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91586

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Mathematics and Statistics (Statistics), 2015

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 20

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Excellence Exemplar
Paper total 20

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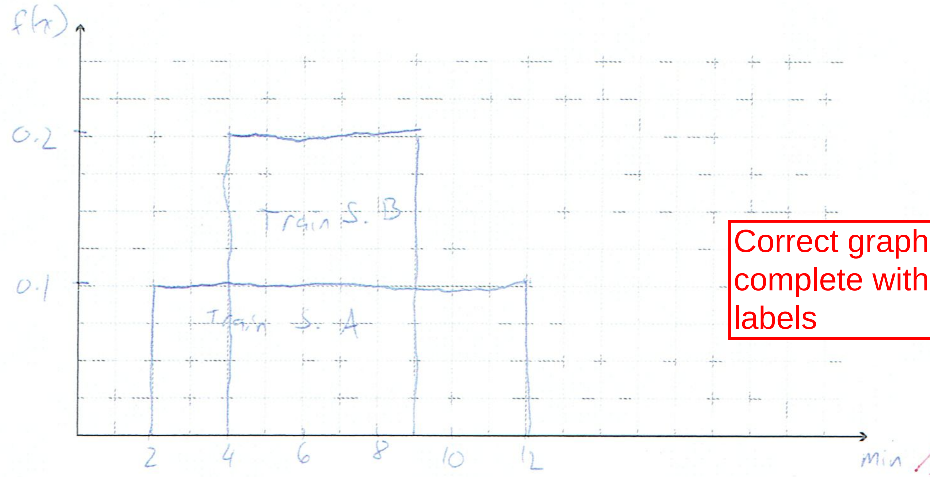
QUESTION ONE

(a) At train station A, the time it will take for the next train to arrive is between 2 and 12 minutes, with all times in between equally likely.

At train station B, the time it will take for the next train to arrive is between 4 and 9 minutes, with all times in between equally likely.

(i) Using an appropriate probability distribution model, sketch each distribution on the same axes below.

Add as much relevant information as possible and clearly label each distribution.



Correct graph complete with labels

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(ii) Calculate the probability that it takes more than 8 minutes for the next train to arrive at train station A and more than 8 minutes for the next train to arrive at train station B.

Give any assumption(s) that needs to be made.

~~4X~~ Let x = time for next train to arrive

$P(x>8 \text{ at } A) = 4 \times 0.1$

$= 0.4$

$P(X>8 \text{ at } B) = 1 \times 0.2$

$= 0.2$

numerical error should be 0.08

$P(x>8 \text{ at } A \text{ and } B) = 0.2 \times 0.4 = 0.04$

we assume that the time for next train to arrive is independent between stations A and B

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Poisson
independent
occur at random
proportion to size & interval

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(b) It is estimated that 13% of the cars driven on New Zealand roads are red.

Suppose that the colours of the next seven cars that pass in an opposite lane to a driver are observed.

(i) Using an appropriate probability distribution model, calculate the probability that at least two of the seven cars are red.

Using Binomial distribution, let x be car being red

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - 0.7719 = 0.2281$

(ii) Justify your selection of a probability distribution model.

There is a fixed probability of a car being red at 13%.

There is 2 outcomes of being red or not red.

There is a fixed number of seven cars being observed.

Assuming that the colour of the car is independent of other cars.

Correct probability in (i)
At least 2 correct Binomial conditions in context in (ii)

(iii) For n number of cars that pass in an opposite lane to a driver, the probability that at least one of the cars is red is 0.965 (rounded to 3 decimal places).

Determine the value of n.

Support your answer with appropriate statistical statements and calculations.

$P(x \geq 1) = 0.965$

$P(x = 0) = 1 - 0.965$

$\binom{n}{0} 0.13^0 (1 - 0.13)^n = 0.035$

$0.87^n = 0.035$

$n = \frac{\ln 0.035}{\ln 0.87}$

$= 24.07$

≈ 24

Correct answer of 24 supported with working

t

E8

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QUESTION TWO

(a) A car-driving training company prepares customers for the restricted licence test. The company has recorded information about customers who were successful at passing the test.

The table below shows the probability distribution of the random variable N, the number of attempts at the restricted licence test by customers of this company. The number of attempts includes the attempt where the customer was successful at passing the test.

n	1	2	3	4
$P(N = n)$	0.82	0.14	0.03	0.01

(i) Calculate the mean number of attempts at the restricted licence test by customers of this company who were successful at passing the test.

