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91586



NEW ZEALAND QUALIFICATIONS AUTHORITY
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SUPERVISOR'S USE ONLY

Level 3 Mathematics and Statistics (Statistics), 2015

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Not Achieved

TOTAL

5

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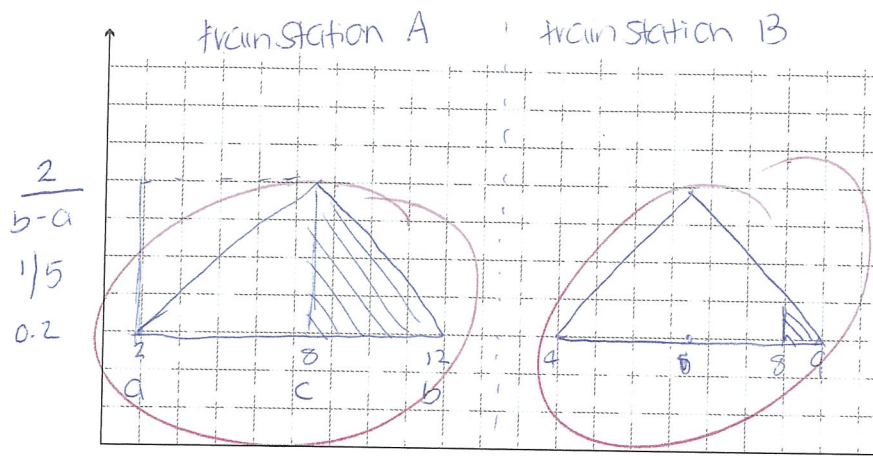
QUESTION ONE

- (a) At train station A, the time it will take for the next train to arrive is between 2 and 12 minutes, with all times in between equally likely.

At train station B, the time it will take for the next train to arrive is between 4 and 9 minutes, with all times in between equally likely.

- (i) Using an appropriate probability distribution model, sketch each distribution on the same axes below.

Add as much relevant information as possible and clearly label each distribution.



- (ii) Calculate the probability that it takes more than 8 minutes for the next train to arrive at train station A and more than 8 minutes for the next train to arrive at train station B.

Give any assumption(s) that needs to be made.

train station A

$$P(X > 8) = 0.5 \times 4 \times 0.2 = 0.4 \quad \text{RAWW} \quad 2/5$$

train station B

$$P(X > 8) = 0.5 \times 1 \times 0.2 = 0.1 \quad 1/10$$

both probabilities needed for u - first is from incorrect working in (i)

- (b) It is estimated that 13% of the cars driven on New Zealand roads are red.

Suppose that the colours of the next seven cars that pass in an opposite lane to a driver are observed.

- (i) Using an appropriate probability distribution model, calculate the probability that at least two of the seven cars are red.

- (ii) Justify your selection of a probability distribution model.

- (iii) For n number of cars that pass in an opposite lane to a driver, the probability that at least one of the cars is red is 0.965 (rounded to 3 decimal places).

Determine the value of n .

Support your answer with appropriate statistical statements and calculations.

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QUESTION TWO

- (a) A car-driving training company prepares customers for the restricted licence test. The company has recorded information about customers who were successful at passing the test. The table below shows the probability distribution of the random variable N , the number of attempts at the restricted licence test by customers of this company. The number of attempts includes the attempt where the customer was successful at passing the test.

n	1	2	3	4
$P(N = n)$	0.82	0.14	0.03	0.01

- (i) Calculate the mean number of attempts at the restricted licence test for customers of this company who were successful at passing the test.

correct answer
is 1.23

$$1 \times 0.82 + 2 \times 0.14 + 3 \times 0.03 + 4 \times 0.01 = 3.19$$

mean number of successful passing restricted is 3.19

- (ii) The cost of the restricted license test is \$137 per attempt.

In addition to this, the company charges a fixed price for customers to prepare them for the test, regardless of how many attempts the customer takes.

For customers of this company that were successful at passing the test, the mean amount paid for tests and driving training was \$468.51.

Calculate the fixed price charged by the company.

- (b) A local transport authority has been monitoring the number of bus breakdowns over a long period of time. Based on the data collected, the mean number of breakdowns per hour is 0.3.

The operations manager for the transport authority uses a Poisson distribution to model the number of bus breakdowns during a set period of time.

- (i) Using this model, calculate the probability that there are no more than two bus breakdowns during any four-hour period.

Give any assumption(s) that needs to be made.

