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# Level 3 Calculus, 2015

## 91578 Apply differentiation methods in solving problems

2.00 p.m. Wednesday 25 November 2015
Credits: Six

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply differentiation methods in solving problems. | Apply differentiation methods, using relational thinking, in solving problems. | Apply differentiation methods, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Achievement

TOTAL 9

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fill max when < 0

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## QUESTION ONE

(a) Differentiate $y = 6\tan(5x)$.

$6\sec^2(5x) \times 5$

$= 30\sec^2(5x)$

4

(b) Find the gradient of the tangent to the function $y = (4x - 3x^2)^3$ at the point (1,1).

*You must use calculus and show any derivatives that you need to find when solving this problem.*

$\frac{dy}{dx} = 3(4x-3x^2)^2 \cdot 4 - 6x$

$= (12-18x)(4x-3x^2)^2$

when $x=1$

$\frac{dy}{dx} = (12-6)(4-9)^2$

$= -150$

(c) Find the values of $x$ for which the function $f(x) = 8x - 3 + \frac{2}{x+1}$ is increasing.

*You must use calculus and show any derivatives that you need to find when solving this problem.*

increasing when $f'(x) < 0$

$f'(x) = 8 - 2(x+1)^{-2} = 8 - \frac{2}{(x+1)^2}$

$\cancel{8 - 2(x+1)^{-2} < 0}$

$\cancel{8 - 2(x+1)^{-2}}$ increasing when $f'x < 0$

$8 - \frac{2}{(x+1)^2} < 0$

$\frac{-2}{(x+1)^2} > -8$

$-2 > 8(x+1)^2$

$-2 > 8(x^2+2x+1)$

$-2 > 8x^2 + 16x + 8$

$\cancel{0 > 8x^2+16x+10}$

$8x^2 + 16x + 6 < 0$

$-0.5 < x < 0.5$

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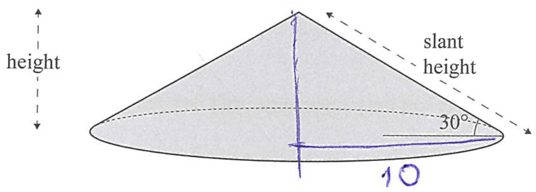
(d) For what value(s) of $x$ is the tangent to the graph of the function $f(x)=\dfrac{x+4}{x(x-5)}$ parallel to the $x$-axis?

*You must use calculus and show any derivatives that you need to find when solving this problem.*

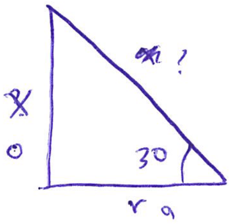
(e) Salt harvested at the Grassmere Saltworks forms a cone as it falls from a conveyor belt.
The slant of the cone forms an angle of 30° with the horizontal.
The conveyor belt delivers the salt at a rate of 2 m$^3$ of salt per minute.




https://bronzblog.files.wordpress.com/2013/07/imgp1182.jpg

Find the rate at which the slant height is increasing when the radius of the cone is 10 m.

*You must use calculus and show any derivatives that you need to find when solving this problem.*




$\frac{dh}{dt} = ?$

tan(30) x

~~dm/dt~~

$r = 10$

$x = \tan^{-1}(30) \times 10$

$= 15.37475331$

$= 15.37\text{ m}$ (4sf)

slant height (h) $= \sqrt{15.37^2 + r^2}$

~~$\frac{dh}{dt}$ slant height $= \sqrt{15.37^2 + 10m^2}$ (at 10 cm)~~

~~$= 18.33$ m (4sf)~~

~~$\frac{dh}{dr} = \sqrt{(x^2+y^2)}$~~

$\frac{dh}{dr} = -\frac{1}{2}(15.37^2 + r^2)^{-1/2} \cdot 2$

$= -(15.37^2 + r^2)^{-1/2}$

when $r = 10$

$\frac{dh}{dr} = -(15.37^2 + 10^2)^{-1/2}$

$= -0.0545$ (3sf)

∴ increasing at rate of 0.0545 m per minute

A3

## QUESTION TWO

(a) Differentiate $f(x)=\sqrt[5]{x-3x^2}$.

$f(x) = (x-3x^2)^{1/5}$

$f'(x) = \frac{1}{5}(x-3x^2)^{-4/5} \cdot (1-6x)$

(b) Find the gradient of the normal to the curve $y = x - \dfrac{16}{x}$ at the point where $x = 4$.

