1

SUPERVISOR'S USE ONLY

91031

910310

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 1 Mathematics and Statistics, 2015

91031 Apply geometric reasoning in solving problems

9.30 a.m. Monday 9 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 12

ASSESSOR'S USE ONLY

Annotated Exemplar Template

Achieved exemplar for 91031 2015		Total score	12
Q	**Grade score**	**Annotation**	
1	A4	The candidate has selected and use methods to find angles. Their solution in a) i) was not fully reasoned. To gain a higher grade score for this question they would need to use similar triangles find lengths and compare areas. As well their solutions would need to be fully reasoned.	
2	A4	The candidate has selected and used methods to find angles involving one step. To gain a higher grade the candidates would need to carry out a logical sequence involving 2 to 3 steps (in parts a) ii and b) ii)) for merit evidence.	
3	A4	The candidate has selected and used methods to find angles involving one step. To gain a higher grade the candidates would need to carry out a logical sequence involving 2 to 3 steps (in parts a) iii b) ii and c) for merit evidence.	

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A clothes drying rack has two horizontal levels on which the clothes can be hung as shown by lines AE and HI on the diagram below.

AE is parallel to HI and parallel to the ground JN.

The rack is symmetrical around the line CL.

BC = CF

Angle KCL = 24°

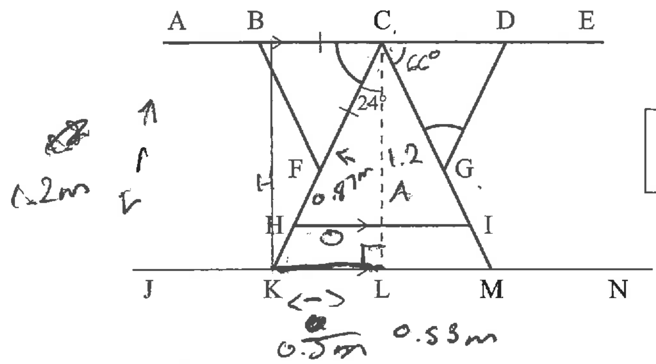


Diagram is NOT to scale

S O H C A H T O A

(i) Find the size of angle BCF.

Justify your answer with clear geometric reasoning.

90 − 24 = ∠BCF ∠'s on a straight line add to 180 and the rack is symmetrical through CL so both sides have to be the same ∠BCF = 66°

r

(ii) Find the size of angle DGC.

Justify your answer with clear geometric reasoning.

DCG = 66° ∠'s on a straight line

DGI = 66 corresponding ∠'s

DGC = 114° ∠'s on a straight line

n

(iii) The height of AE above the ground is 1.2 m.

Pippa says the length KL is 0.53 m.

Show that she is correct.

~~$0.53^2 + 1.2^2 = c^2$ $c^2 = 1.7209$~~

~~$a^2 + b^2 = c^2$~~ yes she is correct

~~$c = 1.31$ m (2DP)~~ $\tan^{-1}\left(\frac{0.53}{1.2}\right) = 23.83°$

which is then rounded to 24°

4

ASSESSOR'S USE ONLY

(iv) What is the length of CK?

$a^2 + b^2 = c^2$

$1.2^2 + 0.58^2 = 1.7209$

$\sqrt{1.7209} = 1.311930792$

CK = 1.3m

u

(v) CH is two-thirds of CK.

Find the length of HI.

Justify your answer with clear geometric reasoning.

$1.3 \times \frac{2}{3} = 0.87m$

CH is two-thirds of CK everything would be $\frac{2}{3}$

HI = $\frac{2}{3}$ of KL

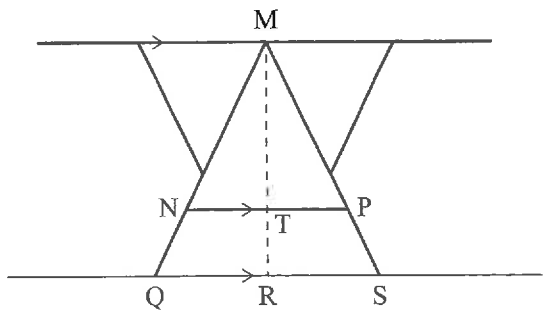
HI = 0.33m

k

(b) For another clothes drying rack:

MN : NQ = a : b

Compare the area of triangles MNP and MQS.



