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SUPERVISOR'S USE ONLY

**91585**

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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# Level 3 Mathematics and Statistics (Statistics), 2015

## 91585 Apply probability concepts in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply probability concepts in solving problems. | Apply probability concepts, using relational thinking, in solving problems. | Apply probability concepts, using extended abstract thinking, in solving problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Achievement

TOTAL 12

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## QUESTION ONE

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(a) The following table shows the number of vehicles reported to the NZ Police as stolen over 2011 to 2013, and the number of vehicles registered with the NZ Transport Agency in each of these years.

| | 2011 | 2012 | 2013 |
|---|---|---|---|
| **Number of vehicles reported as stolen** | 20724 | 17807 | 19221 |
| **Number of vehicles registered** | 4210511 | 4248612 | 4315539 |

(i) Which of these years had the greatest overall risk of a vehicle being stolen in New Zealand?

Support your answer with appropriate calculations.

2011: $\frac{20724}{4210511} = 0.0049$

2012: $\frac{17807}{4248612} = 0.0042$

2013: $\frac{19221}{4315539} = 0.0045$

2011 had the greatest risk of a vehicle being stolen with 0.0049.

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(ii) Give ONE reason why the risks calculated in part (i) are only estimates of the true overall risk of a vehicle being stolen in that year.

Some vehicles may be stolen, but not reported. Some cars may not be registered. Therefore we can only estimate the actual risk of a vehicle being stolen in that year.

(iii) A car owner wants to use the overall risk of a car being stolen in New Zealand during 2013 to estimate the risk of their own car being stolen during 2015.

Discuss what else the car owner should consider to estimate this risk.

In 2015, more cars will be registered than in 2013, so that will have to be considered when making an estimate whether their own car will be stolen. Also, the car type should be considered as some are stolen more often than others.

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(b) An importer of second-hand cars into New Zealand has recorded whether each car has the petrol cap on the left-hand side or the right-hand side of the car, in addition to other information about the cars.

For the last shipment of second-hand cars imported, $\frac{13}{21}$ of the cars had the petrol cap on the left-hand side and 22.8% of the cars were silver.

(i) One car is chosen at random from this shipment of imported second-hand cars.

Determine the probability that this car is silver and has the petrol cap on the left-hand side.

State the assumption you need to make to determine this probability.

P(petrol cap on left) = 0.619
P(silver car) = 0.228
$0.619 \times 0.228 = 0.141$
There is a 14.1% chance that a randomly chosen car will be silver and have the petrol cap on the left side.

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(ii) A customer at a petrol station has observed that of the ten cars currently getting petrol, seven of these cars have petrol caps on the left-hand side.

Explain to the customer why a generalisation should not be made that cars in New Zealand are more likely to have petrol caps on the left-hand side, based on what the customer has observed.

The sample size for this observation is only 10 cars. This is a very small population, so is probably just coincidence that $\frac{7}{10}$ have their petrol cap on the left. This cannot be used to make generalisations about all cars in New Zealand because it is not a representative sample.

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## QUESTION TWO

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(a) An importer of cars into New Zealand is suspected of rounding the odometer reading (the measure of the total kilometres the car has driven) to the nearest 10 kilometres for some of the advertisements on their website.

The car importer currently has 20 cars listed for sale on their website.

The odometer readings for these cars are listed below.

| | | | | |
|---|---|---|---|---|
| 1485 | 25384 | 25499 | 26890 | 29568 |
| 35279 | 47872 | 49200 | 64788 | 68050 |
| 72690 | 75730 | 84457 | 91575 | 92297 |
| 93033 | 109532 | 113395 | 137209 | 142980 |

(i) What proportion of cars advertised by the importer has 0 as the last digit of the odometer reading?

$6/20 = 0.3$ 30% of cars.

(ii) Assuming that the last digit of an odometer reading for a car is determined by chance alone, give a model (theoretical) estimate for the probability that the last digit of an odometer reading is 0.

~~Whn~~ $1/10 = 0.1$ 10%

(iii) A concerned customer conducted a simulation to investigate the variability in the proportion of cars in sets of 20 that have 0 as the last digit of the odometer reading, based on an assumption that the last digit of an odometer reading for a car is determined by chance alone.

A summary of the simulation results is shown below (1000 trials).

| **Proportion with 0 last digit** | $\frac{0}{20}$ | $\frac{1}{20}$ | $\frac{2}{20}$ | $\frac{3}{20}$ | $\frac{4}{20}$ | $\frac{5}{20}$ | $\frac{6}{20}$ | $\frac{7}{20}$ | $\frac{8}{20}$ or higher |
|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | 130 | 260 | 289 | 187 | 92 | 32 | 9 | 1 | 0 |

Based on these simulation results, what conclusion could the customer make in respect to whether or not the last digit of an odometer reading for the cars advertised is determined by chance alone?

The odometer reading ending in 0 is very likely to be by chance alone, as 2/20 cars has the highest frequency of numbers ending in 0, so there are ~~not~~ 10% of cars, which is the same percentage as the theoretical estimate.

(b) In 2013, 63.9% of imported cars registered with the New Zealand Transport Agency were manufactured in Japan. Of these cars manufactured in Japan, 80.3% were used cars.

Suppose that one of the imported cars registered with the New Zealand Transport Agency in 2013 was selected at random.

(i) Explain why the events "The car was manufactured in Japan" and "The car is a used car" are not mutually exclusive.

Include statistical reasoning in your explanation.

For events to be statistically mutually exclusive, $P(A \cap B)$ must $= 0$. The events A (man. in Japan) and B (imp. car) are ~~ma~~ not mutually exclusive because $P(A \cap B)$ does not equal to 0.

