3

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91586

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Mathematics and Statistics (Statistics), 2015

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 16

ASSESSOR'S USE ONLY

Merit Exemplar #1
Paper Total 16

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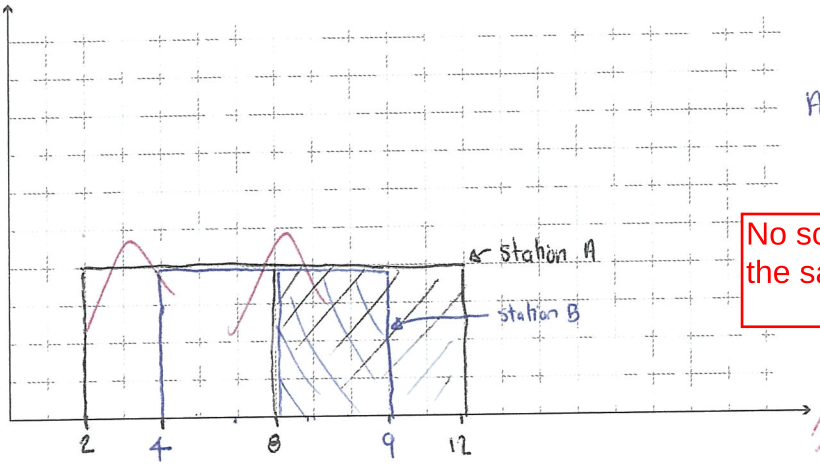
QUESTION ONE

(a) At train station A, the time it will take for the next train to arrive is between 2 and 12 minutes, with all times in between equally likely.

At train station B, the time it will take for the next train to arrive is between 4 and 9 minutes, with all times in between equally likely.

(i) Using an appropriate probability distribution model, sketch each distribution on the same axes below.

Add as much relevant information as possible and clearly label each distribution.



$A = b \cdot h$

No scale on Y axis, B the same height as A

n

(ii) Calculate the probability that it takes more than 8 minutes for the next train to arrive at train station A and more than 8 minutes for the next train to arrive at train station B.

Give any assumption(s) that needs to be made.

Station A. $P(x > 8) \Rightarrow A = b \cdot h \Rightarrow A = (12-8) \times 0.1 = 0.4$

~~A~~ $A = b \cdot h \Rightarrow 1.0 = (12-2) \times h \Rightarrow h = 0.1$

Station B. $P(x > 8) \Rightarrow A = b \cdot h \Rightarrow A = (9-8) \times 0.2 = 0.2$

$A = b \cdot h \Rightarrow 1.0 = (9-4) \times h \Rightarrow h = 0.2$

$\Rightarrow P(\text{station A} > 8\text{min and station B} > 8\text{min}) = 0.4 \times 0.2$

$= 0.08$

u

Correct probabilities but no assumptions given

(b) It is estimated that 13% of the cars driven on New Zealand roads are red.

Suppose that the colours of the next seven cars that pass in an opposite lane to a driver are observed.

$\pi = 0.13$ red
$n = 7$

(i) Using an appropriate probability distribution model, calculate the probability that at least two of the seven cars are red.

$\pi = 0.13$ $P(x \geq 2) = 1 - P(x \leq 1)$

$n = 7$ $= 1 - 0.7719$

$x = 1$ $= 0.2281$ (4 s.f.)

(ii) Justify your selection of a probability distribution model.

Binomial distribution -

- Fixed number of trials (7 cars observed)
- Each car colour is independent of any other.
- Only two possible outcomes (red or not red)
- Probability that a car is red remains constant ($\pi = 0.13$)

(iii) For n number of cars that pass in an opposite lane to a driver, the probability that at least one of the cars is red is 0.965 (rounded to 3 decimal places).

$P(x \geq 1) = 1 - P(x = 0)$

Determine the value of n.

Support your answer with appropriate statistical statements and calculations.

