3

SUPERVISOR'S USE ONLY

91585

915850

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Mathematics and Statistics (Statistics), 2015

91585 Apply probability concepts in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 17

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) The following table shows the number of vehicles reported to the NZ Police as stolen over 2011 to 2013, and the number of vehicles registered with the NZ Transport Agency in each of these years.

	2011	2012	2013
Number of vehicles reported as stolen	20724	17807	19221
Number of vehicles registered	4210511	4248612	4315539

(i) Which of these years had the greatest overall risk of a vehicle being stolen in New Zealand?

Support your answer with appropriate calculations.

20724 ÷ 4210511 = 0.004921...

17807 ÷ 4284612 = 0.004156...

19221 ÷ 4315539 = 0.004453...

The Greatest risk is 2011, with a higher proportion of cars stolen

U

(ii) Give ONE reason why the risks calculated in part (i) are only estimates of the true overall risk of a vehicle being stolen in that year.

Because not every vehicle stolen will have been reported, just the majority. Also not every car will have been registered.

U

(iii) A car owner wants to use the overall risk of a car being stolen in New Zealand during 2013 to estimate the risk of their own car being stolen during 2015.

Discuss what else the car owner should consider to estimate this risk.

The number of registered vehicles is rising so it is likely that more cars will be stolen although the risk might be higher or lower. The overall risk would be roughly around 1/4500 cars being stolen..

NS

ASSESSOR'S USE ONLY

(b) An importer of second-hand cars into New Zealand has recorded whether each car has the petrol cap on the left-hand side or the right-hand side of the car, in addition to other information about the cars.

For the last shipment of second-hand cars imported, $\frac{13}{21}$ of the cars had the petrol cap on the left-hand side and 22.8% of the cars were silver.

(i) One car is chosen at random from this shipment of imported second-hand cars.

Determine the probability that this car is silver and has the petrol cap on the left-hand side.

State the assumption you need to make to determine this probability.

Assuming ~~all the cars~~ we only take the last shipment

13/21 = .619... = 61.9%

.619... × .228 = 0.14114... = 14.1% (1dp)

P(silver and left hand side) = 14.1%

(ii) A customer at a petrol station has observed that of the ten cars currently getting petrol, seven of these cars have petrol caps on the left-hand side.

Explain to the customer why a generalisation should not be made that cars in New Zealand are more likely to have petrol caps on the left-hand side, based on what the customer has observed.

As shown above, 13/21 cars is 61.9%. 7/10 cars getting petrol were on the left hand side. That is 70%. This observation shows it sway quite easily from side to side

QUESTION TWO

ASSESSOR'S USE ONLY

(a) An importer of cars into New Zealand is suspected of rounding the odometer reading (the measure of the total kilometres the car has driven) to the nearest 10 kilometres for some of the advertisements on their website.

The car importer currently has 20 cars listed for sale on their website.

The odometer readings for these cars are listed below.

1 485	25 384	25 499	26 890	29 568
35 279	47 872	49 200	64 788	68 050
72 690	75 730	84 457	91 575	92 297
93 033	109 532	113 395	137 209	142 980

(i) What proportion of cars advertised by the importer has 0 as the last digit of the odometer reading?

6/20 = 3/10

(ii) Assuming that the last digit of an odometer reading for a car is determined by chance alone, give a model (theoretical) estimate for the probability that the last digit of an odometer reading is 0.

1/10

(iii) A concerned customer conducted a simulation to investigate the variability in the proportion of cars in sets of 20 that have 0 as the last digit of the odometer reading, based on an assumption that the last digit of an odometer reading for a car is determined by chance alone.

A summary of the simulation results is shown below (1000 trials).

Proportion with 0 last digit	$\frac{0}{20}$	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{4}{20}$	$\frac{5}{20}$	$\frac{6}{20}$	$\frac{7}{20}$	$\frac{8}{20}$ or higher
Frequency	130	260	289	187	92	32	9	1	0

Based on these simulation results, what conclusion could the customer make in respect to whether or not the last digit of an odometer reading for the cars advertised is determined by chance alone?

The customer could conclude that it is not determined by chance alone. One would expect a 1/10 chance for every car to have any number at the end. As there is nothing at an 8 or above given 1000 trials, it is too unlikely that they are determined by chance alone. Statistically there should be a relatively even amount of each number of the end of the odometers.

(b) In 2013, 63.9% of imported cars registered with the New Zealand Transport Agency were manufactured in Japan. Of these cars manufactured in Japan, 80.3% were used cars.

Suppose that one of the imported cars registered with the New Zealand Transport Agency in 2013 was selected at random.

ASSESSOR'S USE ONLY

(i) Explain why the events "The car was manufactured in Japan" and "The car is a used car" are not mutually exclusive.

Include statistical reasoning in your explanation.

They are not mutually exclusive because over half of used cars were manufactured in Japan, so it is likely that choosing a random will be a Japanese used car.

