91031

910310

NEW ZEALAND QUALIFICATIONS AUTHORITY
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Level 1 Mathematics and Statistics, 2015

91031 Apply geometric reasoning in solving problems

9.30 a.m. Monday 9 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 15

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Annotated Exemplar Template

Merit exemplar for 91031 2015		Total score	15
Q	**Grade score**	**Annotation**	
1	M5	Candidate has correctly selected and used geometric concepts. They have answered a series of questions demonstrating an understanding of angle properties of isosceles triangles and trigonometric relationships in right angles triangles. To gain an E7 or E8 the candidate would need to use similar triangles to find lengths and compare areas.	
2	M6	Candidate has correctly selected and used geometric properties of angles, polygons and circles. They have clear reasons for the angles they have found. To gain an E7 or E8 the candidate would have needed to develop further a chain of logical reasoning. They were not able to fully prove that YR was equal to RZ in c). In 2b) iii) the candidate could not form a trigonometric relationship for NL using x and a.	
3	A4	Candidate has correctly selected and used geometric properties of angles and circles and trigonometric relationships. In a) iii) The student was not able to correctly interpret the solution in context. The candidate was not able to correctly find a correct total distance or bearing.	

ASSES USE

QUESTION ONE

(a) A clothes drying rack has two horizontal levels on which the clothes can be hung as shown by lines AE and HI on the diagram below.

AE is parallel to HI and parallel to the ground JN.

The rack is symmetrical around the line CL.

BC = CF

Angle KCL = 24°

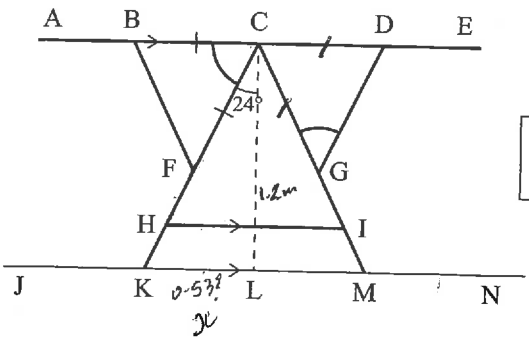

Diagram is NOT to scale

(i) Find the size of angle BCF.

Justify your answer with clear geometric reasoning.

$\angle KCM = 24 \times 2 = 48°$ because they are symmetrical.

$\angle BCF = \angle DCG$ because symmetrical.

$\angle BCF = \frac{180-48}{2} = 66°$ $\angle$'s on a Line $= 180°$

(ii) Find the size of angle DGC.

Justify your answer with clear geometric reasoning.

$\angle DCG = 66°$ $\angle$'s on a Line

$\angle DGC = \angle CDG$ base $\angle$'s isos Δ

$\angle DGC = \frac{180-66}{2} = 57°$ Sum of $\Delta = 180°$

(iii) The height of AE above the ground is 1.2 m. SOH CAH TOA

Pippa says the length KL is 0.53 m.

Show that she is correct.

$\tan 24 \overset{\times 1.2}{=} \frac{x}{1.2} \times 1.2$

$\tan 24 \times 1.2 = x$

$x = 0.53\text{m}$

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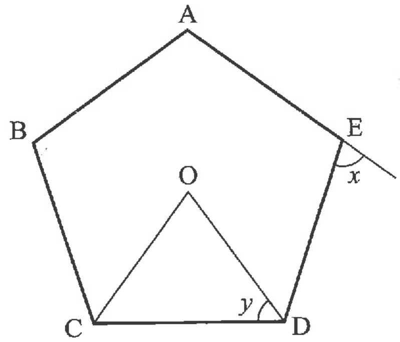
QUESTION TWO

(a) ABCDE is a regular pentagon with centre O.



(i) Find the value of x and explain your answer.

$x = \frac{360}{5}$

$72°$ ext ∠'s of pollygon $= 360°$

(ii) Find the value of y.

Justify your answer with clear geometric reasoning.

