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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Mathematics and Statistics (Statistics), 2015

91585 Apply probability concepts in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 24

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QUESTION ONE

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(a) The following table shows the number of vehicles reported to the NZ Police as stolen over 2011 to 2013, and the number of vehicles registered with the NZ Transport Agency in each of these years.

	2011	2012	2013
Number of vehicles reported as stolen	20724	17807	19221
Number of vehicles registered	4210511	4248612	4315539

(i) Which of these years had the greatest overall risk of a vehicle being stolen in New Zealand?

Support your answer with appropriate calculations.

P(vehicle being stolen 2011) = $\frac{20724}{4210511}$ = 0.00492

P(vehicle being stolen 2012) = $\frac{17807}{4248612}$ = 0.00419

P(vehicle ~~stolen~~ being stolen 2013) = $\frac{19221}{4315539}$ = 0.00445

2013 has the highest risk of a vehicle being stolen in New Zealand of these 3 years (0.00445)

(ii) Give ONE reason why the risks calculated in part (i) are only estimates of the true overall risk of a vehicle being stolen in that year.

Not all cars are registered, and also possibly not every car that got stolen was reported as stolen. If both or one of these numbers is different it changes the probability

(iii) A car owner wants to use the overall risk of a car being stolen in New Zealand during 2013 to estimate the risk of their own car being stolen during 2015.

Discuss what else the car owner should consider to estimate this risk.

The risk may increase or decrease each year (however it went ~~up~~ down from 2011 then up from 2012 so it is very hard to judge whether it will go up of down. The number of registered cars is going up each year while the number of stolen cars is varying, maybe this will mean the ~~[illegible]~~ risk will decrease if the number of cars keeps decreasing. Also, there will be areas in New Zealand where a car ~~is~~ has more or less risk of being stolen, so the car owner should also definitely consider this. – may be hugely different in different areas.

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(b) An importer of second-hand cars into New Zealand has recorded whether each car has the petrol cap on the left-hand side or the right-hand side of the car, in addition to other information about the cars.

For the last shipment of second-hand cars imported, $\frac{13}{21}$ left of the cars had the petrol cap on the left-hand side and 22.8% of the cars were silver.

(i) One car is chosen at random from this shipment of imported second-hand cars.

Determine the probability that this car is silver and has the petrol cap on the left-hand side.

State the assumption you need to make to determine this probability.

P(car is silver AND has left-hand petrol cap) = $\frac{13}{21} \times$ ~~[illegible]~~ 0.228

= 0.1411

Assumption: the probability of ~~[illegible]~~ the petrol cap being on the left side is independent of the probability of the car being silver.

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(ii) A customer at a petrol station has observed that of the $\boxed{\text{ten cars}}$ currently getting petrol, seven of these cars have petrol caps on the left-hand side.

Explain to the customer why a generalisation should not be made that cars in New Zealand are more likely to have petrol caps on the left-hand side, based on what the customer has observed.

A sample of 10 cars is a very very small compared to all of New Zealand. A sample of 1000 cars might give a better idea of the probability. Because the event of ~~probability that~~ a car that is getting petrol being ~~is~~ silver is ~~[illegible]~~ purely by chance, we should not expect this to be exactly the same as the true probability, and especially not as the sample (10 cars) was very small. It is also possible that cars are more likely to have left-hand sided petrol caps in different areas of the country.

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QUESTION TWO

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(a) An importer of cars into New Zealand is suspected of rounding the odometer reading (the measure of the total kilometres the car has driven) to the nearest 10 kilometres for some of the advertisements on their website.

The car importer currently has 20 cars listed for sale on their website.

The odometer readings for these cars are listed below.

1485	25384	25499	26890	29568
35279	47872	49200	64788	68050
72690	75730	84457	91575	92297
93033	109532	113395	137209	142980

(i) What proportion of cars advertised by the importer has 0 as the last digit of the odometer reading?

