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SUPERVISOR'S USE ONLY

Level 1 Mathematics and Statistics, 2015

91031 Apply geometric reasoning in solving problems

9.30 a.m. Monday 9 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 21

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Annotated Exemplar Template

Excellence exemplar for 91031 2015		Total score	21
Q	**Grade score**	**Annotation**	
1	E7	This candidate has devised a strategy to solve the problem using similar triangles and properties of triangles. The candidate has developed clear and logical solutions with clear reasoning. Has used correct mathematical statements and communicated their findings correctly. The candidate did not get E8 as they were not able to generalise to compare the scale factor for areas.	
2	E7	This candidate has developed a clear and logical chain of reasoning to solve the problems using angle properties of circles, polygons and triangles, as well as trigonometric relationships. The candidate did not get E8 as their reasoning in 2c) was not complete.	
3	E7	This candidate devised a clear strategy to solve the problems. Their answer in 3c) was correct and clear. They have used trig relationships to successfully find the bearing and distance and have used correct mathematical statements. The candidate did not get E8 as they were not able to form a generalisation about isosceles trapeziums inscribed in circles.	

QUESTION ONE

(a) A clothes drying rack has two horizontal levels on which the clothes can be hung as shown by lines AE and HI on the diagram below.

AE is parallel to HI and parallel to the ground JN.

The rack is symmetrical around the line CL.

BC = CF

Angle KCL = 24°

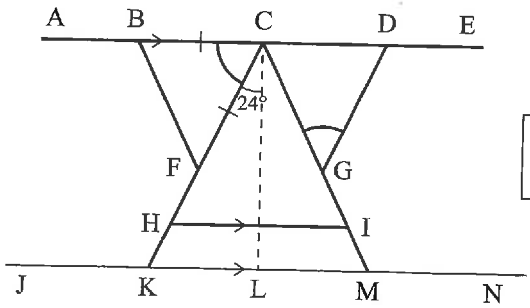


Diagram is NOT to scale

(i) Find the size of angle BCF.

Justify your answer with clear geometric reasoning.

ACL = 90° as it is symetrical

BCF = 90 − 24 = 66 $\quad$ 2(66) + 2(24) = 180

angles on a line = 180°

BCF = 66°

r

(ii) Find the size of angle DGC.

Justify your answer with clear geometric reasoning.

△DGC is an isosceles triangle, meaning both base angles are equal (DGC and CDG)

DCG = 66°

180 − 66 = 114 / 2 = 57 $\quad$ angles in a triangle add to 180°

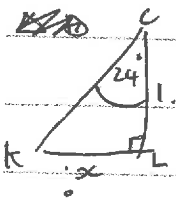
DGC = 57°

r

(iii) The height of AE above the ground is 1.2 m.

Pippa says the length KL is 0.53 m.

Show that she is correct.



TOA

$\frac{O}{T \ \ \tan 24 \ | \ 1.2 \ \ A}$ with $O = x$

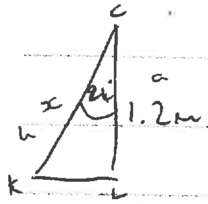
x = Tan24 × 1.2

x = 0.534274422

KL ≈ 0.53 m

u

(iv) What is the length of CK?



CAH

$\frac{1.2}{\cos 24 \mid x}$

$x = 1.2 \div \cos 24$

$x = 1.313563534$

(3sf)

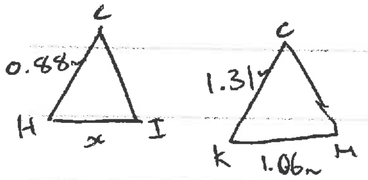
CK ≈ 1.31 m

(v) CH is two-thirds of CK.

Find the length of HI.

Justify your answer with clear geometric reasoning.

$CH = \frac{2}{3} \times 1.31\ldots = 0.875709022$



similar triangles = proportional

$KM = 2 \times KL = 1.06$

$\frac{1.06}{1.31} = \frac{x}{0.88}$

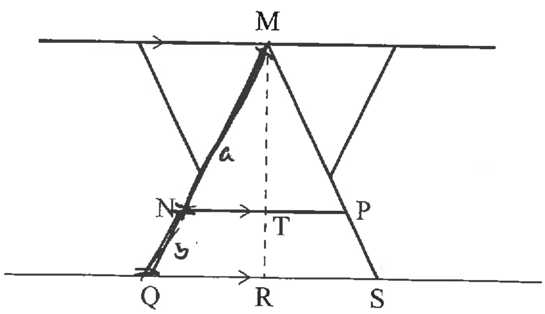
×0.88 ×0.88

$x = 0.88 \times \frac{1.06}{1.31} = 0.712061068$ HI ≈ 0.71 m

(b) For another clothes drying rack:

MN : NQ = a : b

Compare the area of triangles MNP and MQS.