$\cancel{(5-2) \times 180}$ $\cancel{5}$ $y = 108° \div 2 = 54°$

$\frac{1}{2}$ interior ∠ pollygon

$(n-2) \times 180$

$(5-2) \times 180 = \frac{540}{5} = 108$ int ∠s pollygon

(b) A, B, and C are on the circumference of a circle with centre O. BOC is a diameter.

QCP is a tangent to the circle.

Angle ACP = 40°.

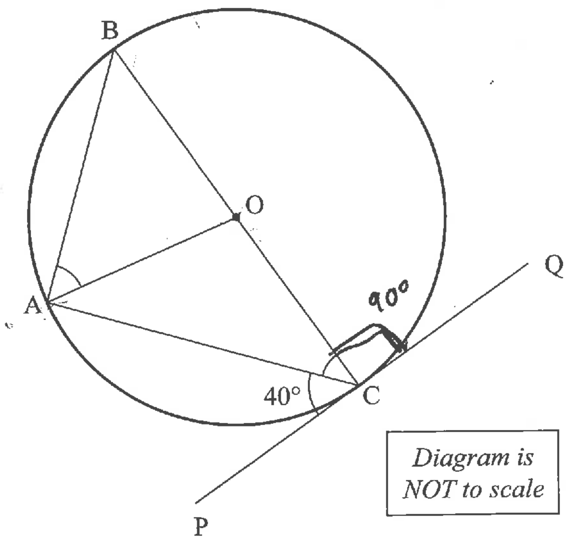


Diagram is NOT to scale

(i) Find the size of angle ACO.

Justify your answer with clear geometric reasoning.

$\angle ACO = 90 - 40 = 50°$ rad ⊥ tgt.

(ii) Find the size of angle OAB.

Justify your answer with clear geometric reasoning.

$\angle AOC = 180 - 50 - 50 = 80°$ sum of isos △

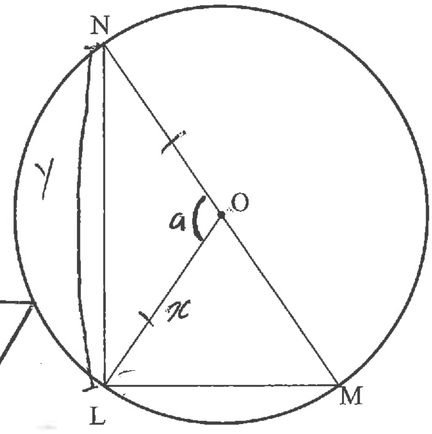
$\angle BOA = 180°$ ∠'s on a Line.

$\angle OAB = \frac{180 - 100}{2} = 40°$ sum of isos △

(iii) The points L, M, and N lie on the circumference of a circle centre O. NOM is a diameter.

If OL = x cm and LOM = $a°$, calculate the length of NL in terms of x and a.

Justify your answer with clear geometric reasoning.



$NO = x$ isos $\triangle$

$a^2 + b^2 = c^2$

$x^2 + x^2 = NL^2$

$2x^2 = NL^2$

$2x = NL$

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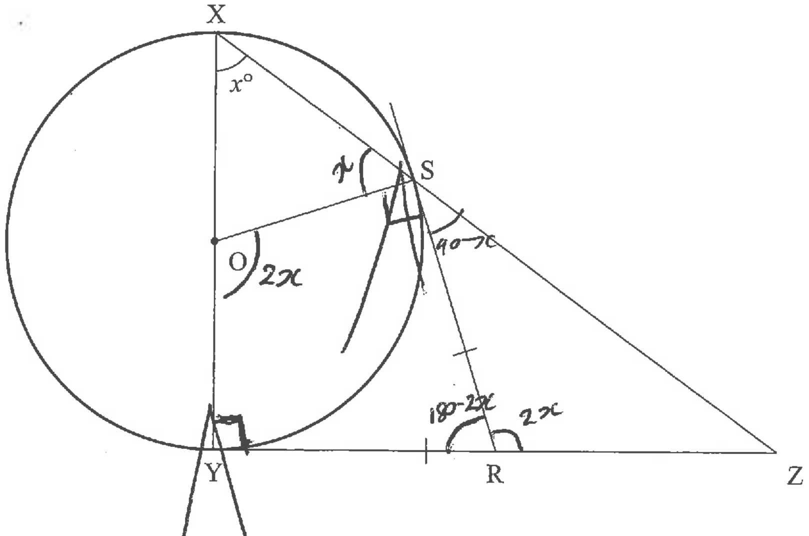
n

n

(c)



