SUPERVISOR'S USE ONLY

Tirohia te uhi o muri e kitea ai te whakapākehātanga o tēnei uhi

3

91579M

915795

Tuhia he (☒) ki te pouaka mēnā kāore koe i tuhi kōrero ki tēnei puka

+

NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Te Tuanaki, Kaupae 3, 2026

91579M Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga

Ngā whiwhinga: E ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā te whakaaro ā-pānga.	Te whakahāngai i ngā tikanga pāwhaitua i te whakaoti rapanga, mā te whakaaro waitara e whānui ana.

Tirohia kia kitea ai e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga ake o tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia kia kitea ai kei a koe te pukapuka mō Ngā Ture Tātai me ngā Tūtohi L3–CALCFM.

Whakaaturia ō whiriwhiringa KATOA.

Ki te hiahia wāhi atu anō koe mō ō tuhinga, whakamahia ngā whārangi kei muri o tēnei pukapuka.

Tirohia kia kitea ai e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

Kaua e tuhi ki ngā paenga (////). Ka poroa taua wāhanga ka mākahia ana te pukapuka.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

© Mana Tohu Mātauranga o Aotearoa, 2026. Pūmau ana te mana.
Kāore e whakaaetia ana kia tāruahia tētahi paku wāhanga o tēnei pukapuka ki te kore te Mana Tohu Mātauranga e mātua whakaae.

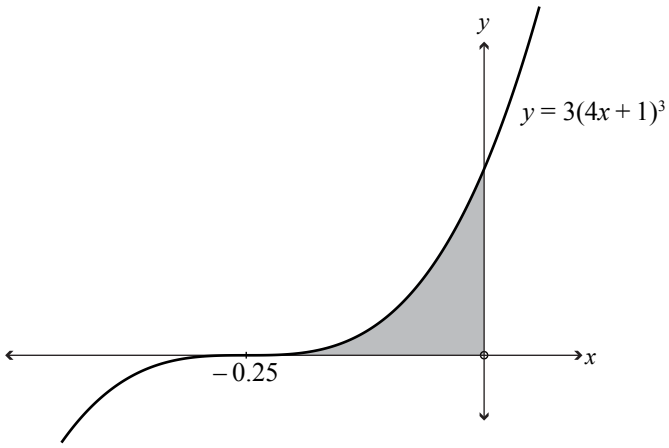
TE TŪMAHI TUATAHI

a) Whiriwhiria te $\int \left(5e^{3x} - \frac{5}{x^3} \right) dx$.

b) E whakaaturia ana i te kauwhata i raro nei te pānga o $y = 3(4x + 1)^3$.

Whiriwhiria te wāhi kua kaurukutia i waenga i ngā uara o $x = -0.25$ me $x = 0$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.



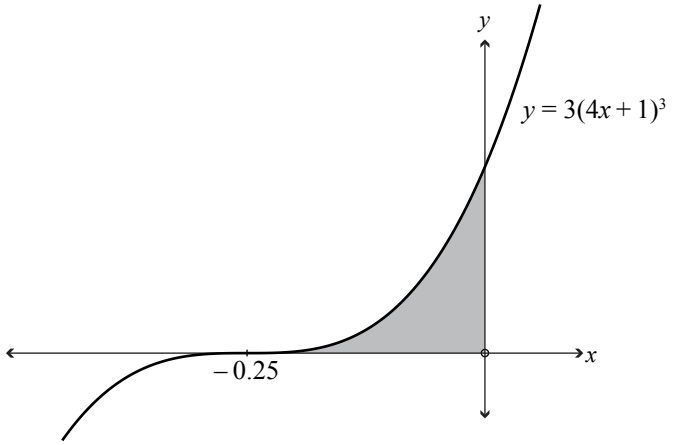
QUESTION ONE

a) Find $\int \left(5e^{3x} - \frac{5}{x^3} \right) dx$.

b) The graph below shows the function $y = 3(4x + 1)^3$.

Find the shaded area between the values of $x = -0.25$ and $x = 0$.

You must use calculus and show the results of any integration needed to solve the problem.



(c) Whakaarohia te whārite pārōnaki o $\frac{\mathrm{d}y}{\mathrm{d}x} = (3y - 2)\cos(3x)$.

Nā te mea ko $y = 1$ i te wā ko $x = \frac{\pi}{6}$, whiriwhiria te/ngā uara o y, i te wā ko $x = \frac{5\pi}{18}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

(c) Consider the differential equation $\frac{dy}{dx} = (3y - 2)\cos(3x)$.