MNP is $\frac{2}{3}$ of MQS

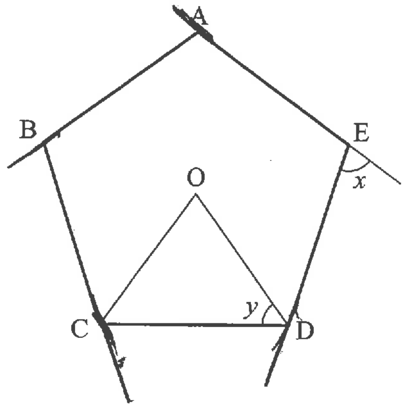
n

A4

ASSESSOR'S USE ONLY

QUESTION TWO

(a) ABCDE is a regular pentagon with centre O.



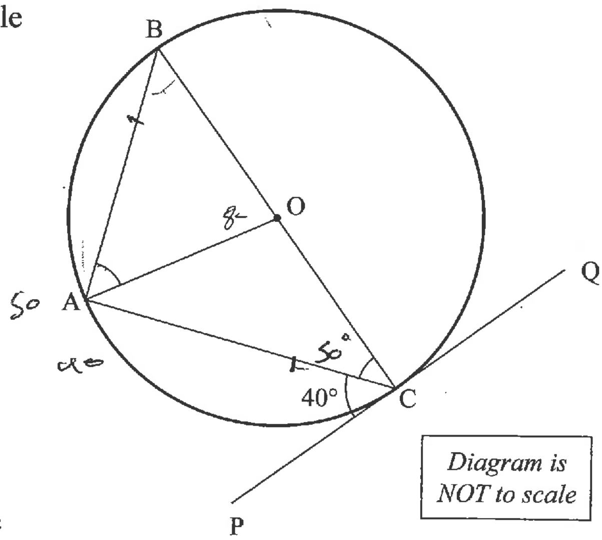
(i) Find the value of x and explain your answer.

exterior ∠'s in a polygon add to 360°
360 ÷ 5 = 72°
x = 72° u

(ii) Find the value of y.
Justify your answer with clear geometric reasoning.

it is a regular pentagon so all sides and angles are the same and (5 − 2) × 180 = 540° interior sum
540 ÷ 5 = 108° of each interior ∠ angle
∠'s on a straight line add to 180° n

(b) A, B, and C are on the circumference of a circle with centre O. BOC is a diameter.
QCP is a tangent to the circle.
Angle ACP = 40°.



(i) Find the size of angle ACO.
Justify your answer with clear geometric reasoning.

∠ACO = 50° (rad ⊥ tangent) u

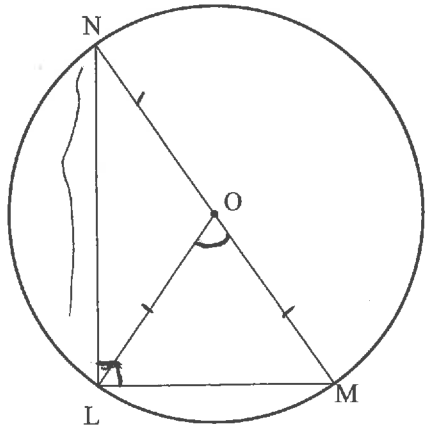
(ii) Find the size of angle OAB.
Justify your answer with clear geometric reasoning.

∠ OAB = 30° ∠'s on the same arc are equal n

(iii) The points L, M, and N lie on the circumference of a circle centre O. NOM is a diameter.

If OL = x cm and LOM = $a°$, calculate the length of NL in terms of x and a.

Justify your answer with clear geometric reasoning.