$P(x \geq 1) = 0.965$ red

$P(x = 0) = 1 - 0.965 = 0.035$

$0.035 = \frac{e^{-\lambda} \lambda^0}{0!}$

$\Rightarrow \ln 0.035 = -\lambda$

$\Rightarrow \lambda = 3.352$ (3 d.p)

$P(x \geq 1) = 1 - P(x = 0)$

$\lambda = 3.352$

$0.035 = \frac{3.352^x e^{-3.352}}{x!}$

Probability atleast one car is red is 3.352

$3.352 = (1 - 0.965)^n$

$\frac{\log 3.352}{\log 0.035} = n$

0.36

$P = {}^nC_x \pi^x (1-\pi)^{n-x}$

$0.965 = {}^nC_0 \, 3.352^0 (1 - 3.352)^{n-0}$

$\log 0.965 = n \log 1.352$ $\Rightarrow 0.965 = (1 - 3.352)^n$

$=$

$\sqrt[x]{0.965}$

Parts (i) and (ii) correct - no answer given for n in part (iii). Incorrectly using Poisson distribution

M5

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QUESTION TWO

(a) A car-driving training company prepares customers for the restricted licence test. The company has recorded information about customers who were successful at passing the test.

The table below shows the probability distribution of the random variable N, the number of attempts at the restricted licence test by customers of this company. The number of attempts includes the attempt where the customer was successful at passing the test.

n	1 137	2 ×137	3 ×137	4 ×137	no. of attemps.
$P(N = n)$	0.82	0.14	0.03	0.01	Sucess.

$137 ($331.51) $274 ($194.51) $411 ($57.51) $548

(i) Calculate the mean ~~number~~ of ~~attempts at~~ the restricted ~~licence~~ test by customers of this company who were successful at passing the test.

$E(X) = (1 \times 0.82) + (2 \times 0.14) + (3 \times 0.03) + (4 \times 0.01) = 1.23$

(ii) The cost of the restricted license test is $137 per attempt. + fixed price (a)

In addition to this, the company charges a fixed price for customers to prepare them for the test, regardless of how many attempts the customer takes.

For customers of this company that were successful at passing the test, the mean amount paid for tests and driving training was $468.51.

Calculate the fixed price charged by the company.

$\frac{\$468.51}{\$137} = 3.42$ attempts

$468.51 - 168.51 = \$300$.

$137 \times 1.23 = \$168.51$

Fixed price of $300 (assuming success of 1.23 from previous question.)

(b) A local transport authority has been monitoring the number of bus breakdowns over a long period of time. Based on the data collected, the mean number of breakdowns per hour is 0.3.

$\lambda = 0.3$ per hour Poisson.

The operations manager for the transport authority uses a Poisson distribution to model the number of bus breakdowns during a set period of time. RICS

(i) Using this model, calculate the probability that there are no more than two bus breakdowns during any four-hour period.

Give any assumption(s) that needs to be made.

$\lambda = 0.3$ breakdowns per hour $\lambda = 1.2$ 4 hour period.

$P(x \leq 2) = 0.8795$ (4 s.f.)

Assumptions:
- No two breakdowns can occur simultaneously
- Each breakdown is independent of anyother
- The rate of ~~an accident~~ breakdowns remains constant ($\lambda = 1.2$)
- The rate depends upon the time interval ($\lambda = 1.2$ for 4 hours)
- Breakdowns occur at random.