Mean = 1×0.82 + 2×0.14 + 3×0.03 + 4×0.01

= 1.23 //

(ii) The cost of the restricted license test is $137 per attempt.

In addition to this, the company charges a fixed price for customers to prepare them for the test, regardless of how many attempts the customer takes.

For customers of this company that were successful at passing the test, the mean amount paid for tests and driving training was $468.51.

Calculate the fixed price charged by the company.

Let x = fixed price

E(T) = 468.51

1.23×137 + x = 468.51

x = $300.00 //

Both (i) and (ii) correct

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(b) A local transport authority has been monitoring the number of bus breakdowns over a long period of time. Based on the data collected, the mean number of breakdowns per hour is 0.3.

The operations manager for the transport authority uses a Poisson distribution to model the number of bus breakdowns during a set period of time.

P(X>2) P(X ≤ 2)

(i) Using this model, calculate the probability that there are no more than two bus breakdowns during any four-hour period.

Give any assumption(s) that needs to be made.

λ = 0.3 × 4

= 1.2 breakdowns every 4h

P(X ≤ 2) = ~~0.2169~~ 0.8795 (GC)

Assuming that a bus breaking down is independent of another //

Two assumptions needed

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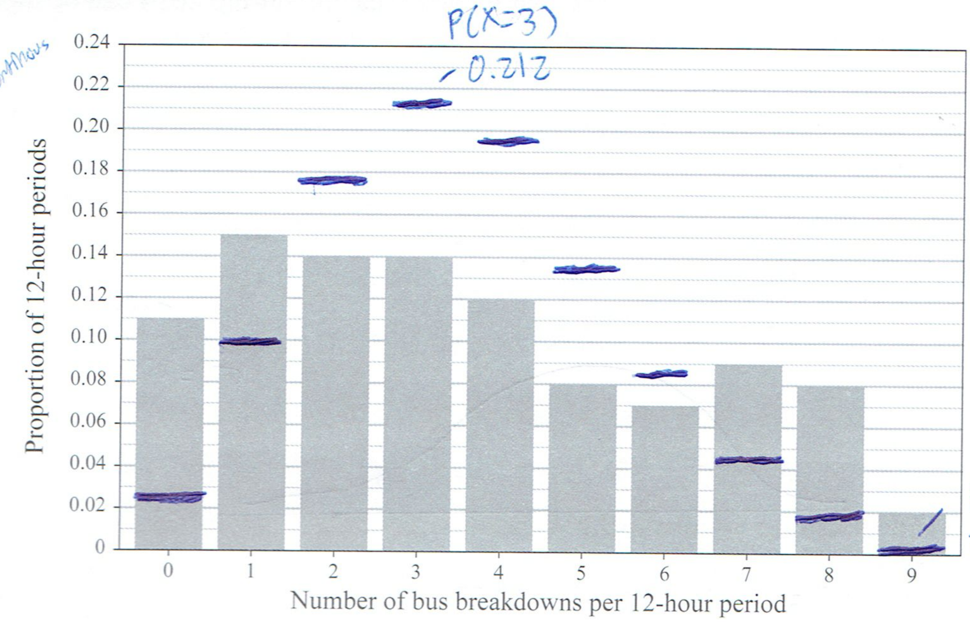
(ii) The operations manager has produced a graph of the data collected on the number of bus breakdowns during 12-hour periods (the experimental distribution).

- Not simultaneous
- Independent
- Discrete values over continuous interval
- Rare and random events

$P(X=3)$
$= 0.212$



$\mu = 0.3 \times 12$
$= 3.6$

$P(X=9)$
$= 7.647 \times 10^{-3}$

Discuss whether a Poisson distribution is a good model for the number of bus breakdowns for any 12-hour period.

As part of your discussion, you should compare the features of the experimental and model (theoretical) distributions. You may wish to draw on the graph above, but should include any probabilities related to these drawings in your working below.

I have drawn up the theoretical distribution for this graph above for the individual values using $\lambda = 3.6$ (ie. 0.3×12 hours). As we can see above the theoretical distribution graph and the experimental graph definitely do not fit eachother well. For the experimental distribution, the data values are rather flat, despite having a unimodal peak at 1 bus breakdown with a probability of 0.15, the values are quite uniformly distributed with data evenly spread out decreasing slightly as the number of bus breakdowns increase. For the theoretical distribution, the graph appears much more sharp with more values around the centre, with a high ~~sharp~~ unimodal peak of 3 bus breakdowns and a probability of 0.212. This curve appears to skew to the right slightly with far less spread compared to the experimental distribution which is spread out alot over the distribution. An example is the number of bus breakdowns being 9, with the experimental probability this is 0.02 while the theoratical probability $P(X=9) = 0.0076$ which is far smaller, therefore the possion distribution is not a good model.