P(last digit is 0) $= \frac{6}{20} = \frac{3}{10} = 0.3$

(ii) Assuming that the last digit of an odometer reading for a car is determined by chance alone, give a model (theoretical) estimate for the probability that the last digit of an odometer reading is 0. $\boxed{P(\text{last digit is } 0) = \frac{1}{10} = 0.1}$

~~Expect: 0.1 × 20 cars = 2 with 0 as the last digit.~~

(iii) A concerned customer conducted a simulation to investigate the variability in the proportion of cars in sets of 20 that have 0 as the last digit of the odometer reading, based on an assumption that the last digit of an odometer reading for a car is determined by chance alone.

A summary of the simulation results is shown below (1000 trials). 20,000

Proportion with 0 last digit	$\frac{0}{20}$	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{4}{20}$	$\frac{5}{20}$	$\frac{6}{20}$	$\frac{7}{20}$	$\frac{8}{20}$ or higher
Frequency	130	260	289	187	92	32	9	1	0

Based on these simulation results, what conclusion could the customer make in respect to whether or not the last digit of an odometer reading for the cars advertised is determined by chance alone?

The probability of getting $\frac{6}{20}$ with 0 as the last digit ~~is~~ ~~is~~ is 0.009 $\left(\frac{9}{1000}\right)$ which is very very small.

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	Japan	Not
used	0.513	
Not	0.126	
	~~0.639~~	0.361

(b) In 2013, 63.9% of imported cars registered with the New Zealand Transport Agency were manufactured in Japan. Of these cars manufactured in Japan, 80.3% were used cars.

Suppose that one of the imported cars registered with the New Zealand Transport Agency in 2013 was selected at random.

(i) Explain why the events "The car was manufactured in Japan" and "The car is a used car" are not mutually exclusive.

Include statistical reasoning in your explanation.

Mutually exclusive means ~~P(A∩B) = 0~~ that the two events cannot occur at the same time. $(P(A \cap B) = 0)$

However, 80.3% of the cars manufactured in Japan were also used cars, so these events do occur at the same time.

$P(A \cap B) = 0.513 \neq 0$ $\quad P(A \cap B) \neq 0$

$P(A \cap B) = 0.513$, not 0.

(ii) Explain why it can be deduced from this information alone that the car selected is more likely to have been manufactured in Japan than not, given the car selected is a used car.

~~P(Made in Japan | used) =~~

$P(A \cap B) = 0.513$

So, 51.3% of cars were made in Japan and also were used cars. This means, as 63.9% were made in Japan, only 36.1% were not made in Japan, and this value 36.1% is both used cars and not used cars (not made in Japan). So the ~~maximum~~ percentage of cars that are used is 36.1% (0.361), although likely lower.

0.513 is still higher ~~lower~~ than 0.361,

$\frac{0.513}{0.361} = 1.42$ (1.4 times higher).

This means, although we dont know ~~how many~~ what proportion of cars were not made in Japan and not used, it is no higher than 0.361, so there is at least 1.42 times more cars made in Japan than not in Japan, given they are used cars. So there is a higher probability they are made in Japan.

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QUESTION THREE

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(a) People take their cars to testing centres for a Warrant of Fitness (WOF).

Three testing centres were recently reviewed over a one-month period: testing centre A, testing centre B, and testing centre C. During this time, all results for tests completed by each of the testing centres were recorded.

40% of the tests reviewed were completed by testing centre A, and 25% of the tests reviewed were completed by testing centre B. $= 0.35$ C

Of the tests completed by testing centre A, 82% were successful (the car passed the WOF).
Of the tests completed by testing centre B, 96% were successful.
Of the tests completed by testing centre C, 94% were successful.

(i) What percentage of tests completed during the review were successful?

	A	B	C	
Successful	0.328	0.240	0.329	0.897
Unsuccessful	0.072	0.01	0.021	0.103
	0.4	0.25	0.35	

$P(\text{successful test}) = (0.4 \times 0.82) + (0.25 \times 0.96) + (0.35 \times 0.94)$

$= 0.328 + 0.24 + 0.329$

$= 0.897$

So, 89.7% of the tests during the review were successful.

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(ii) Of the tests that were unsuccessful, what proportion were completed at testing centre C?

You may wish to assume that there were 10 000 tests completed during the review of the three testing centres.