 $\lambda = 3.6$ for 12 hours

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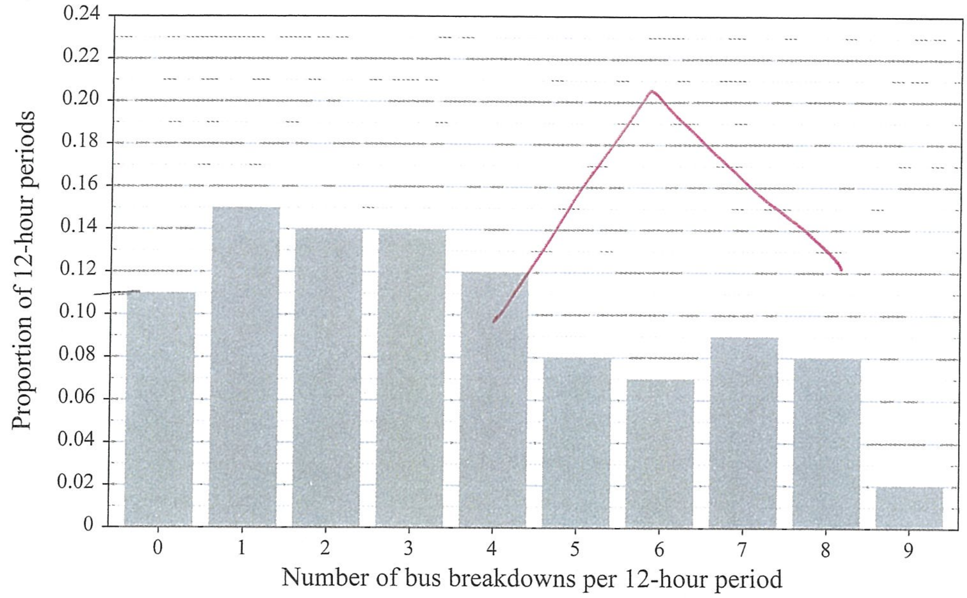
(ii) The operations manager has produced a graph of the data collected on the number of bus breakdowns during 12-hour periods (the experimental distribution).



Discuss whether a Poisson distribution is a good model for the number of bus breakdowns for any 12-hour period.

As part of your discussion, you should compare the features of the experimental and model (theoretical) distributions. You may wish to draw on the graph above, but should include any probabilities related to these drawings in your working below.

Theoretical $\lambda = 3.6$ for 12 hour period.

$P(x=0) = 0.02732$ $\quad P(x=1) = 0.09837$ $\quad P(x=2) = 0.1771$ $\quad P(x=3) = 0.2125$

$P(x \leq 4) = 0.7064$ $\quad$ 10% range. $0.7064 \div 10 \Rightarrow 0.6358 - 0.77704$

Experimental $\lambda = (0\times0.11)+(1\times0.15)+(2\times0.14)+(3\times0.14)+(4\times0.12)+(5\times0.08)+(6\times0.07)+(7\times0.09)+(8\times0.08)+(9\times0.02) = 3.6$

$P(x=0) = 0.11$ $\quad P(x=1) = 0.15$ $\quad P(x=2) = 0.14$ $\quad P(x=3) = 0.14$

$P(x \leq 4) = 0.66$ (Experimental Probabilities are similar to theoretical, therefore good model.)

- When looking at the theoretical probability example that $x \leq 4$, which gives a probability of 0.7064, the experimental probability for $x \leq 4$ is shown to be 0.66. Since this experimental probability is within the 10% range of the theoretical value (0.6358 ⟷ 0.7770), we can say that poisson is a good model for this distribution.

Credit for correct comparison, however conclusion is incorrect

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M6

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QUESTION THREE

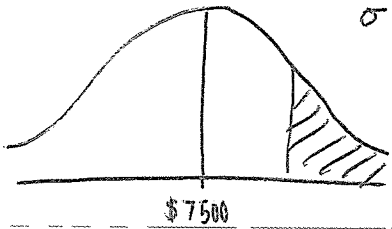
(a) The prices of ten-year-old hatchback cars sold in New Zealand during 2014 can be modelled by a normal distribution, with mean $7500 and standard deviation $2000.

(i) Ten-year-old hatchback cars could be considered "over-priced" if they sold for more than $9000.

Based on the model provided, calculate the percentage of "over-priced" cars that sold for more than $9500.