The points S, X, and Y are on the circumference of a circle centre O.

XY is a diameter of the circle.

YZ and SR are tangents to the circle.

RS = RY

Angle YXZ = $x°$

Prove that YR = RZ

∠XSO = $x°$ base ∠'s isos △

∠XOS = 180 − 2x sum of isos △

∠YRS = 360 − 180 − 2x = 180 − 2x sum of □

∠SOY = 180 − (180 − 2x)
= 2x ∠'s on Line

∠SRZ = 180 − (180 − 2x) = 2x ∠'s on Line

ASSESSOR'S USE ONLY

M6

M6

CAH SOH

QUESTION THREE

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(a) (i) A farmer wants to climb a ladder to check the water in a tank.

He uses a 3 metre ladder and places it so that the top of the ladder just reaches the top of the tank.

The top of the tank is 2.9 metres from the ground.

He wants the angle of the ladder to the ground to be less than 80°.

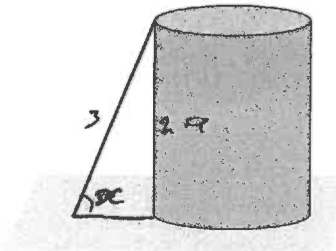


Diagram is NOT to scale

Is the ladder long enough to meet this requirement?

~~$\cos x = \frac{2.9}{3}$~~

~~$x = \cos^{-1}\left(\frac{2.9}{3}\right)$~~ ~~$x =$~~

$\sin x = \frac{2.9}{3}$

$x = \sin^{-1}\left(\frac{2.9}{3}\right)$

$x = 75.16°$ Yes the Ladder is long enough.

r

r

(ii) How far is the foot of this ladder from the base of the tank?

Assume that the tank is sitting on level ground.

$a^2 + b^2 = c^2$

$c^2 - a^2 = b^2$

$3^2 - 2.9^2 = b^2$

$9 - 8.41 = b^2$

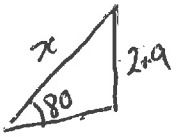
$\sqrt{0.59} = b$

~~base~~ ~~b 7.8 ...~~ $b = 0.77m$

u

u

(iii) If the farmer places the ladder at 80° to the ground, how much of the ladder is above the top of the tank?



$x \times \sin 80 = \frac{2.9}{x} \times x$

$\frac{x \times \sin 80}{\sin 80} = \frac{2.9}{\sin 80}$

$x = \frac{2.9}{\sin 80}$

$x = 2.94m$

~~The la~~

none it is below the tank

u

u

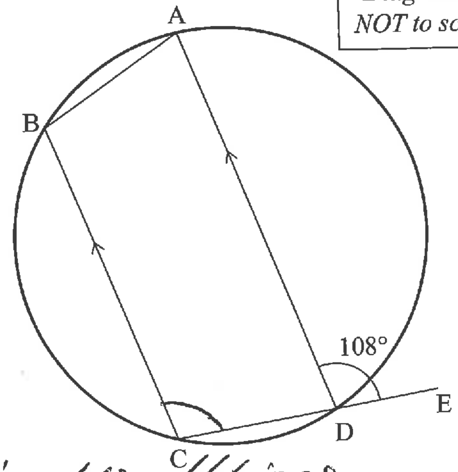
ASSESSOR'S USE ONLY

(b) (i) A trapezium has two sides that are parallel.