Given that $y = 1$ when $x = \frac{\pi}{6}$, find the value of y when $x = \frac{5\pi}{18}$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Ka taea te whakaterenga o tētahi mea te whakaatu ki te whārite o $a(t) = 32\cos(5t)\sin(3t)$, arā, ko ***a*** te whakaterenga o te mea hei m s^{-2}, ā, ko ***t*** te wā ā-hēkona mai i te tīmatanga o te inenga o te wā.

I te tīmatanga, ko te 10 m s^{-1} te tere whaiahu o taua mea.

Tātaihia te tere whaiahu o taua mea i te $\frac{\pi}{8}$ hēkona i muri i te tīmatanga o te ine i te wā.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

(d) An object's acceleration can be modelled by the equation $a(t) = 32\cos(5t)\sin(3t)$, where ***a*** is the acceleration of the object in m s^{-2} and ***t*** is the time in seconds since the start of timing.

The initial velocity of the object was 10 m s^{-1}.

Calculate the velocity of this object $\frac{\pi}{8}$ seconds after the start of timing.

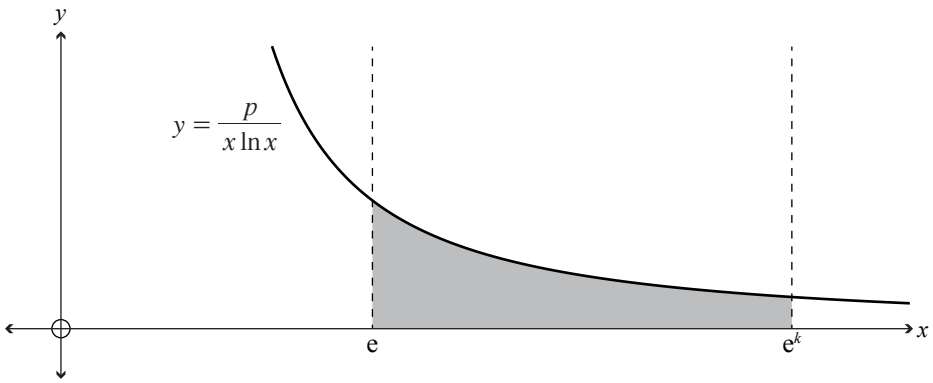
You must use calculus and show the results of any integration needed to solve the problem.

(e) E whakaaturia ana i te kauwhata i raro nei te pānga o $y = \dfrac{p}{x \ln x}$.

E whakaaturia ana i te hoahoa i raro nei te wāhi kua kaurukutia i waenga i te tuaka-x, i ngā rārangi o $x = \text{e}$ me $x = \text{e}^k$, me te kōpiko o $y = \dfrac{p}{x \ln x}$, arā, he tau pūmau tōrunga te k me te p, ā, ko te $k > 1$.

Hāponotia ko te wāhi kua kaurukutia, ko te $p \ln k$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

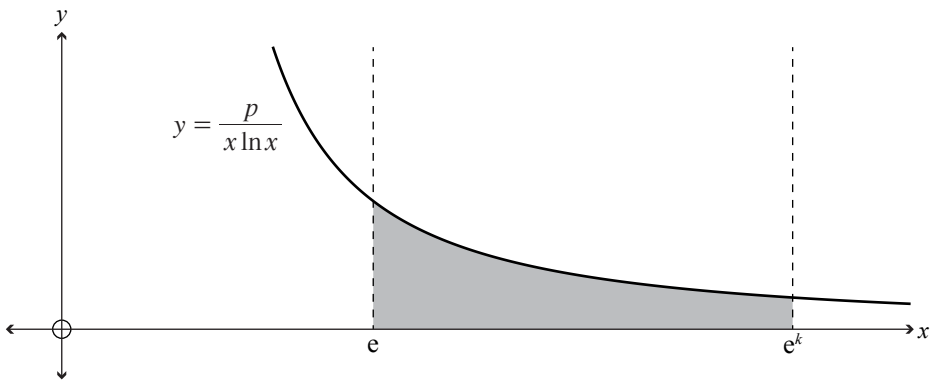


(e) The graph below shows the function $y = \dfrac{p}{x \ln x}$.

The diagram below shows the shaded area between the x-axis, the lines $x = \mathrm{e}$ and $x = \mathrm{e}^k$, and the curve $y = \dfrac{p}{x \ln x}$, where k and p are both positive constants and $k > 1$.

Prove that the shaded area is $p \ln k$.

You must use calculus and show the results of any integration needed to solve the problem.



TE TŪMAHI TUARUA

(a) Whiriwhiria te $\int (\operatorname{cosec}^2(4x) - \operatorname{cosec} x \cot x)\,\mathrm{d}x$.