*You must use calculus and show any derivatives that you need to find when solving this problem.*

$M_T \times M_N = -1$

$\frac{dy}{dx} = 1 + 16x^{-2}$ $\quad \frac{dy}{dx} = 0$

when $x = 4$

$\frac{dy}{dx} = 1 + 16(4)^{-2}$

$= 2$

$\therefore M_T = 2$ so $M_N = \frac{-1}{2}$

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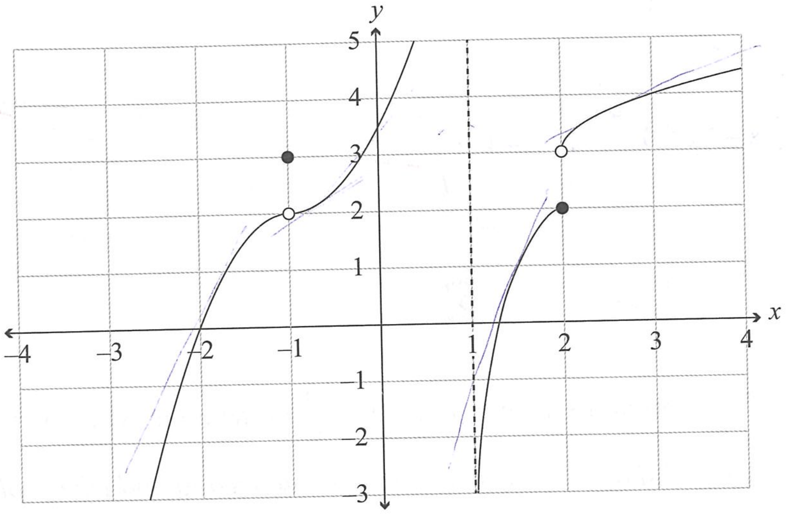
(c) The graph below shows the function $y = f(x)$.




For the function above:

(i) Find the value(s) of $x$ that meet the following conditions:

1. $f(x)$ is not defined: $x = 1$ and $x = 2$

2. $f(x)$ is not differentiable: were $x = -1$, $x = 1$ and $x = 2$

3. $f''(x) > 0$: $-3 \leq x < 1$ , & $2 < x < 4$

(ii) What is the value of $f(-1)$? 3

State clearly if the value does not exist.

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(iii) What is the value of $\lim\limits_{x \to 2} f(x)$? does not exist

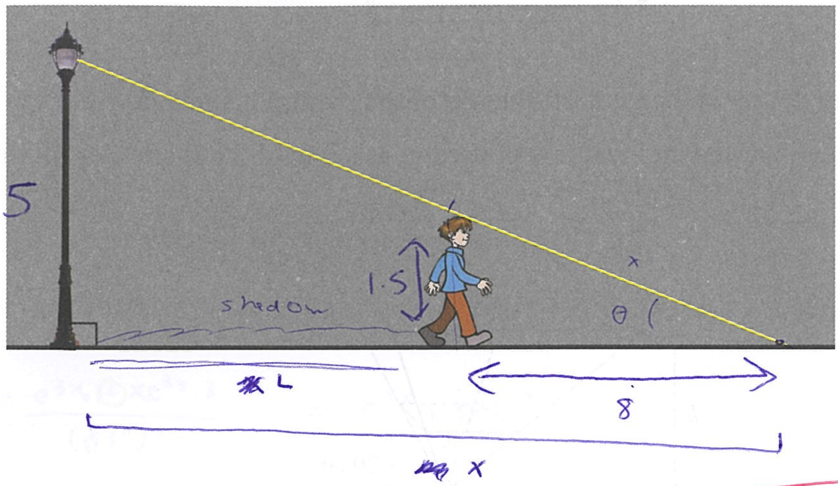
State clearly if the value does not exist.

(d) A street light is 5 m above the ground, which is flat.

A boy, who is 1.5 m tall, is walking away from the point directly below the streetlight at 2 metres per second.

At what rate is the length of his shadow changing when the boy is 8 m away from the point directly under the light?

*You must use calculus and show any derivatives that you need to find when solving this problem.*



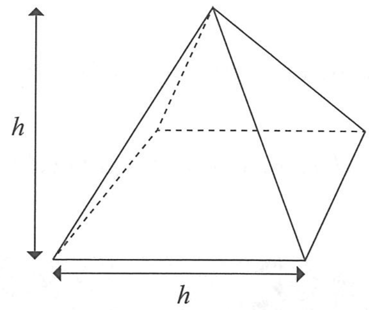

$\frac{dl}{dt} = ?$

$\frac{dm}{dt} = 2$ $\cancel{\frac{dl}{dt}}$ $\theta = \tan\left(\frac{1.5}{8}\right) = 0.003272504°$

$x = 8 + L$ $= 0.00327°$

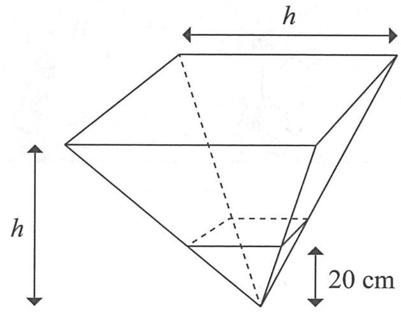
$L = x - 8$

(e) A water container is constructed in the shape of a square-based pyramid. The height of the pyramid is the same as the length of each side of its base.




A vertical height of 20 cm is then cut off the top of the pyramid, and a new flat top added.

The pyramid is then inverted and water is poured in at a rate of 3000 $cm^3$ per minute.




Find the rate at which the surface area of the water is increasing when the depth of the water is 15 cm.

