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3

91579



915790



NEW ZEALAND QUALIFICATIONS AUTHORITY  
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD  
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SUPERVISOR'S USE ONLY

## Level 3 Calculus, 2015

### 91579 Apply integration methods in solving problems

2.00 p.m. Wednesday 25 November 2015  
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

**Not Achieved**

**TOTAL**

**6**

ASSESSOR'S USE ONLY

## QUESTION ONE

ASSESSOR'S  
USE ONLY

- (a) Find
- $\int(\sqrt{x} + 6\cos 2x)dx$
- .

$$\int (x^{\frac{1}{2}} + 6\cos 2x) dx$$

$$= \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + \frac{6}{2} \sin 2x + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} + 3 \sin 2x + C$$

- (b) Solve the differential equation
- $\frac{dy}{dx} = \frac{2}{x}$
- , given that when
- $x = 1, y = 3$
- .

$$\int \frac{2}{x} \cdot dx$$

$$y = 2 \ln|x| + C = 3$$

$$\text{where } x = 1, y = 2 \ln|1| + C = 3$$

$$\Rightarrow y = 3 = 2 \cdot 0 + C$$

$$\Rightarrow C = 3$$

- (c) If
- $\frac{dy}{dx} = \frac{e^{2x}}{4y}$
- and
- $y = 4$
- when
- $x = 0$
- , find the value of
- $y$
- when
- $x = 2$
- .

$$\int \frac{e^{2x}}{4y} \cdot dx$$

$$y = e^{2x} \ln|4y| + C$$

$$\text{where } y = 4 \text{ and } x = 0 \quad e^{2(0)} \ln|4(4)| + C = 4$$

$$4 = 1 \cdot 2.773 + C$$

$$C = 4 - 2.773$$

$$= 1.227$$

$$\Rightarrow y = e^{2x} \ln|4y| + 1.227$$

$$\text{where } x = 2 \quad y = e^{2(2)} \ln|4y| + 1.227$$

$$= 54.59 \ln|4y| + 1.227$$

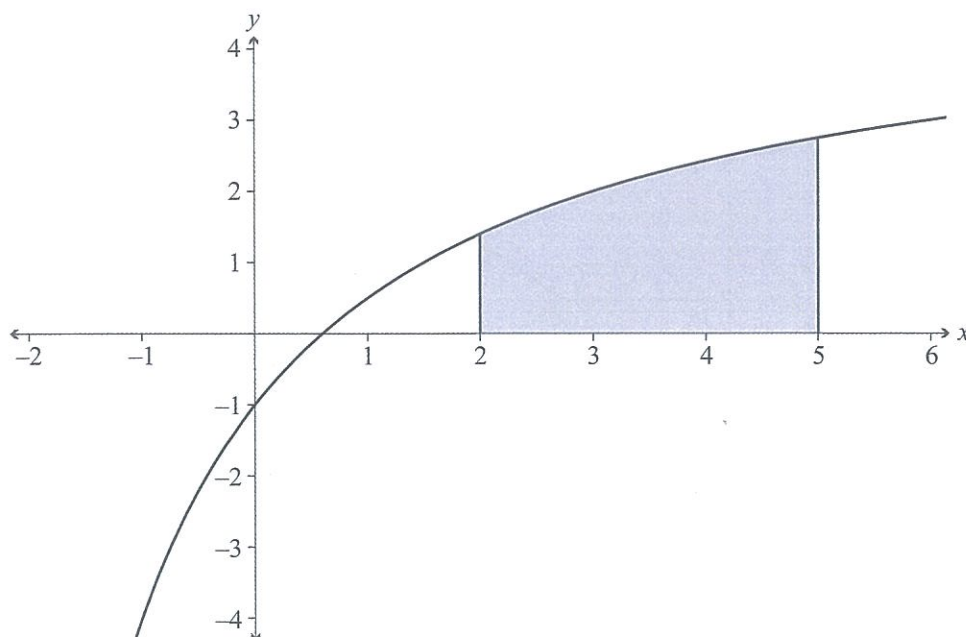
$$\Rightarrow 54.59 \ln|4y| = 1.227$$

$$\Rightarrow 54.59 \cdot 4y = e^{1.227} = 3.4109$$

$$\Rightarrow 4y = \frac{3.4109}{54.59} = 0.0624 \Rightarrow y = \frac{0.0624}{4} = 0.0156$$

- (d) Use integration to find the area enclosed between the curve  $y = \frac{5x-3}{x+3}$  and the lines  $y = 0$ ,  $x = 2$  and  $x = 5$ .

The area is shown shaded in the diagram below.



Show your working.