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(ii) Explain why it can be deduced from this information alone that the car selected is more likely to have been manufactured in Japan than not, given the car selected is a used car.

P(Japan) = 0.639
P(used) = 0.803

~~0.639~~ $\frac{0.803}{0.639} = 1.26$

Given the car is a used car, it is 1.26 times more likely to have been made in Japan than elsewhere.

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## QUESTION THREE

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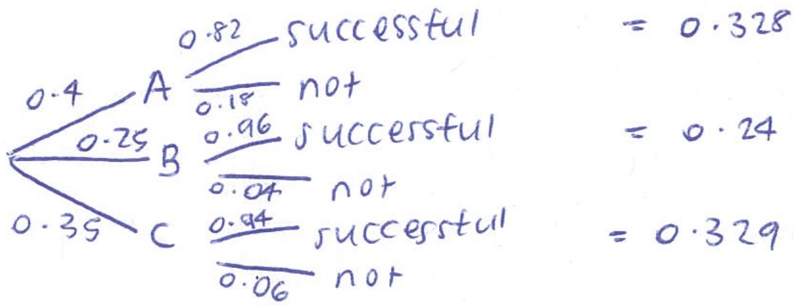
(a) People take their cars to testing centres for a Warrant of Fitness (WOF).

Three testing centres were recently reviewed over a one-month period: testing centre A, testing centre B, and testing centre C. During this time, all results for tests completed by each of the testing centres were recorded.

40% of the tests reviewed were completed by testing centre A, and 25% of the tests reviewed were completed by testing centre B.

Of the tests completed by testing centre A, 82% were successful (the car passed the WOF).
Of the tests completed by testing centre B, 96% were successful.
Of the tests completed by testing centre C, 94% were successful.

(i) What percentage of tests completed during the review were successful?



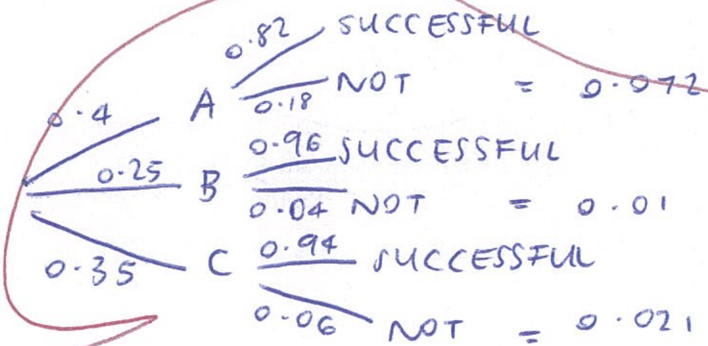

0 . 328 + 0.24 + 0 . 329

= 0 . 897

89.7% of tests completed during the review were successful.

(ii) Of the tests that were unsuccessful, what proportion were completed at testing centre C?

You may wish to assume that there were 10 000 tests completed during the review of the three testing centres.




0.021 was the proportion of unsuccessful tests at centre C.

0 . 021 x 10,000 = 210

210 unsuccessful tests were completed at testing centre C.

(iii) Based on the results of the review, a car owner has decided that they should take their car to testing centre B to increase their chances of having a successful WOF test.

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Is this decision justified?

This decision isn't justified because testing centre B had the least tests performed there, so it is not reliable that 0.96 were successful. Centre A has the highest chance of the WOF being successful; taking into account the amount of tests performed and the success rate.

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(b) Information about the ages of cars and motorcycles registered with the New Zealand Transport Agency (NZTA) at the end of 2013 is presented in the table below. This table shows information about only cars or motorcycles less than 5 years old at the end of 2013.

| | Age of vehicles registered with NZTA at the end of 2013 | | | | |
|---|---|---|---|---|---|
| | **0 years old** | **1 year old** | **2 years old** | **3 years old** | **4 years old** |
| **Proportion of cars** | 0.238 | 0.223 | 0.188 | 0.186 | 0.165 |
| **Proportion of motorcycles** | 0.215 | 0.181 | 0.177 | 0.183 | 0.244 |

One car and one motorcycle are chosen at random from vehicles registered with NZTA at the end of 2013.

Given that both vehicles are less than five years old, estimate the probability that the motorcycle is at least two years older than the car.

Support your answer with appropriate statistical statements and calculations.

A7

## Annotated Exemplar: Level 3 Probability (91585) 2015

| "Achieved" exemplar for 91585 2015 | | Total score | 12 |
|---|---|---|---|
| **Q** | **Grade score** | **Annotation** | |
| 1 | M5 | (a) (i) Calculated the three risks, coped with values in standard form and made the correct decision.<br>(ii) Gave examples of why the collected data may not be accurate.<br>(iii) Identified that historical data would be useful, but missed that there are other factors that affect the risk of having a car stolen.<br>(b) (i) Calculated the combined probability, but did not state the assumption of independent events.<br>(ii) Vague about how the true probability will be different from the observed results, and that a further sample is likely to have different results because of sampling variation. | |
| 2 | A3 | (a) (i) Found the observed proportion.<br>(ii) Calculated the chance of the number ending in a zero.<br>(iii) Misinterpreted what the results of the simulation represented so was unable to make the conclusion that it was likely that the importer was rounding the odometer readings.<br>(b) (i) Understand the concept of mutually exclusive events and the test, but did not do the necessary calculation to show this.<br>(ii) Misunderstood the concept of conditional probability. | |
| 3 | A4 | (a) (i) Calculated the probability of the combined event.<br>(ii) Calculated the proportion of unsuccessful tests at testing centre C, but misunderstood the conditional probability aspect of the question.<br>(iii) Made the decision that even though testing centre B had the highest pass rate, it had the smallest test rate. However, did not give a possible reason for this.<br>(b) Did not attempt this question. | |