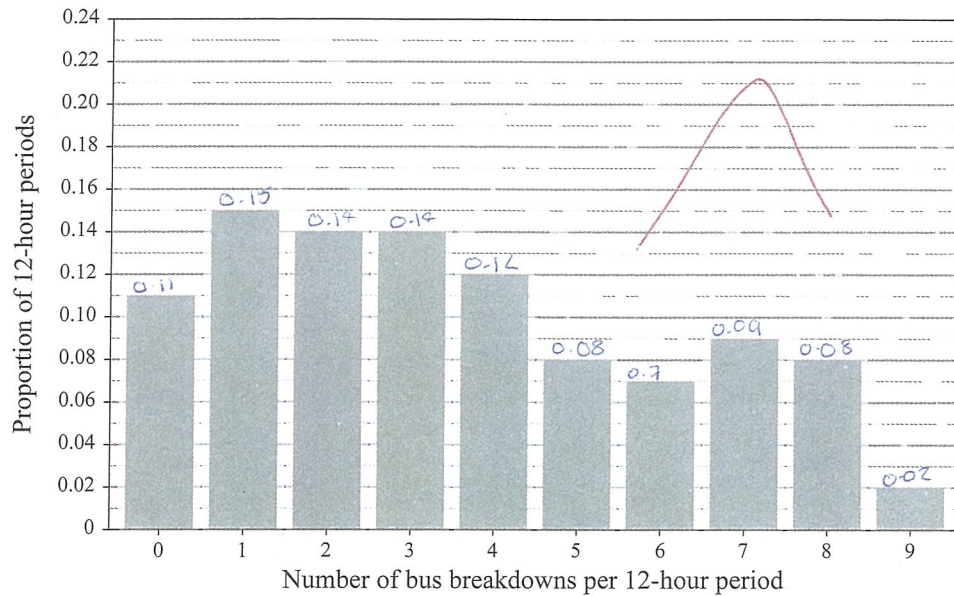
$$p = 0.9964$$

incorrectly used mean
0.3 instead of 1.2

it is highly likely there will be no bus breakdowns during the set period of time

- (ii) The operations manager has produced a graph of the data collected on the number of bus breakdowns during 12-hour periods (the experimental distribution).

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Discuss whether a Poisson distribution is a good model for the number of bus breakdowns for any 12-hour period.

As part of your discussion, you should compare the features of the experimental and model (theoretical) distributions. You may wish to draw on the graph above, but should include any probabilities related to these drawings in your working below.

Poisson would be the preferred model for the numbers of bus breakdowns for any 12 hour period as there is a certain time frame and we can assume that the bus company cannot guess when there will be a breakdown, the bus breakdowns will be independent of each other.

incorrect conclusion - no comparisons made

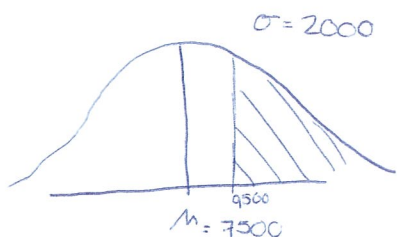
N2

QUESTION THREE

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- (a) The prices of ten-year-old hatchback cars sold in New Zealand during 2014 can be modelled by a normal distribution, with mean \$7500 and standard deviation \$2000.
- (i) Ten-year-old hatchback cars could be considered “over-priced” if they sold for more than \$9000.

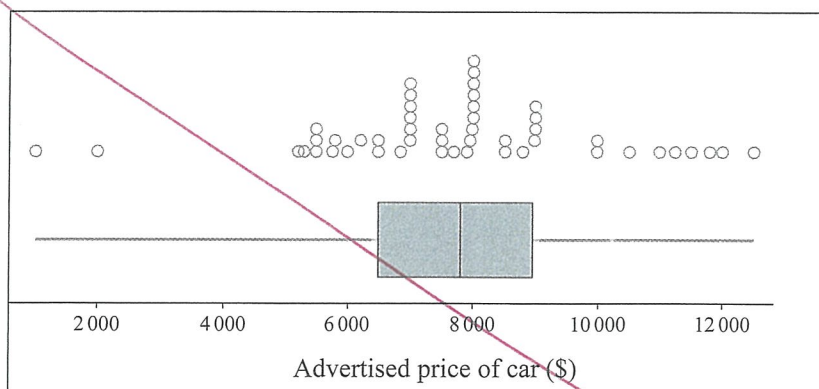
Based on the model provided, calculate the percentage of “over-priced” cars that sold for more than \$9500.



one probability calculated,
need both plus the
conditional probability

The probability of over-priced cars that sold for more than \$9500 is $0.158 = 15\%$ are sold over \$9500

- (ii) A Year 13 student obtained data on a random sample of 49 ten-year-old hatchbacks from a New Zealand online trading website during 2014. The advertised prices of these cars are shown in the figure below.



The student claims that the prices of ten-year-old hatchback cars are not normally distributed, as the distribution of car prices in the sample is negatively skewed.

Discuss why the student may be incorrect in their reasoning.

Give at least TWO discussion points.

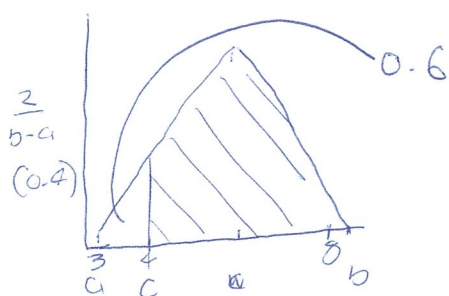
The student may be incorrect in their reasoning as the graph dot plots are bell shaped curve just like the normal distribution graph and the dot plot is ~~symmetrical~~ it shows us clearly where the peaks are as does a normally distributed graph. ~~///~~

no valid points made

- (b) The time taken to travel along a stretch of motorway in Dunedin by car can be modelled by a random variable that takes on values between 3 and 8 minutes. The most likely time taken is 4 minutes.

Using a triangular probability distribution model:

- (i) Calculate the probability that it will take more than 4 minutes to travel along this stretch of motorway by car.



correct answer is
0.8

$$h = 1.2$$

$$0.4 - \frac{2(x-3)}{(8-3)(4-3)} = 1.2 \quad 0.5 \times 1 \times 1.2 = 0.6$$

$$a, c \quad \frac{2(x-3)}{(8-3)(4-3)} \quad \text{---} \quad \text{---} \quad \text{---}$$

When figured out height do $\frac{1}{2} b \times h$ then minus 1 to find shaded triangle. ~~///~~

- (ii) Explain why the median time taken to travel this stretch of motorway by car is not 6 minutes.