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E7

Correct conclusion.
Compared values between theoretical and experimental distribution.
Compared features of the two graphs, theoretical and experimental.
For E8 they needed to make another comparison - could have compared means or standard deviations. Could have discussed how the Poisson conditions , may not be valid, (eg independence etc)

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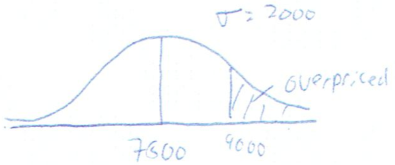
QUESTION THREE

(a) The prices of ten-year-old hatchback cars sold in New Zealand during 2014 can be modelled by a normal distribution, with mean \$7500 and standard deviation \$2000.

(i) Ten-year-old hatchback cars could be considered "over-priced" if they sold for more than \$9000.

Based on the model provided, calculate the percentage of "over-priced" cars that sold for more than \$9500.



Let x = price of cars

$P(x > 9000) = 0.2266$ (GC)

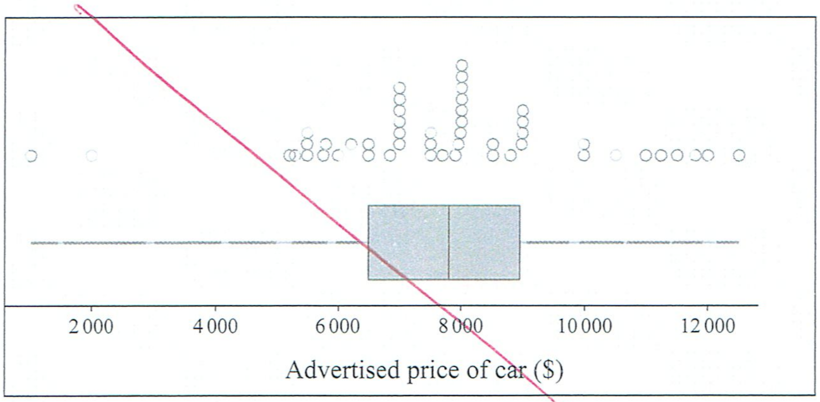
$P(x > 9500) = 0.1587$ (GC)

Percentage $= \frac{0.1587}{0.2266}$

$= 0.7604$

Both probabilities correct plus the conditional probability calculated correctly

(ii) A Year 13 student obtained data on a random sample of 49 ten-year-old hatchbacks from a New Zealand online trading website during 2014. The advertised prices of these cars are shown in the figure below.



The student claims that the prices of ten-year-old hatchback cars are not normally distributed, as the distribution of car prices in the sample is negatively skewed.

Discuss why the student may be incorrect in their reasoning.

Give at least TWO discussion points.

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it does not matter how the sample is skewed and what matter is whether it is unimodal. From the data we can see that it has only one mode at $8200.

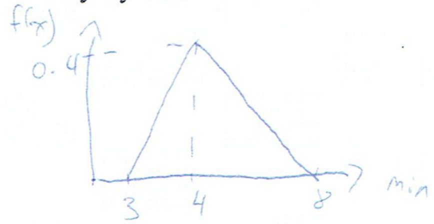
The data can be considered normally distributed because there is a much lower probability at the extreme ends of the data.

Haven't recognised outliers as cause of skew
Could have discussed small size of sample.
Could have discussed fact that data is only from one website
Could have discussed the difference between advertised price and selling price

(b) The time taken to travel along a stretch of motorway in Dunedin by car can be modelled by a random variable that takes on values between 3 and 8 minutes. The most likely time taken is 4 minutes.

Using a triangular probability distribution model:

(i) Calculate the probability that it will take more than 4 minutes to travel along this stretch of motorway by car.



let x be travel time of car

$P(x>4) = \frac{1}{2} \times (8-4) \times 0.4$

$= 0.8.$

(ii) Explain why the median time taken to travel this stretch of motorway by car is not 6 minutes.

The median time is 5.5 minutes. The median time taken is the value in the middle of the two extreme points 3 and 8 min. To calculate the median we use $3 + \frac{8-3}{2}$

5.5 is not the median

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