ASSESSOR'S USE ONLY

N

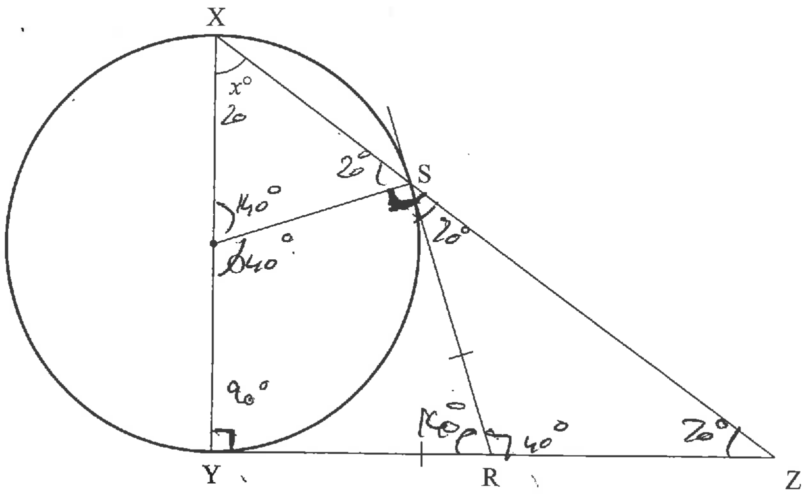
∠ NLM = 90° two points from a semicircle that connect at the circumference make a 90° ∠

OL ~~x~~ = ~~NL~~ OM isos triangle any two points from the radius forms an isos triang

OL = ON

~~NL~~

(c)



The points S, X, and Y are on the circumference of a circle centre O.

XY is a diameter of the circle.

YZ and SR are tangents to the circle.

RS = RY

Angle YXZ = $x°$

Prove that YR = RZ

~~X + S = 180° opp ∠ cyclic quad~~

∠ OYR = ∠RSO which is 90° tangents from a point

let x = 20°

~~x~~ ~~X50~~ XSO = 20° isos triangle form from 2 points on a semi circle connected

XOS = 140° sum of triangle SOY = 40° ∠'s on a straight line

YRS = 140° sum of ∠'s in a polygon ZRS = 40° ∠s on a straight line

ZSR = 70° ∠'s on a straight line SZR = 20° sum of triangle

YR = RZ

n

A4

O

S· H

QUESTION THREE

ASSESSOR'S USE ONLY

(a) (i) A farmer wants to climb a ladder to check the water in a tank.

He uses a 3 metre ladder and places it so that the top of the ladder just reaches the top of the tank.

The top of the tank is 2.9 metres from the ground.

He wants the angle of the ladder to the ground to be less than 80°.

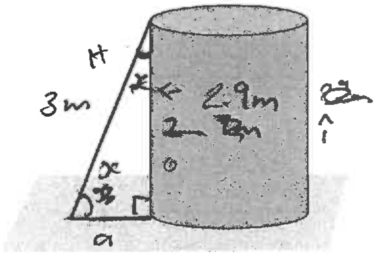


Diagram is NOT to scale

Is the ladder long enough to meet this requirement?

yes

$\sin^{-1}\left(\frac{2.9}{3}\right)$ $= \cancel{64.16°}$ (2DP)

$= 75.16°$ (2DP)

r

(ii) How far is the foot of this ladder from the base of the tank?

Assume that the tank is sitting on level ground.

$a^2 = c^2 - b^2$

$a^2 = 9 - \cancel{7.29}$ 8.41 $\sqrt{0.59} = 0.768114574 8m$

foot of ladder $= \cancel{1.3m}$ (1DP) $0.768114574 8m$

u

(iii) If the farmer places the ladder at 80° to the ground, how much of the ladder is above the top of the tank?

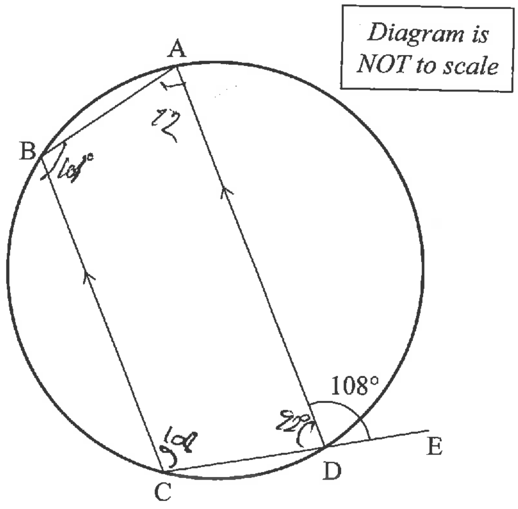
$\cancel{75.16} = 2.9$

n

(b) (i) A trapezium has two sides that are parallel.