You must use calculus and give the results of any integration needed to solve this problem.

$$\int_2^5 \frac{5x-3}{x+3} dx = \int_2^5 \frac{5x-3}{x+3} dx$$

$$= \int_2^5 \frac{5x-3}{x+3} dx$$

$$= \int_2^5 \frac{5x-3}{x+3} dx$$

$$= \int_2^5 \frac{5x-3}{x+3} dx$$

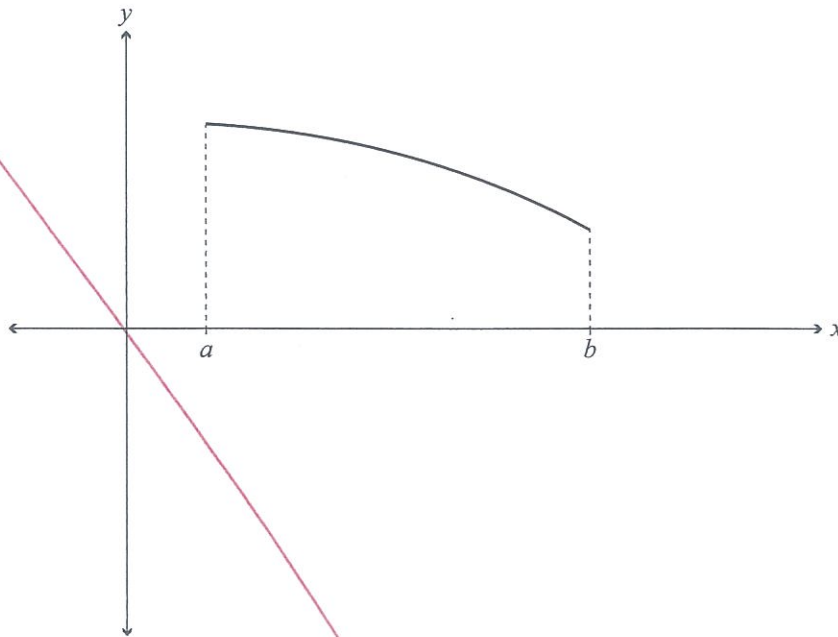
$$= \int_2^5 \frac{5x-3}{x+3} dx$$

$$= \int_2^5 \frac{5x-3}{x+3} dx$$

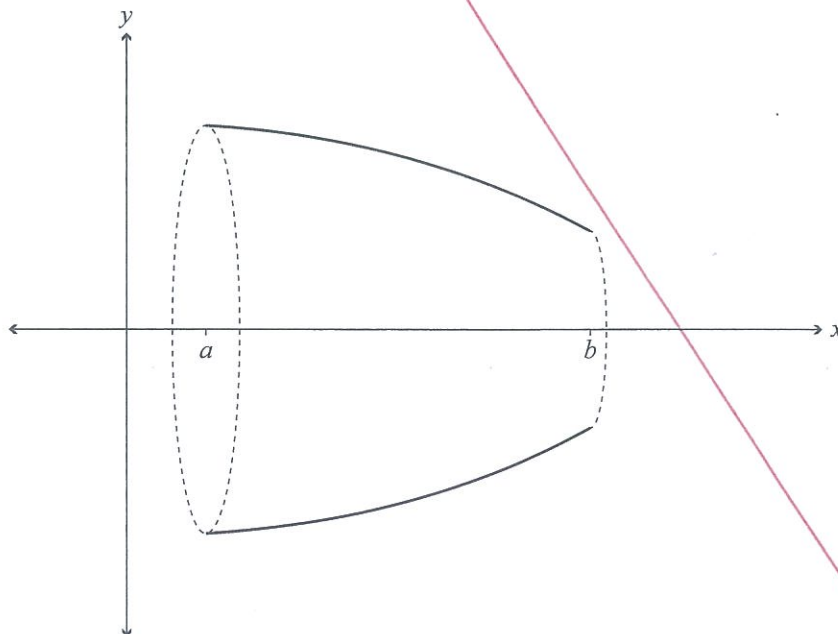
(e)

ASSESSOR'S  
USE ONLY

Consider the curve defined by the function  $y = f(x)$ , bounded by  $x = a$  and  $x = b$ .



This portion of the curve is rotated around the x-axis, as shown below.



The volume created by this rotation is given by the formula

$$\text{Volume} = \pi \int_a^b (f(x))^2 dx$$

*You must use calculus and give the results of any integration needed to solve this problem.*

[illegible]

A3

## QUESTION TWO

ASSESSOR'S  
USE ONLY

- (a) Find  $\int \left(3 - \frac{5}{x^2}\right) dx$ .

$$y = 3x - 5 \ln|x^2| + C$$

- (b) Use the values given in the table below to find an approximation to  $\int_1^{2.5} f(x) dx$ , using the Trapezium Rule.