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

*You must use calculus and show any derivatives that you need to find when solving this problem.*

~~SA~~ $V = \frac{1}{3} \times h^2 h$ $\quad \frac{dV}{dt} = 3000$ $\quad \frac{dV}{dh} = \frac{2}{3}h = \frac{2}{3} \times 15 = 10$

$SA = h^2 + 4 \times (\frac{1}{2} h \times (h-20))$ $\quad \frac{dSA}{dh} = 2h + 4(\frac{1}{2} \times (h-20))$

$\frac{dSA}{dt} = \frac{dSA}{dh} \times \frac{dh}{dV} \times \frac{dV}{dt}$

$= \cancel{2(15) \times \frac{1}{10} \times 3000}$

$(2 \times 15 + 4(\frac{1}{2} \times (15-20))) \times \frac{1}{10} \times 3000$

$= 6000$ $cm^2$ per minute.

A4

## QUESTION THREE

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(a) For what value(s) of $x$ does the tangent to the graph of the function $f(x) = 5\ln(2x-3)$ have a gradient of 4?

*You must use calculus and show any derivatives that you need to find when solving this problem.*

$f'(x) = \frac{5}{2x-3} \times 2 = \frac{10}{2x-3}$

when $f'(x) = 4$

$4 = \frac{10}{2x} - 3$

$\frac{10}{2x} = 1 \quad \therefore 2x = 10 \quad \therefore x = 5$

(b) If $f(x) = \frac{x}{e^{3x}}$, find the value(s) of $x$ such that $f'(x) = 0$.

*You must use calculus and show any derivatives that you need to find when solving this problem.*

$f'(x) = \frac{e^{3x} \cdot 1 + x e^{3x} \cdot 3}{(e^{3x})^2} = \frac{e^{3x} + 3x e^{3x}}{e^{6x}}$

When $f'(x) = 0$

$e^{3x} + 3x e^{3x} = 0$

$e^{3x}(3x+1) = 0$

$3x + 1 = 0 \quad \therefore 3x = -1 \quad x = \frac{-1}{3}$

(c) A curve is defined parametrically by the equations $x = 3\cos t$ and $y = \sin 3t$.

Find the gradient of the normal to the curve at the point where $t = \frac{\pi}{4}$.

*You must use calculus and show any derivatives that you need to find when solving this problem.*

$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} \qquad \frac{dx}{dt} = -3\sin t \qquad \frac{dy}{dt} = 3\cos 3t$

$dy/dx = \frac{3\cos 3t}{-3\sin t} \qquad M_N \times M_T = -1$

when $t = \pi/4$

$\frac{3\cos 3(\pi/4)}{-3\sin(\pi/4)} = -0.707\ (3sf) \quad \therefore M_N = \frac{-1}{M_T} = \frac{-1}{-0.707}$

$= 1.41\ (3sf)$

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(d) The equation of motion of a particle is given by the differential equation

$$\frac{d^2x}{dt^2} = -k^2x$$

where $x$ is the displacement of the particle from the origin at time $t$, and k is a positive constant.

(i) Show that $x = A\cos kt + B\sin kt$, where A and B are constants, is a solution of the equation of motion.

(ii) The particle was initially at the origin and moving with velocity 2k.

Find the values of A and B in the solution $x = A\cos kt + B\sin kt$.

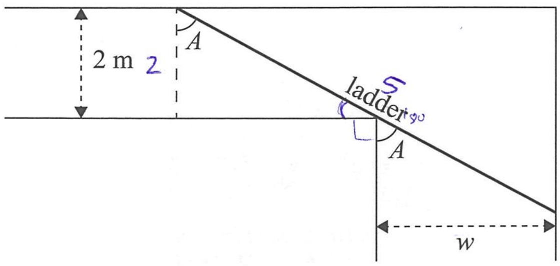
$(0,0)$

$2k$

$\frac{d^2x}{dt^2} = k^2x$

(e) A corridor is 2 m wide.

At the end it turns 90° into another corridor.




What is the minimum width, $w$, of the second corridor if a ladder of length 5 m can be carried horizontally around the corner?

*You must use calculus and show any derivatives that you need to find when solving this problem.*

minimum were =0

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N2

**Extra paper if required.**
**Write the question number(s) if applicable.**

QUESTION NUMBER

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## Annotated Exemplar Template

| Achieved exemplar for 91578 2015 | | | Total score | 9 |
|---|---|---|---|---|
| **Q** | **Grade score** | **Annotation** | | |
| 1 | A3 | The candidate could differentiate a trig function using the chain rule, but was unable to find the gradient of the tangent in 1b because they omitted brackets.<br>The candidate correctly differentiated the function in 1c but was unable to solve correctly.<br>The candidate was unable to make progress on 1d or 1e. | | |
| 2 | A4 | The candidate was able to use the chain rule, differentiate functions with negative powers and has some knowledge of the properties of graphs (limits, differentiability, continuity, concavity). | | |
| 3 | N2 | The candidate correctly differentiated 3a but made an algebraic error in attempting to solve. The candidate did not apply the quotient rule correctly in 3b. The candidate demonstrated the differentiation of a pair of parametric equations but was unable to solve for the gradient of the tangent. | | |