$\mu = 7500$

$\sigma = 2000$



$P(x > 9500) = 0.1587$ (4 s.f)

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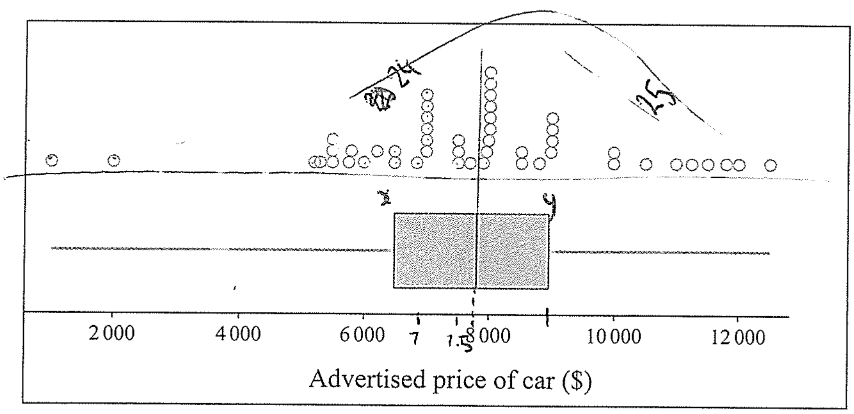
Only one probability calculated

(ii) A Year 13 student obtained data on a random sample of 49 ten-year-old hatchbacks from a New Zealand online trading website during 2014. The advertised prices of these cars are shown in the figure below.

$n = 49$



$\mu \approx \$7700 \sim$

The student claims that the prices of ten-year-old hatchback cars are not normally distributed, as the distribution of car prices in the sample is negatively skewed.

Discuss why the student may be incorrect in their reasoning.

Give at least TWO discussion points.

normal distribution is a continuous data set with an average price for 10 year old hatch backs shown to be approximately $7,700 on the graph. ~~We can Normal distribution data~~ also has this ~~spread~~ ~~a median~~

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looking at the box, we can see that upper (2) quartile ~~quartile~~ and lower (4) quartile are approximately equal distances from the middle line, suggesting middle 50% is normally distributed.

looking at the spread of the data it is reasonably symmetrical distributed with 24 and 25 data points either side of (the ($7,700) central box line), suggesting normal distribution.

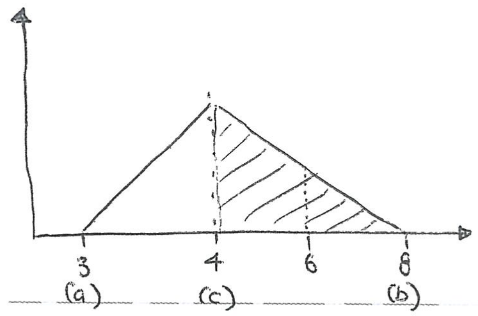
Outliers identified and recognised as cause of skew

The two points at ≤ $2,000 may be possible outliers which is causing the skewed graph. If removed, the data distribution would be more symmetrical & bell-shaped to ressemble normal distribution.

(b) The time taken to travel along a stretch of motorway in Dunedin by car can be modelled by a random variable that takes on values between 3 and 8 minutes. The most likely time taken is 4 minutes.

Using a triangular probability distribution model:

(i) Calculate the probability that it will take more than 4 minutes to travel along this stretch of motorway by car.



$f(x) = \frac{2(8-4)}{(8-3)(8-4)} = 0.4$

$P(X > 4) = A$

$A = \frac{1}{2} b.h = 0.5 \times (8-4) \times 0.4 \Rightarrow P(X > 4) = 0.8$

(ii) Explain why the median time taken to travel this stretch of motorway by car is not 6 minutes.

$P(X = 6)$ $f(x) = \frac{2(8-6)}{(8-3)(8-4)} = 0.2$

$A = \frac{1}{2}.b.h$

Since the most common time taken is 4 and the frequency of 6 minutes is ½ that for 4 min this means 6 ~~cannot~~ is not likely to be the median

Median is the middle number when all data is ordered.