ABCD is an isosceles trapezium with its vertices on the circumference of a circle.

Angle EDA = 108°.



Find the size of angle ECB.

Justify your answer with clear geometric reasoning.

ADC = 72° ∠'s on a straight line

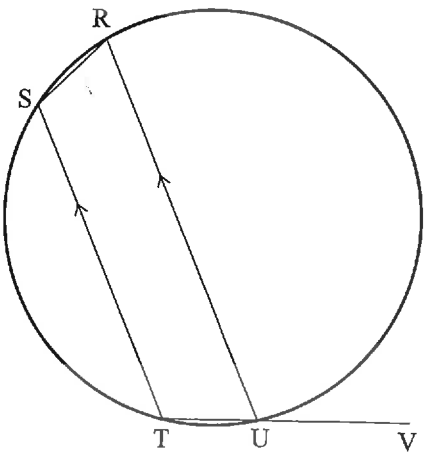
ECB = 108° opp ∠'s cyclic quad

DAB = 72 isos trapezium

(ii) RSTU is any trapezium with its vertices on the circumference of a circle.

Determine any geometrical facts about RSTU and prove that these are true for all such trapeziums.

Justify your answers with clear geometric reasoning.



RUV = RST ~~(a~~ alt ∠'s in a segment)

interior ∠'s will sum to 360° for any 4 sided shape

SRU and STU will sum to 180° (opp ∠'s in a cyclic quad) same as RST and RUT

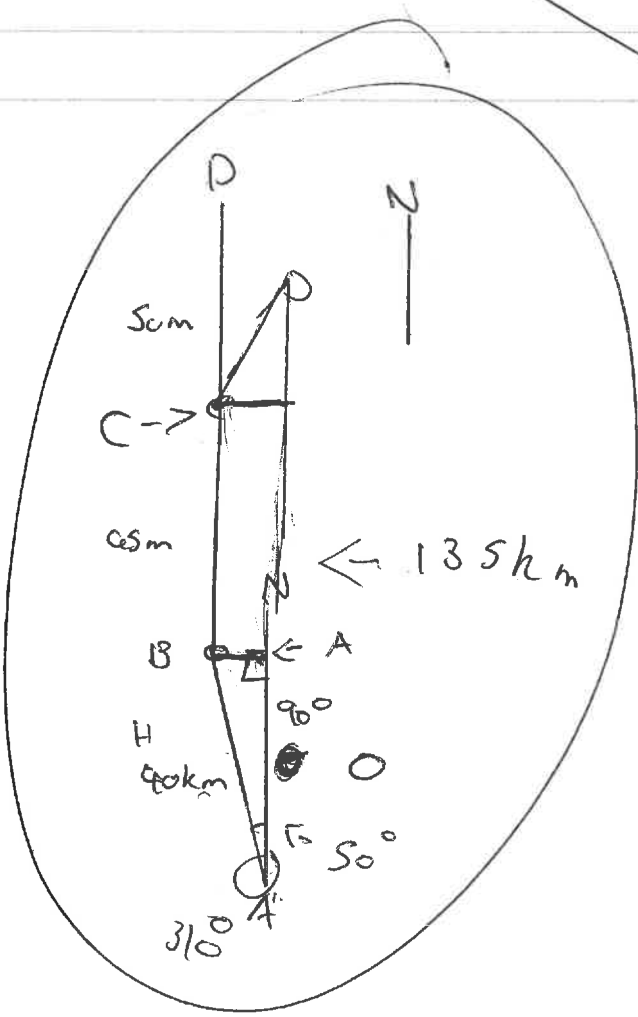
(c) An aeroplane is flown 40 km on a bearing of 310° from airstrip A to airstrip B.

It then turns and flies 45 km due north to airstrip C.

The plane then heads directly to airstrip D, which is 135 km due north of its starting point at airstrip A.

For the final leg of the flight path of the plane, how far does it need to fly, and what is the bearing of the final leg of ~~the~~ flight path?

Final leg is 50km and its bearing is 050°



ASSESSOR'S USE ONLY

N

A4