(ii) Explain why it can be deduced from this information alone that the car selected is more likely to have been manufactured in Japan than not, given the car selected is a used car.

63.9% of registered cars in NZ were made in Japan.
80.3% are used cars.

$.639 \times .803 = .513117$
$= 51.3\%$ (1dp)

This shows that over half the used cars are likely to have been made in Japan.

M5

QUESTION THREE

ASSESSOR'S USE ONLY

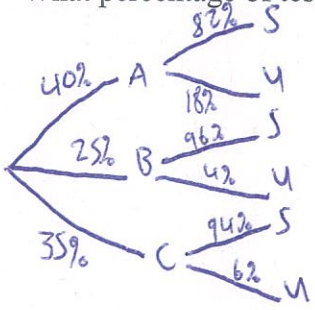
(a) People take their cars to testing centres for a Warrant of Fitness (WOF).

Three testing centres were recently reviewed over a one-month period: testing centre A, testing centre B, and testing centre C. During this time, all results for tests completed by each of the testing centres were recorded.

40% of the tests reviewed were completed by testing centre A, and 25% of the tests reviewed were completed by testing centre B.

Of the tests completed by testing centre A, 82% were successful (the car passed the WOF).
Of the tests completed by testing centre B, 96% were successful.
Of the tests completed by testing centre C, 94% were successful.

(i) What percentage of tests completed during the review were successful?



.4 × .82 = .328
.4 × .18 = .072
.25 × .96 = .24
.25 × .04 = 0.01
.35 × .94 = .329
.35 × .06 = 0.021

.328 + .24 + .329 = .897
.072 + .01 + .021 = .103
= 89.7% test were successful

(ii) Of the tests that were unsuccessful, what proportion were completed at testing centre C?

You may wish to assume that there were 10 000 tests completed during the review of the three testing centres.

Test Centre A = .072 × 10000 = 720
Test Centre B = 0.01 × 10000 = 100
Test Centre C = 0.021 × 10000 = 216

720 + 100 + 210 = 1030
210 ÷ 1030 = 0.2
Proportion = 210/1030
= 1/5

ASSESSOR'S USE ONLY

(iii) Based on the results of the review, a car owner has decided that they should take their car to testing centre B to increase their chances of having a successful WOF test.

Is this decision justified?

His decision is justified because out of all the testing centres, centre B has the highest success rate

(b) Information about the ages of cars and motorcycles registered with the New Zealand Transport Agency (NZTA) at the end of 2013 is presented in the table below. This table shows information about only cars or motorcycles less than 5 years old at the end of 2013.

	Age of vehicles registered with NZTA at the end of 2013				
	0 years old	**1 year old**	**2 years old**	**3 years old**	**4 years old**
Proportion of cars	0.238	0.223	0.188	0.186	0.165
Proportion of motorcycles	0.215	0.181	0.177	0.183	0.244

One car and one motorcycle are chosen at random from vehicles registered with NZTA at the end of 2013.

Given that both vehicles are less than five years old, estimate the probability that the motorcycle is at least two years older than the car.

Support your answer with appropriate statistical statements and calculations.

.244 x .188 = 0.045872 +
.244 x .223 = 0.054412 +
.244 x .238 = 0.058072 +
.183 x .223 = 0.040809 +
.183 x .238 = 0.043554 +
.177 x .238 = 0.042126

0.28484

P(motorcycle 2 years older) = 28.5% (1dp)

Annotated Exemplar: Level 3 Probability (91585) 2015

"Merit" exemplar for 91585 2015		Total score	17

Q	Grade score	Annotation
1	A4	(a) (i) Calculated the three risks, coped with values in standard form and made the correct decision. (ii) Gave examples of why the collected data may not be accurate. (iii) Missed the point that historical data would be useful and that there are other factors that affect the risk of having a car stolen. (b) (i) Calculated the combined probability, but did not state the assumption of independent events. (ii) Vague about how the true probability will be different from the observed results, and that a further sample is likely to have different results because of sampling variation.
2	M5	(a) (i) Found the observed proportion. (ii) Calculated the chance of the number ending in a zero. (iii) Misinterpreted what the results of the simulation represented so was unable to make the conclusion that it was likely that the importer was rounding the odometer readings. (b) (i) Misunderstand the concept of mutually exclusive events. (ii) Calculated the proportion of cars that were used and manufactured in Japan and realised that because this was greater than 50% that I would be greater than the proportion of used cars not manufactured in Japan. However did not consider the maximum proportion of used cars not manufactured in Japan.
3	E8	(a) (i) Calculated the probability of the combined event. (ii) Calculated the proportion of cars from testing centre C that were unsuccessful tests. Ignore minor transfer error (1030). (iii) Made the decision that even though testing centre B had the highest pass rate, it had the smallest test rate. (b) Calculated the probability that a motorcycle was at least two years older than a car.