3, 4, 5, 6, 7, 8

From ~~3~~ 4 min to ~~8~~ 8 min there is a decrease in frequency, the most likely median will be 5 minutes.

f(x) for 3 = 0
for 4 = 0.4
for 5 = 0.3
for 6 = 0.2

4, 4, 4, 4, 5, 5, 5, 6, 6, 7

median is 4.84 not 5

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Merit

TOTAL 16

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Merit Exemplar #2
Paper total 16

QUESTION ONE

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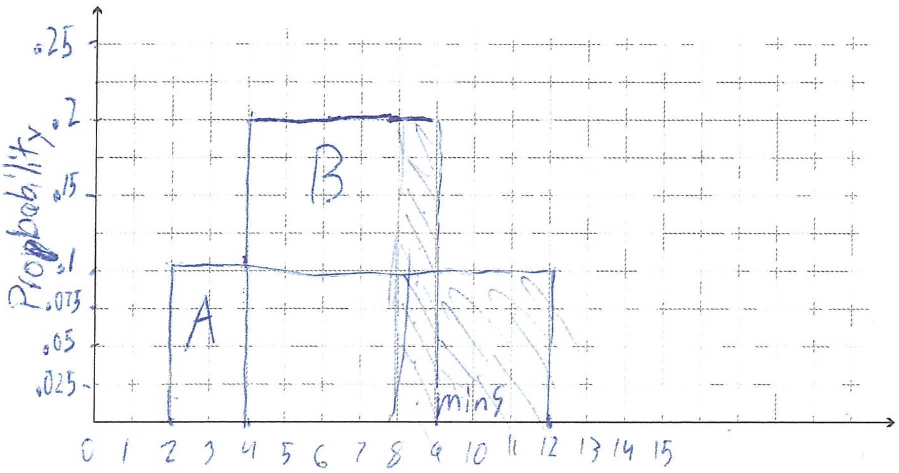
(a) At train station A, the time it will take for the next train to arrive is between 2 and 12 minutes, with all times in between equally likely.

At train station B, the time it will take for the next train to arrive is between 4 and 9 minutes, with all times in between equally likely.

(i) Using an appropriate probability distribution model, sketch each distribution on the same axes below.

Add as much relevant information as possible and clearly label each distribution.



(ii) Calculate the probability that it takes more than 8 minutes for the next train to arrive at train station A and more than 8 minutes for the next train to arrive at train station B.

Give any assumption(s) that needs to be made.

.2 .4

the probability that ~~a~~ it will take more than 8 mins for a train to arrive in station A is .4 or 40% with station B at .2 or 20%.

Probability of the "and" event not calculated
No assumptions given

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(b) It is estimated that 13% of the cars driven on New Zealand roads are red.

Suppose that the colours of the next seven cars that pass in an opposite lane to a driver are observed.

(i) Using an appropriate probability distribution model, calculate the probability that at least two of the seven cars are red.

13% does not exist on the Binomial Distribution table although I can say it is between 0.1497 and 0.2834
(10%) (15%)

incorrect probability

(ii) Justify your selection of a probability distribution model.

I used binomial distribution model because there were only two outcomes of car. These being red and non-red

Only one condition given

(iii) For n number of cars that pass in an opposite lane to a driver, the probability that at least one of the cars is red is 0.965 (rounded to 3 decimal places).

Determine the value of n.

Support your answer with appropriate statistical statements and calculations.

or greater than 10

n is 6 because that is the only number to have a probability of ~0.045 of 0 cars being red

n

correct answer is 24

MS

ASSESSOR'S USE ONLY

QUESTION TWO

(a) A car-driving training company prepares customers for the restricted licence test. The company has recorded information about customers who were successful at passing the test.

The table below shows the probability distribution of the random variable N, the number of attempts at the restricted licence test by customers of this company. The number of attempts includes the attempt where the customer was successful at passing the test.

n	1	2	3	4
$P(N = n)$	0.82	0.14	0.03	0.01

(i) Calculate the mean number of attempts at the restricted licence test by customers of this company who were successful at passing the test.