MNP : MQS

a : a+b

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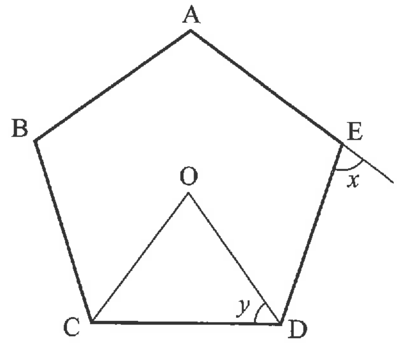
QUESTION TWO

(a) ABCDE is a regular pentagon with centre O.



(i) Find the value of x and explain your answer.

sum of exterea angles of polygon = 360°

360 ÷ 5 = 72 (bc its regular)
all equal

x = 72°

u

(ii) Find the value of y.

Justify your answer with clear geometric reasoning.

the exterior age = 72° sum ext ∠s = 360°

CDE = 180 − 72 = 108° ∠s on a line = 180°

y = 108 ÷ 2 = 54

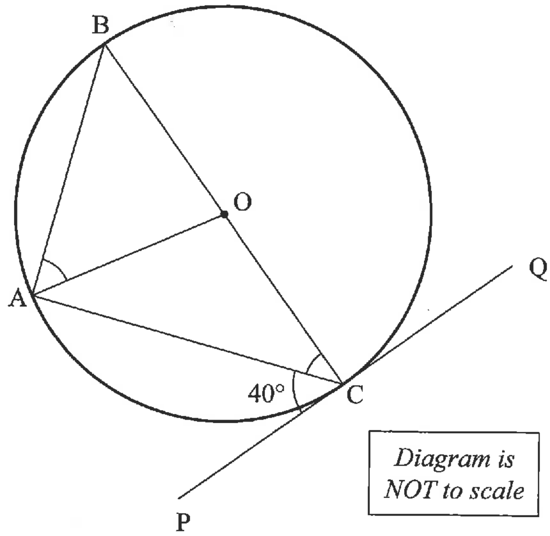
y = 54°

u

(b) A, B, and C are on the circumference of a circle with centre O. BOC is a diameter.

QCP is a tangent to the circle.

Angle ACP = 40°.



(i) Find the size of angle ACO.

Justify your answer with clear geometric reasoning.

ACO = 90 − 40 = 50°

ACO = 50° angle tangent to radius = 90°

u

(ii) Find the size of angle OAB.

Justify your answer with clear geometric reasoning.

BAC = 90° angle in a semi-circle = 90°

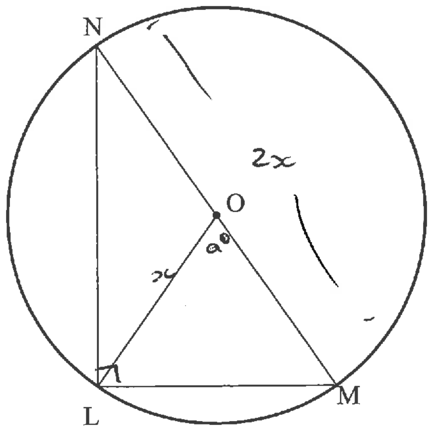
OAC = 50° = issos triangle base angles in isos Δ =

BAC − OAC = OAB

90 − 50 = 40 OAB = 40°

r

(iii) The points L, M, and N lie on the circumference of a circle centre O. NOM is a diameter.



If OL = x cm and LOM = a°, calculate the length of NL in terms of x and a.

Justify your answer with clear geometric reasoning.

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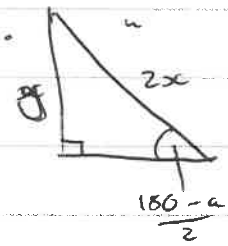
NLM = 90°   ∠ in semi ⊙ = 90°

