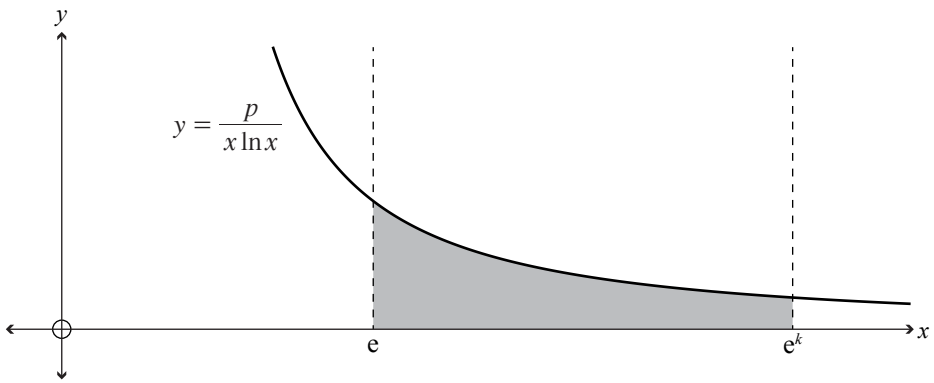
You must use calculus and show the results of any integration needed to solve the problem.

(e) The graph below shows the function $y = \dfrac{p}{x \ln x}$.

The diagram below shows the shaded area between the x-axis, the lines $x = \text{e}$ and $x = \text{e}^k$, and the curve $y = \dfrac{p}{x \ln x}$, where k and p are both positive constants and $k > 1$.

Prove that the shaded area is $p \ln k$.

You must use calculus and show the results of any integration needed to solve the problem.



QUESTION TWO

(a) Find $\int (\text{cosec}^2(4x) - \text{cosec}\,x\,\cot x)\,\text{d}x$.

(b) Solve the differential equation $\dfrac{\text{d}y}{\text{d}x} = \dfrac{2x^3 + 4x^2 - x}{x^2}$ given that $y = 4$ when $x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Find the value of the constant p, given that $\int_2^p (6e^{3x-6})dx = 8$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Evaluate $\int_0^{\frac{\pi}{6}} \frac{10\cos x}{(5+\sin x)^2} dx$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Consider the differential equation $(1+\mathrm{e}^{2v})\dfrac{\mathrm{d}v}{\mathrm{d}t} = 2\mathrm{e}^{v}sin(2t)$.

Given that $v = 0$ when $t = \dfrac{\pi}{6}$, **prove** that $v = \ln 2$ when $t = \dfrac{3\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

QUESTION THREE

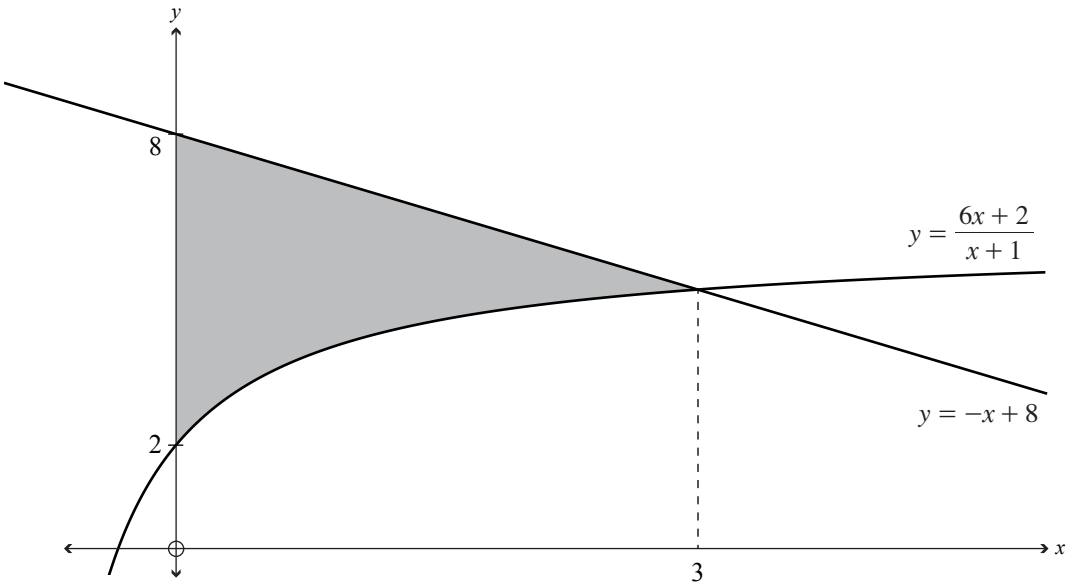
(a) Find $\int \frac{20x}{5x^2 - 1} \mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_1^4 f(x)\mathrm{d}x$ using Simpson's Rule.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	12	15	21	22	18	15	14

(c) Find the shaded area enclosed by the lines $y = \dfrac{6x + 2}{x + 1}$, $y = -x + 8$, and the y-axis, as shown in the diagram below.

You must use calculus and show the results of any integration needed to solve the problem.



(d) Evaluate $\int_0^{\frac{\pi}{3}} 3\sin^2(6x)\,dx$.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the following page.

(e) A banana skin is thrown onto a flower bed where it begins to decompose.

The rate at which the banana skin decomposes, at any instant, is proportional to the **square root** of the weight of the banana skin present.

The banana skin initially weighed h^2 grams.

It is recorded that 19% of the **initial** weight of the banana skin has decayed in 5 days.

Form and solve a differential equation to calculate how many days it would take for 91 % of the **initial** weight of the banana skin to decay.

You must use calculus and show the results of any integration needed to solve the problem.

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