1.23

(ii) The cost of the restricted license test is $137 per attempt.

In addition to this, the company charges a fixed price for customers to prepare them for the test, regardless of how many attempts the customer takes.

For customers of this company that were successful at passing the test, the mean amount paid for tests and driving training was $468.51.

Calculate the fixed price charged by the company.

1.23 × $137 = 168.51   468.51 − 168.51

fixed price = $300

√

(b) A local transport authority has been monitoring the number of bus breakdowns over a long period of time. Based on the data collected, the mean number of breakdowns per hour is 0.3.

The operations manager for the transport authority uses a Poisson distribution to model the number of bus breakdowns during a set period of time.

.04667 + .05 + .03667

(i) Using this model, calculate the probability that there are no more than two bus breakdowns during any four-hour period.

.9963

Give any assumption(s) that needs to be made.

assuming they have the same chance every hour .8088

or 80.9%

Incorrect probability

n

(ii) The operations manager has produced a graph of the data collected on the number of bus breakdowns during 12-hour periods (the experimental distribution).

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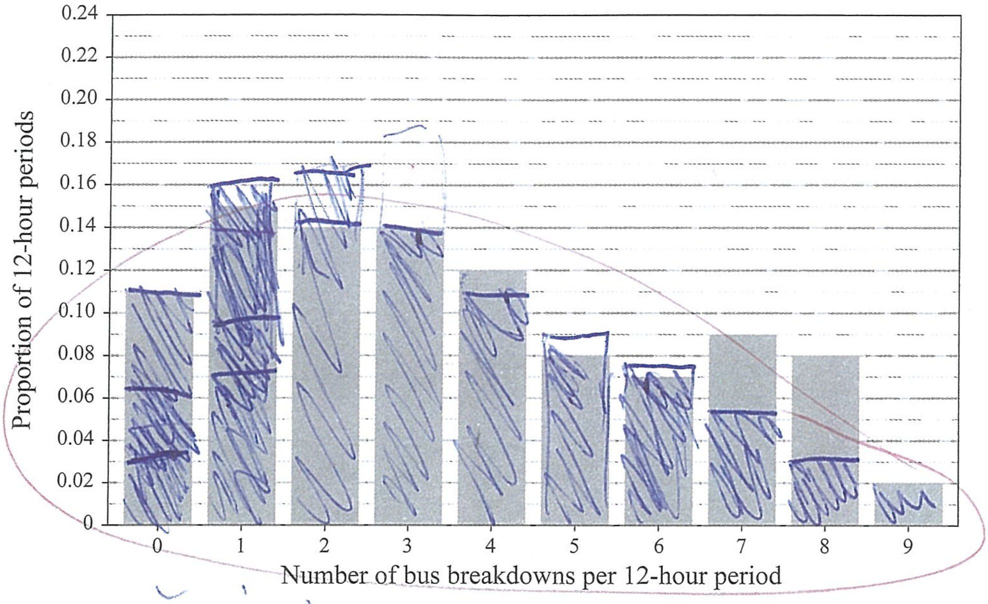

Discuss whether a Poisson distribution is a good model for the number of bus breakdowns for any 12-hour period.

As part of your discussion, you should compare the features of the experimental and model (theoretical) distributions. You may wish to draw on the graph above, but should include any probabilities related to these drawings in your working below.