$\frac{0.021}{0.103} = 0.204$

$P(\text{test completed at centre C} \mid \text{unseccessful})$

$= \frac{P(\text{centre C} \cap \text{unseccessful})}{P(\text{unsuccessful})}$

$= \frac{0.021}{0.103} = 0.2039$ (this would be 2038.8 or 2039 in 10000 tests)

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(iii) Based on the results of the review, a car owner has decided that they should take their car to testing centre B to increase their chances of having a successful WOF test.

Is this decision justified?

P(successful | centre A) = 0.82
P(successful | centre B) = 0.96
P(successful | centre C) = 0.94

It is true that ~~[illegible]~~ a car tested at centre B has a higher chance of passing, but ~~[illegible]~~ it is not necessary completely because of the way that centre tests. The people / type of cars / customers that go to centre C might be more likely to pass anyway than ones that go to centre C. But it is still probably more likely, based on this information, that they pass if they go to centre B, so yes, it is justified.

ASSESSOR'S USE ONLY

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(b) Information about the ages of cars and motorcycles registered with the New Zealand Transport Agency (NZTA) at the end of 2013 is presented in the table below. This table shows information about only cars or motorcycles less than 5 years old at the end of 2013.

	Age of vehicles registered with NZTA at the end of 2013				
	0 years old	1 year old	2 years old	3 years old	4 years old
Proportion of cars	0.238	0.223	0.188	0.186	0.165
Proportion of motorcycles	0.215	0.181	0.177	0.183	0.244

One car and one motorcycle are chosen at random from vehicles registered with NZTA at the end of 2013.

Given that both vehicles are less than five years old, estimate the probability that the motorcycle is at least two years older than the car.

Support your answer with appropriate statistical statements and calculations.

~~P~~ {motorcycle is at least 2 yrs older than car} = (C=0, M=2), (C0, M3) (C0, M4), (C1, M3), (C1 M4), (C2, M4)

P(Car 0 ∧ motorcycle 2) = 0.238 × 0.177 = 0.0421
P(Car 0 ∧ motorcyle 3) = 0.238 × 0.183 = 0.0436
P(Car 0 ∧ motorcyle 4) = 0.238 × 0.244 = 0.0581
P(Car 1 ∧ motorcyle 3) = 0.223 × 0.183 = 0.0408
P(Car 1 ∧ motorcyle 4) = 0.223 × 0.244 = 0.0544
P(Car 2 ∧ motorcycle 4) = 0.188 × 0.244 = 0.0459

P(motorcyle is at least 2yrs older than car) = 0.0421 + 0.0436 + 0.0581 + 0.0408 + 0.0544 + 0.0459 = 0.2849

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Annotated Exemplar: Level 3 Probability (91585) 2015

“Excellence” exemplar for 91585 2015		Total score	24
Q	**Grade score**	**Annotation**	
1	E8	(a) (i) Calculated the three risks and coped with values in standard form, but made the incorrect decision. (ii) Gave examples of why the collected data may not be accurate. (iii) Discussed that historical data would be useful and that there are other factors that affect the risk of having a car stolen. (b) (i) Calculated the combined probability, but stated the assumption was about of independent probabilities rather than independent events. (ii) Vague about how the true probability will be different from the observed results, and that a further sample is likely to have different results because of sampling variation.	
2	E8	(a) (i) Found the observed proportion. (ii) Calculated the chance of the number ending in a zero. (iii) Understood the results of the simulation, but did not make the conclusion that it was likely that the importer was rounding the odometer readings. (b) (i) Understood the concept of mutually exclusive events and made the necessary calculation to make the test. (ii) Calculated the proportion of cars that were used and manufactured in Japan and realised that because this was greater than 50% that I would be greater than the proportion of used cars not manufactured in Japan. Also considered the maximum proportion of used cars not manufactured in Japan.	
3	E8	(a) (i) Calculated the probability of the combined event. (ii) Calculated the proportion of cars from testing centre C that were unsuccessful tests. (iii) Made the decision that testing centre B had the highest pass rate, but did not go into enough detail about reasons why you may still not use testing centre B. (b) Calculated the probability that a motorcycle was at least two years older than a car.	