$x$	1	1.25	1.5	1.75	2	2.25	2.5
$f(x)$	0.3	0.7	1.65	1.9	2.35	1.7	1.1

$$= \frac{1}{2} [0.3 + 1.1 + 2(0.7 + 1.65 + 1.9 + 2.35 + 1.7)]$$

$$= \frac{1}{2} [1.4 + 2(9.4)]$$

$$= \frac{1}{2} [1.4 + 18.8]$$

$$= \frac{1}{2} [20.2]$$

$$= 10.1 \text{ units}^2$$

- (c) An object originally moving at a constant velocity suddenly starts to accelerate. From the start of the object's acceleration the motion of the object can be modelled by the differential equation

$$\frac{dv}{dt} = \frac{50t^2 - 80\sqrt{t}}{5\sqrt{t}} \text{ for } 0 \leq t \leq 20$$

where  $v$  is the velocity of the object in  $\text{m s}^{-1}$

and  $t$  is the time in seconds after the object starts to accelerate.

If the original velocity of the object was  $6 \text{ m s}^{-1}$ , find the velocity of the object when  $t = 4$ .

*You must use calculus and give the results of any integration needed to solve this problem.*

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- (d) In the town of Clarkeville, the rate at which the population,  $P$ , of the town changes at any instant is proportional to the population of the town at that instant.

- (i) Write a differential equation which models this situation.

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- (ii) At the start of 2000, the population of the town was 12 000.

At the start of 2010, the population of the town was 16 000.

Solve the differential equation in (i) to find the population the town will have at the start of 2025.

*You must use calculus and give the results of any integration needed to solve this problem.*

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- A Cartesian coordinate system showing the graph of the function  $y = \frac{1}{x}$  (a hyperbola with branches in the first and third quadrants) and its tangent line at the point  $(k, \frac{1}{k})$ . The tangent line is a straight line with a positive slope. The region bounded by the curve and the tangent line in the first quadrant is shaded in light blue. The x-axis is labeled with  $x$  and the y-axis with  $y$ . The point of tangency is marked with a vertical line segment from the x-axis at  $k$  to the curve.

Find the value of  $k$ .

This image shows a blank sheet of white paper with horizontal blue or grey ruling lines. A single diagonal red line runs from the top center towards the bottom right corner, creating a margin. The paper appears to be part of a notebook or a template for writing.

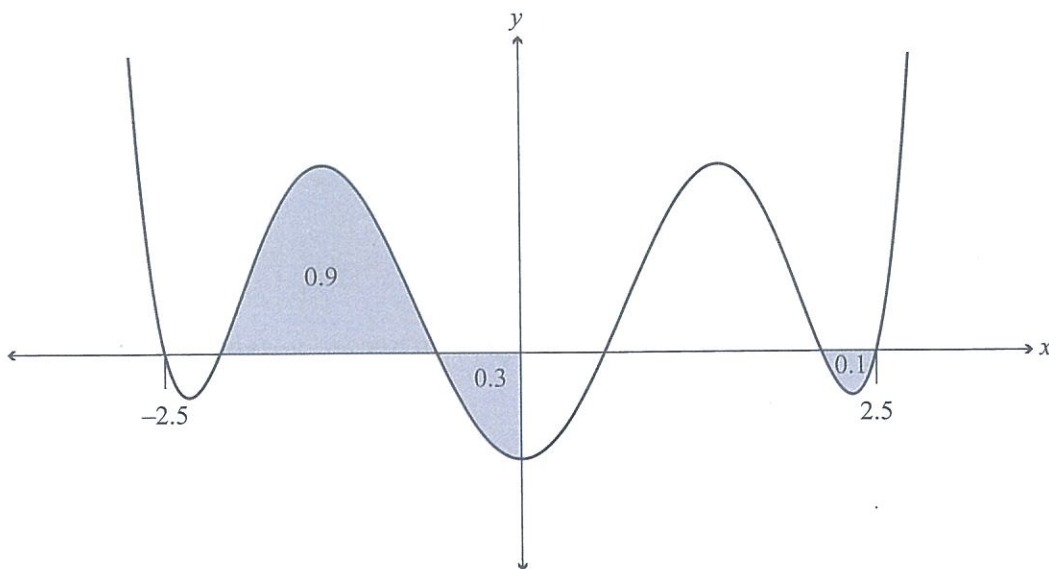
## QUESTION THREE

ASSESSOR'S  
USE ONLY

- (a) Find
- $\int ((x+4)^2 + 8e^{4x}) dx$
- .