(b) Whakaotihia te whārite pārōnaki o $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x^3 + 4x^2 - x}{x^2}$ i runga i te mōhio ko $y = 4$ i te wā ko $x = 1$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

QUESTION TWO

(a) Find $\int (\text{cosec}^2(4x) - \text{cosec}\,x\,\cot x)\,dx$.

(b) Solve the differential equation $\frac{dy}{dx} = \frac{2x^3 + 4x^2 - x}{x^2}$ given that $y = 4$ when $x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) Whiriwhiria te uara o te pūmau o p, i runga i te mōhio ko $\int_2^p (6e^{3x-6})dx = 8$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

(d) Whiriwhiria te $\int_0^{\frac{\pi}{6}} \frac{10\cos x}{(5+\sin x)^2} dx$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

(c) Find the value of the constant p, given that $\int_2^p (6e^{3x-6})dx = 8$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Evaluate $\int_0^{\frac{\pi}{6}} \frac{10\cos x}{(5+\sin x)^2} dx$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Whakaarohia te whārite pārōnaki o $(1+\mathrm{e}^{2v})\dfrac{\mathrm{d}v}{\mathrm{d}t} = 2\mathrm{e}^{v}sin(2t)$.

Nā te mea ko $v = 0$ i te wā ko $t = \dfrac{\pi}{6}$, **hāponotia** ko $v = \ln 2$ i te wā ko $t = \dfrac{3\pi}{2}$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

(e) Consider the differential equation $(1+e^{2v})\dfrac{dv}{dt} = 2e^{v}sin(2t)$.

Given that $v = 0$ when $t = \dfrac{\pi}{6}$, **prove** that $v = \ln 2$ when $t = \dfrac{3\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

TE TŪMAHI TUATORU

(a) Whiriwhiria te $\int \frac{20x}{5x^2 - 1} \mathrm{d}x$.

(b) Whakamahia ngā uara kei te tūtohi i raro nei hei whiriwhiri i tētahi āwhiwhitanga mō $\int_1^4 f(x)\mathrm{d}x$ mā te whakamahi i te Ture a Simpson.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	12	15	21	22	18	15	14

QUESTION THREE

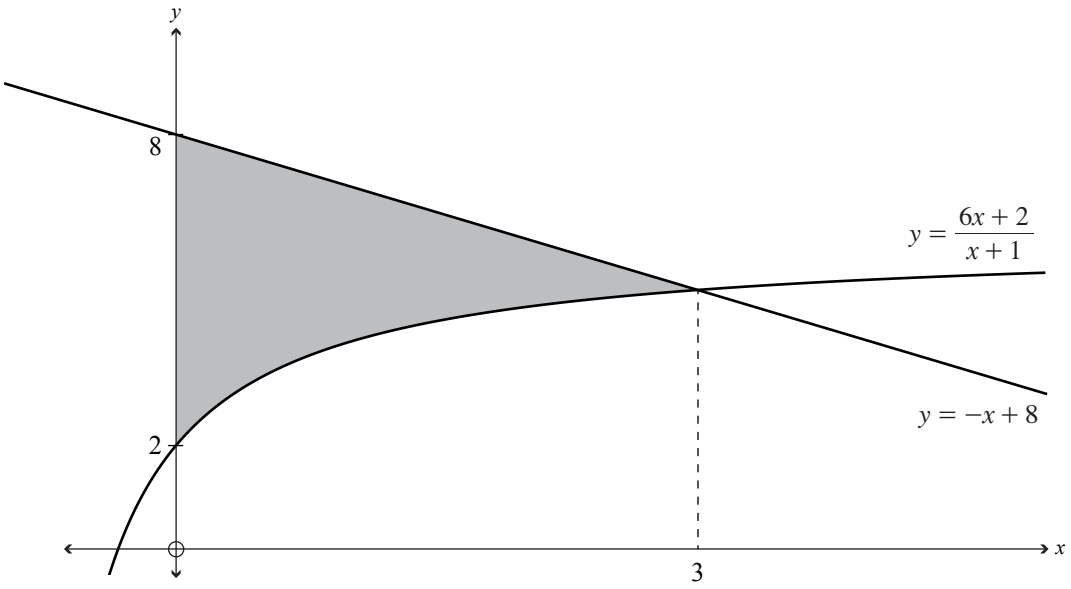
(a) Find $\int \frac{20x}{5x^2 - 1} \mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_1^4 f(x)\mathrm{d}x$ using Simpson's Rule.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	12	15	21	22	18	15	14

