3

SUPERVISOR'S USE ONLY

91586

915860

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
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Level 3 Mathematics and Statistics (Statistics), 2015

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 19 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

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Achievement

TOTAL 11

ASSESSOR'S USE ONLY

Achieved Exemplar #1
Paper Total 11

ASSESSOR'S USE ONLY

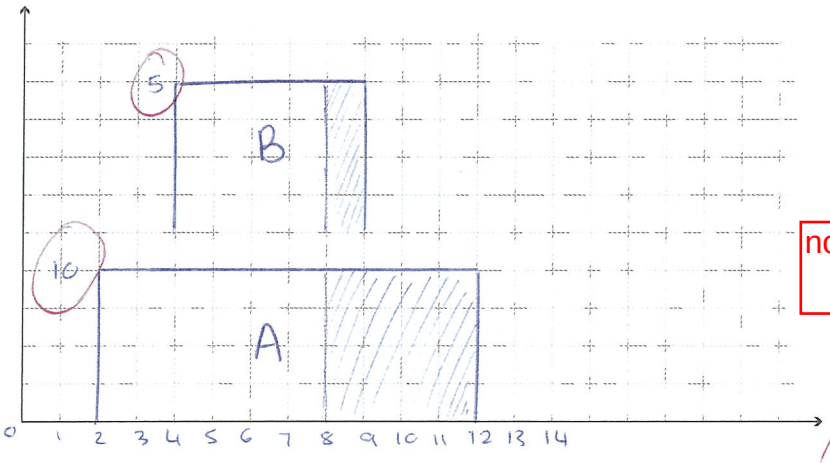
QUESTION ONE

(a) At train station A, the time it will take for the next train to arrive is between 2 and 12 minutes, with all times in between equally likely.

At train station B, the time it will take for the next train to arrive is between 4 and 9 minutes, with all times in between equally likely.

(i) Using an appropriate probability distribution model, sketch each distribution on the same axes below.

Add as much relevant information as possible and clearly label each distribution.



no scale on Y axis

(ii) Calculate the probability that it takes more than 8 minutes for the next train to arrive at train station A and more than 8 minutes for the next train to arrive at train station B.

Give any assumption(s) that needs to be made.

A. $P(X > 8)$ $a \leq x \leq b$ = 0 elsewhere.

$b \times h = 4 \times 10 = 40$ rectangular distribution

B. $P(X > 8)$

$b \times h = 1 \times 5 = 5$

Assuming the time it will take for the next train to arrive at A does not effect the time it will take for the next train to arrive at B and vice versa. (Assuming the events are independent).

A: $1/b-a$
$= 1/12-2$
$= 10$

B: $1/b-a$
$= 1/9-4$
$= 5$

incorrect probabilities

(b) It is estimated that 13% (0.13) of the cars driven on New Zealand roads are red.

Suppose that the colours of the next seven cars that pass in an opposite lane to a driver are observed.

ASSESSOR'S USE ONLY

(i) Using an appropriate probability distribution model, calculate the probability that at least two of the seven cars are red.

0 1 2 3 4 5 6 7

$P(X \leq 1) = 0.77185463$

$1 - 0.77185463 = 0.228$ (3dp).

$\pi = 0.13 \quad x = 1 \quad n = 7$ Binomial Bcd

(ii) Justify your selection of a probability distribution model.

Binomial because it has the parameters π, n, x.

$\pi = 0.13 \quad x = 1 \quad n = 7$

There is a set number of identical trials, n, of 7

The probability of the event for each trial is constant, 0.13.

Each trial is independent, and the outcome of one does not affect the other.

There are two possible outcomes, red or not red.

only one condition in context

(iii) For n number of cars that pass in an opposite lane to a driver, the probability that at least one of the cars is red is 0.965 (rounded to 3 decimal places).

0 1 2 3 4 5

Determine the value of n.

$x = 1$

$\pi = 0.13$

Support your answer with appropriate statistical statements and calculations.

$P(X \geq 1) = 0.965$

$0.965 = \binom{n}{x} n^x (1-\pi)^{n-x}$

$= 2.$

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A3

QUESTION TWO

ASSESSOR'S USE ONLY

(a) A car-driving training company prepares customers for the restricted licence test. The company has recorded information about customers who were successful at passing the test.

The table below shows the probability distribution of the random variable N, the number of attempts at the restricted licence test by customers of this company. The number of attempts includes the attempt where the customer was successful at passing the test.

n	1	2	3	4
$P(N = n)$	0.82	0.14	0.03	0.01

(i) Calculate the mean number of attempts at the restricted licence test by customers of this company who were successful at passing the test.

f(n) = (1 x 0.82) + (2 x 0.14) + (3 x 0.03) + (4 x 0.01) = 1.23 ≈ 1 attempt

(ii) The cost of the restricted license test is $137 per attempt.

In addition to this, the company charges a fixed price for customers to prepare them for the test, regardless of how many attempts the customer takes.

For customers of this company that were successful at passing the test, the mean amount paid for tests and driving training was $468.51.

Calculate the fixed price charged by the company.

1.37 x 1.23 = 168.51

468.51 - 168.51 = $300

Rounding to a whole number not accepted above, however correct answer here

(b) A local transport authority has been monitoring the number of bus breakdowns over a long period of time. Based on the data collected, the mean number of breakdowns per hour is 0.3.

The operations manager for the transport authority uses a Poisson distribution to model the number of bus breakdowns during a set period of time.

(i) Using this model, calculate the probability that there are no more than two bus breakdowns during any four-hour period.