NM = 2x   LO = OM = NO

OLM = isosceles triangle

= isos Δ = base ∠s equal   ∠s in Δ = 180°

$\angle OML = \frac{180 - a}{2}$



SOH

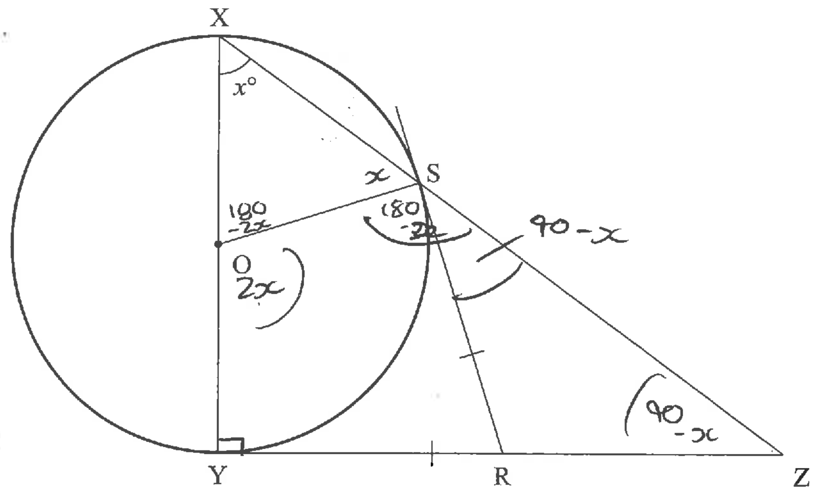
O
y

S $\sin\left(\frac{180-a}{2}\right)$ 2x   H

$y = \sin\left(\frac{180-a}{2}\right) \times 2x$

$NL = \sin\left(\frac{180-a}{2}\right) \times 2x$

(c)



The points S, X, and Y are on the circumference of a circle centre O.

XY is a diameter of the circle.

YZ and SR are tangents to the circle.

RS = RY

Angle YXZ = $x°$

Prove that YR = RZ

XSO = $x°$ → base ∠s isos △ =

XOS = 180 − 2x

OSZ = 180 − x

$\boxed{RSZ = 90 - x}$ → SRZ = isos △

YOS = 2x

$\boxed{YZX = 90 - x}$

SR = RZ

and because YR = SR

YR = RZ //

∠s in △ add to 180°

∠s on st line = 180°

tgt ⊥ ~~to~~ to radius //

4B

r

E7

QUESTION THREE

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(a) (i) A farmer wants to climb a ladder to check the water in a tank.

He uses a 3 metre ladder and places it so that the top of the ladder just reaches the top of the tank.

The top of the tank is 2.9 metres from the ground.

He wants the angle of the ladder to the ground to be less than 80°.

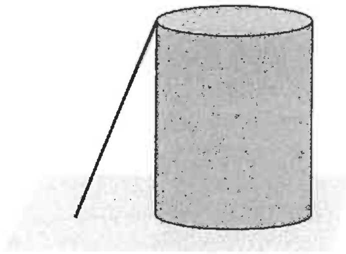
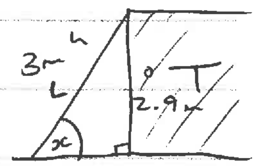


Diagram is NOT to scale

Is the ladder long enough to meet this requirement?

SOH

$S \quad \frac{2.9}{\sin x \mid 3} \quad H \qquad \sin x = 2.9 \div 3$



$x = \sin^{-1}(2.9 \div 3)$ less than 80°

$x = 75.16488842$ = Yes, it is long enough.

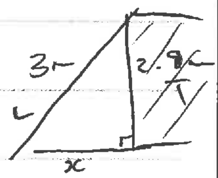
(ii) How far is the foot of this ladder from the base of the tank?

Assume that the tank is sitting on level ground.

$x = \sqrt{(3^2 - 2.9^2)}$

$x = 0.768114574$ (2 dp)

≈ 0.77 m



(iii) If the farmer places the ladder at 80° to the ground, how much of the ladder is above the top of the tank?

SOH

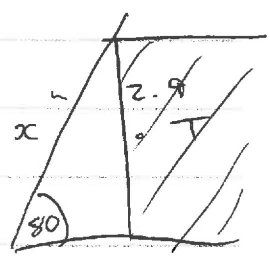
$S \quad \frac{2.9}{\sin 80 \mid x} \quad H$

$x = 2.9 \div \sin 80$

$x = 2.944737$

$3 - \quad " \quad = 0.055262825$ (2 d.p)

$= 0.06$ m of the ladder is above the top of the tank.

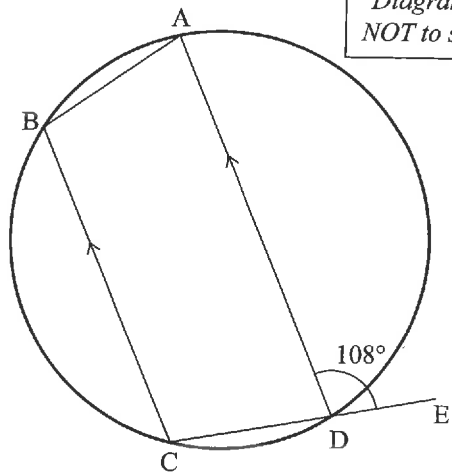


(b) (i) A trapezium has two sides that are parallel.