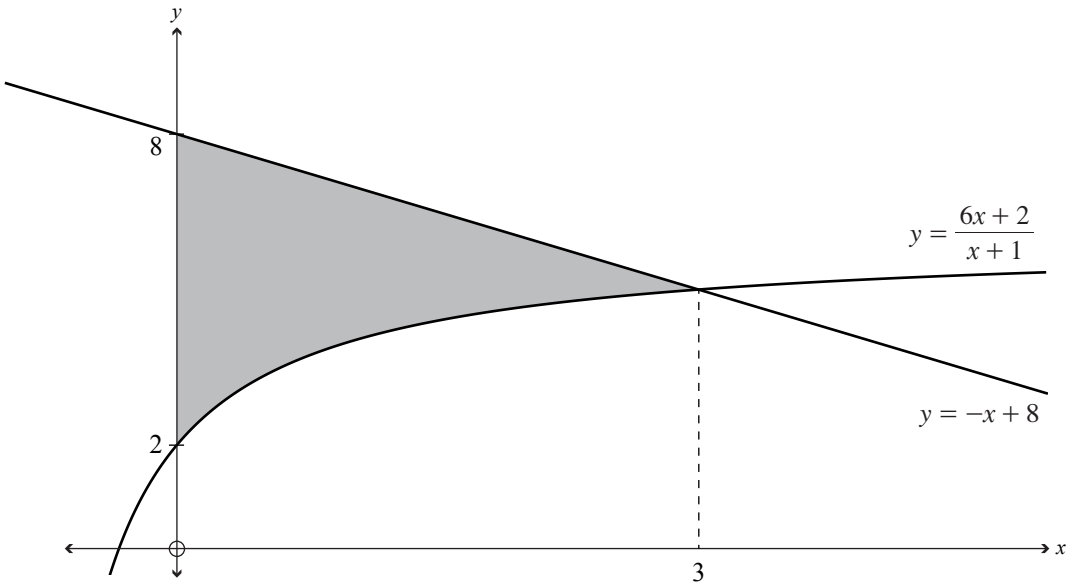
(c) Whiriwhiria te wāhi kua kaurukutia e rohea ana ki ngā rārangi o $y = \dfrac{6x + 2}{x + 1}$, $y = -x + 8$, me te tuaka-y, e whakaaturia ana i te hoahoa i raro nei.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.



(c) Find the shaded area enclosed by the lines $y = \dfrac{6x + 2}{x + 1}$, $y = -x + 8$, and the y-axis, as shown in the diagram below.

You must use calculus and show the results of any integration needed to solve the problem.



(d) Whiriwhiria te $\int_0^{\frac{\pi}{3}} 3\sin^2(6x)\,\mathrm{d}x$.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

Ka rere tonu te Tūmahi Tuatoru i te whārangi e whai ake nei.

(d) Evaluate $\int_0^{\frac{\pi}{3}} 3\sin^2(6x)\,dx$.

You must use calculus and show the results of any integration needed to solve the problem.

Question Three continues on the following page.

(e) Ka rukea tētahi kiri panana ki tētahi māra putiputi, ā, ka tīmata te popo haere i reira.

Ko te pāpātanga e popo ai te kiri panana, ahakoa te wā, e hāngai ana ki te **pūtakerua** o te taumaha o te kiri o taua panana.

I te tīmatanga, ko te taumaha o te kiri panana, ko te $\boldsymbol{h}^2$ karamu.

I rekoatahia ko te 19% o te taumaha o te kiri panana i te **tīmatanga** kua popo i te 5 rā.

Waihangahia, whakaotia hoki tētahi whārite pārōnaki hei tātai i te maha o ngā rā ka pau e popo ai te 91 % o te taumaha o te kiri panana i te **tīmatanga**.

Me whakamahi rawa koe i te tuanaki, me whakaatu hoki i ngā otinga o te mahi pāwhaitua me whai hei whakaoti i te rapanga.

(e) A banana skin is thrown onto a flower bed where it begins to decompose.

The rate at which the banana skin decomposes, at any instant, is proportional to the **square root** of the weight of the banana skin present.

The banana skin initially weighed $\boldsymbol{h}^2$ grams.

It is recorded that 19% of the **initial** weight of the banana skin has decayed in 5 days.

Form and solve a differential equation to calculate how many days it would take for 91 % of the **initial** weight of the banana skin to decay.

You must use calculus and show the results of any integration needed to solve the problem.

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

He whārangi anō ki te hiahiatia.
Tuhia te tau tūmahi mēnā e hāngai ana.

TE TAU TŪMAHI

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

English translation of the wording on the front cover

Level 3 Calculus 2026

91579M Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

91579M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCFM.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in the margins (////). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

© New Zealand Qualifications Authority, 2026. All rights reserved.
No part of this publication may be reproduced by any means without the prior permission of the New Zealand Qualifications Authority.