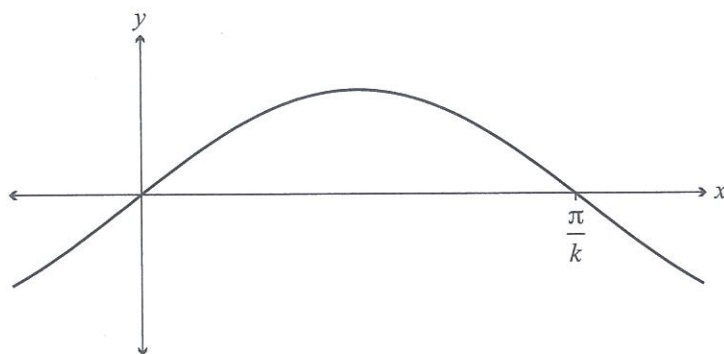
$$\int = \frac{1}{3}(x+4)^3 + 2e^{4x} + C$$

- (b) The graph of the function  $y = f(x)$  below is symmetrical about the  $y$ -axis. The areas of the shaded regions are given.

Find  $\int_{-2.5}^{2.5} f(x) dx$ .

$$\begin{aligned} \int_{-2.5}^{2.5} x \, dx &= \left[ \frac{1}{2}x + C \right]_{-2.5}^{2.5} \\ &= \left( \frac{1}{2}2.5 + C \right) - \left( \frac{1}{2}(-2.5) + C \right) \\ &= 1.25 - (-1.25) \\ &= 2.5 \end{aligned}$$

- (c) Find an expression in terms of  $k$  for the area bounded by the function  $y = \sin kx$  and the  $x$ -axis, between  $x = 0$  and  $x = \frac{\pi}{k}$ .



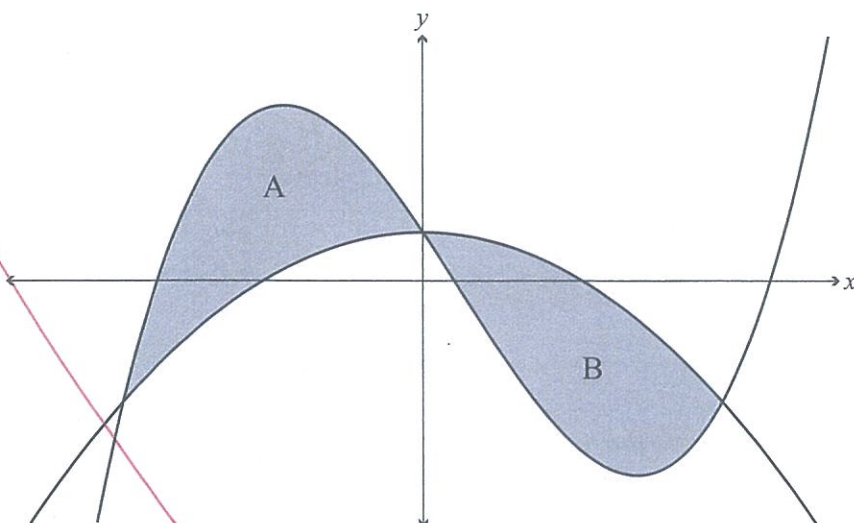
*You must use calculus and give the results of any integration needed to solve this problem.*

Handwritten solution area with horizontal lines for writing. A large red diagonal line is drawn across the page, and a red oval is drawn around the first line.

- (d) The graphs of  $f(x) = -x^2 + 2$  and  $g(x) = x^3 - x^2 - kx + 2$  are shown below.

The graphs intersect and create two closed regions, A and B.

ASSESSOR'S  
USE ONLY



Show that these two regions have the same area.

*You must use calculus and give the results of any integration needed to solve this problem.*

- (e) An object starts from rest.

The object's acceleration is given by the formula  $a = B(e^{kt})^2$

where  $a$  is the acceleration of the object in  $\text{m s}^{-2}$

and  $t$  is the time, in seconds, from when the object started moving.

Show that the time that it takes the object to reach velocity  $v_0$  is

$$t = \frac{1}{2k} \ln \left( \frac{2v_0 k + B}{B} \right)$$

*You must use calculus and give the results of any integration needed to solve this problem.*

ASSESSOR'S  
USE ONLY

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Extra paper if required.  
Write the question number(s) if applicable.

ASSESSOR'S  
USE ONLY

QUESTION  
NUMBER

Extra paper if required.  
Write the question number(s) if applicable.

ASSESSOR'S  
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