Poisson distribution is good when each value is independent and discrete. ~~But they need to each have An eqa~~

poisson distribution would not be a good model for the number of bus breakdowns for any 12 hour period. because the data on the graph shows a diffrent amount of breakdowns than what would be shown on the model

No comparisions with theoretical model made

N1

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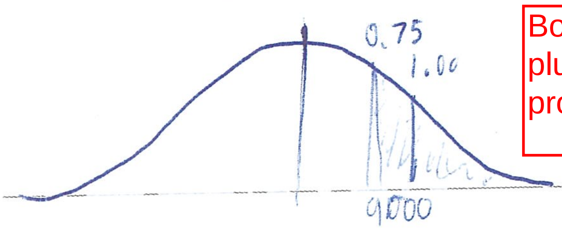
QUESTION THREE

(a) The prices of ten-year-old hatchback cars sold in New Zealand during 2014 can be modelled by a normal distribution, with mean \$7500 and standard deviation \$2000.

(i) Ten-year-old hatchback cars could be considered "over-priced" if they sold for more than \$9000.

Based on the model provided, calculate the percentage of "over-priced" cars that sold for more than \$9500.

μ 7500 σ 2000



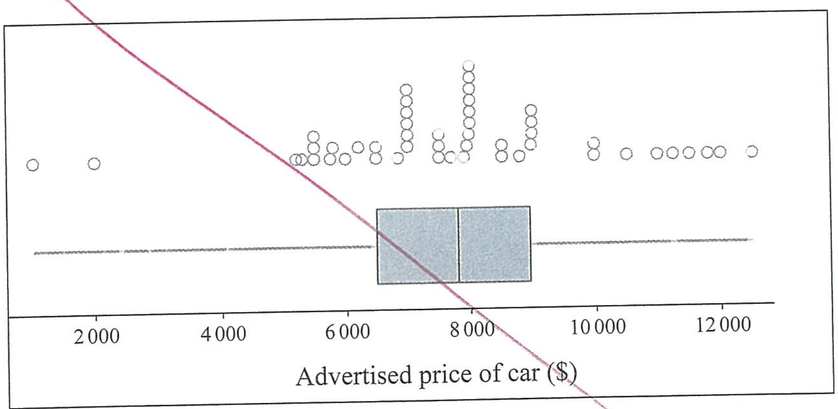
Both probabilities correct plus conditional probability calculated

$.5 - .2734 = 0.2266$

$.5 - .3413 = 0.1587$

70% of "over priced" cars sold for more than \$9500

(ii) A Year 13 student obtained data on a random sample of 49 ten-year-old hatchbacks from a New Zealand online trading website during 2014. The advertised prices of these cars are shown in the figure below.



The student claims that the prices of ten-year-old hatchback cars are not normally distributed, as the distribution of car prices in the sample is negatively skewed.

Discuss why the student may be incorrect in their reasoning.

Give at least TWO discussion points.

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~~A normal distribution curve has no skews and is perfectly simetrical but~~
He only had a sample size of 49, if he had a larger sample size the skew might have evend out. Normal distrabution is just a model estimite of the true results, even if the true results are skewed we can still use normal distrabution

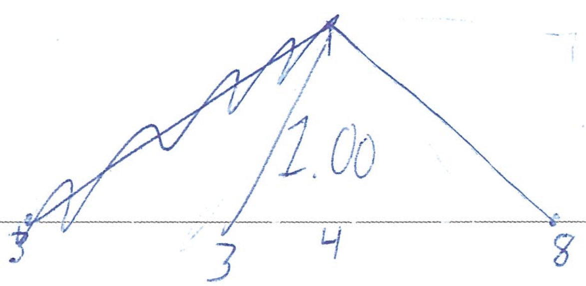
r

first point about sample size is well made
second point not enough

(b) The time taken to travel along a stretch of motorway in Dunedin by car can be modelled by a random variable that takes on values between 3 and 8 minutes. The most likely time taken is 4 minutes.

Using a triangular probability distribution model:

(i) Calculate the probability that it will take more than 4 minutes to travel along this stretch of motorway by car.



~~0.8 or 80%~~ ~~0.8 or 80%~~ 80%

(ii) Explain why the median time taken to travel this stretch of motorway by car is not 6 minutes.

because most cars took 4 ~~minutes~~ or 3 minutes to travel and that balenced out the median

u

Not a valid explanation

M6