ABCD is an isosceles trapezium with its vertices on the circumference of a circle.

Angle EDA = 108°.

Diagram is NOT to scale



Find the size of angle ECB.

Justify your answer with clear geometric reasoning.

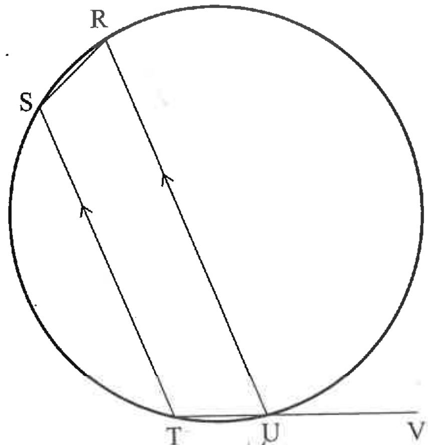
∠ECB = 108° corresponding ∠'s // Lines.

u
u

(ii) RSTU is any trapezium with its vertices on the circumference of a circle.

Determine any geometrical facts about RSTU and prove that these are true for all such trapeziums.

Justify your answers with clear geometric reasoning.



~~P~~ trapeziums have 2 Lines that are parallel ST is parallel to RU

n
n

(c) An aeroplane is flown 40 km on a bearing of 310° from airstrip A to airstrip B.

It then turns and flies 45 km due north to airstrip C.

The plane then heads directly to airstrip D, which is 135 km due north of its starting point at airstrip A.

For the final leg of the flight path of the plane, how far does it need to fly, and what is the bearing of the final leg of the flight path?

$\angle CBA = 130°$ co-int ∠'s // lines.

~~$Y^2\ a^2+b^2$~~

~~$Y^2\ c$~~

$a^2+b^2=c^2$

$c^2+a^2=Y^2$

$40^2-25.71^2=Y^2$

$1600-661=Y^2$

$\sqrt{939}=Y$

$Y=30.6\text{km}$

$a = 30.6\text{km}$ because it's a square.

$a^2+b^2=c^2$

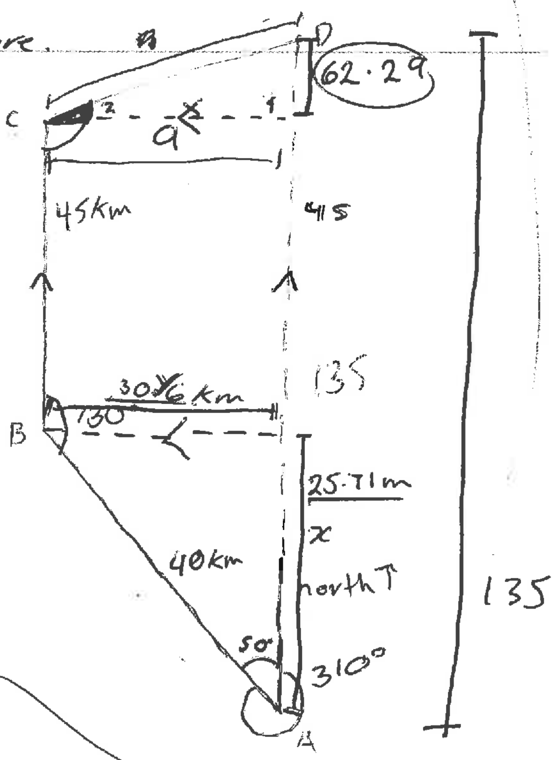
$62.29^2+30.6^2=CD^2$

$3880.04+936.39=CD^2$

$\sqrt{4819.43}=CD$

$CD = 69.42\text{km}$

The plane flys 69.42km on the last leg of it's flight.



~~Z SOH~~

~~$\sin z = \frac{62.29}{30.6}$~~

~~$z = \sin^{-1}\left(\frac{62.29}{30.6}\right)$~~

$\cos 50 = \frac{x}{40}$ ×40 ×4

$x = 25.71\text{m}$

CAH

U

U

A4

A4