ABCD is an isosceles trapezium with its vertices on the circumference of a circle.

Angle EDA = 108°.

Diagram is NOT to scale



Find the size of angle ECB.

Justify your answer with clear geometric reasoning.

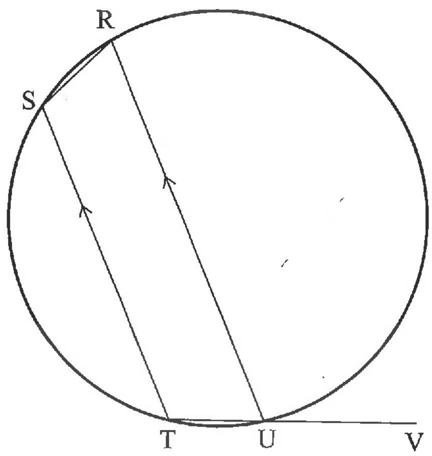
ECB = 108°

corresponding angles on parallel lines are equal.

(ii) RSTU is any trapezium with its vertices on the circumference of a circle.

Determine any geometrical facts about RSTU and prove that these are true for all such trapeziums.

Justify your answers with clear geometric reasoning.



U

N

(c) An aeroplane is flown 40 km on a bearing of 310° from airstrip A to airstrip B.

It then turns and flies 45 km due north to airstrip C.

The plane then heads directly to airstrip D, which is 135 km due north of its starting point at airstrip A.

For the final leg of the flight path of the plane, how far does it need to fly, and what is the bearing of the final leg of the flight path?

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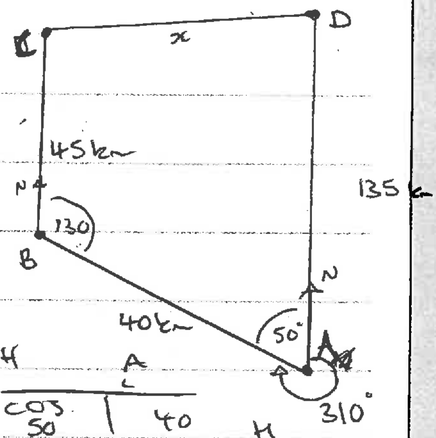


co-int ∠s // lines = 180° $180 - 50 = 130$

a = SOH   O a
S sin 50 | 40 H

$a = \sin 50 \times 40$

$a = 30.641777$

$b = \sqrt{(30.64^2 + 45^2)}$

$= 54.4418224$

c = CAH   A c
c cos 50 | 40 H

$c = \cos 50 \times 40$

$c = 25.7115\ 0439$

$d = \sqrt{(54.4^2 - 30.6^2)}$

$d = 44.97777$

y = SOH   S sin 71.237 / y | 64.31 H

~~y = sin⁻¹(71.23/64.31)~~   135 − = 70.688

$y = \sin^{-1}(64.31 \div 71.23) = 64.312$

$y = 64.38$

$90 - \Downarrow$

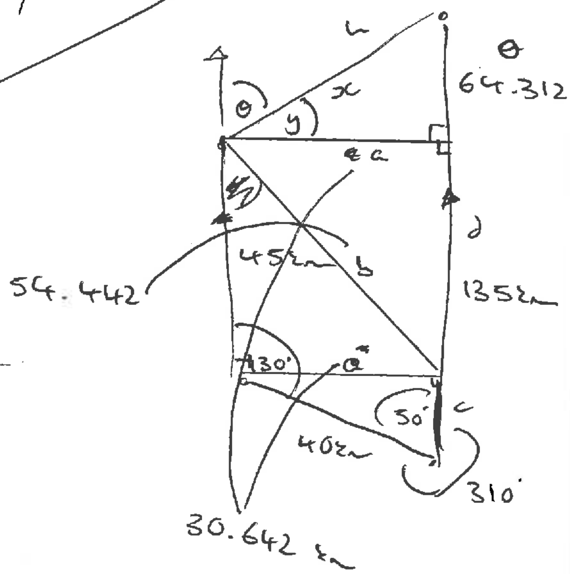
$= 25.616$

$x = \sqrt{(64.312^2 + 30.642^2)}$

$x = 71.236$

distance = 71 km

bearing = 026°



E7