0 1 2] 3

Give any assumption(s) that needs to be made.

λ = 0.3
x = 2

P(x ≤ 2) = 0.9964005

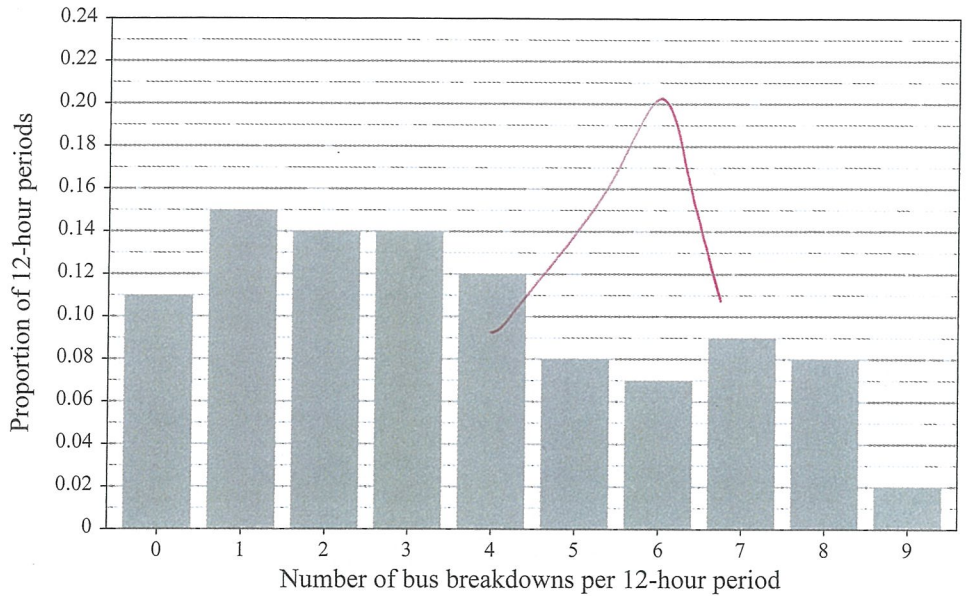
0.9964005 x 0.9964005 x 0.9964005 x 0.9964005

= 0.985679552 = 0.986 (3dp)

Assuming trials are independent of one another, and the outcome of one does not effect the outcome of another.

ASSESSOR'S USE ONLY

(ii) The operations manager has produced a graph of the data collected on the number of bus breakdowns during 12-hour periods (the experimental distribution).



Discuss whether a Poisson distribution is a good model for the number of bus breakdowns for any 12-hour period.

As part of your discussion, you should compare the features of the experimental and model (theoretical) distributions. You ~~may wish to~~ draw on the graph above, but should include any probabilities related to these drawings in your working below.

Poisson distribution is a good model because it allows for the probability of an event occuring to be calculated within a given timeframe, (i.e. per ~~P2 event~~ 12 hours). // or period of time.

no comparisons made

M5

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The prices of ten-year-old hatchback cars sold in New Zealand during 2014 can be modelled by a normal distribution, with mean \$7500 and standard deviation \$2000.

(i) Ten-year-old hatchback cars could be considered "over-priced" if they sold for more than \$9000.

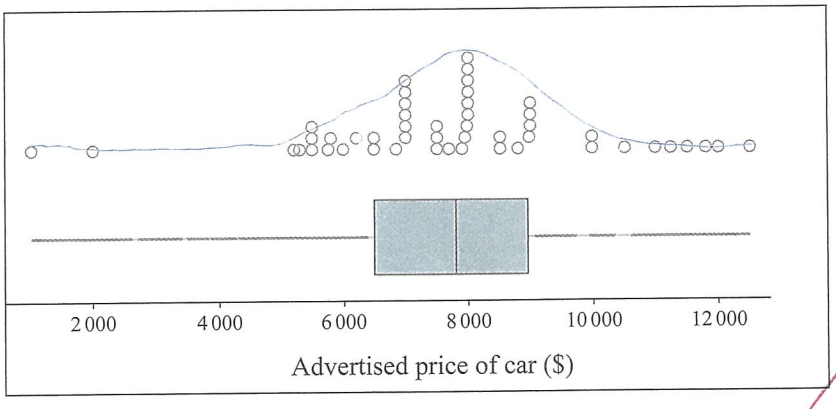
Based on the model provided, calculate the percentage of "over-priced" cars that sold for more than \$9500.

$\mu = 7500$
$\sigma = 2000$
Upper = 10000000000
Lower = 9500

$P(X > 9500) = 0.15865525$
$= 0.159$ (3dp).

U

(ii) A Year 13 student obtained data on a random sample of 49 ten-year-old hatchbacks from a New Zealand online trading website during 2014. The advertised prices of these cars are shown in the figure below.



The student claims that the prices of ten-year-old hatchback cars are not normally distributed, as the distribution of car prices in the sample is negatively skewed.

Discuss why the student may be incorrect in their reasoning.

Give at least TWO discussion points.

needed to identify outliers, (near $2000) and say they could be causing skew

ASSESSOR'S USE ONLY

May be incorrect in their reasoning because the distribution displays a bell-shaped curve and is mostly symmetrical, much as in a normal distribution.
May be incorrect in their reasoning as the sample may be negatively skewed due to the presence of outliers in the data.

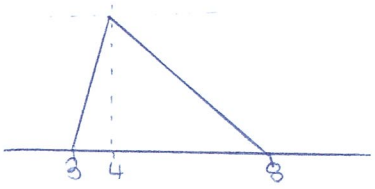
(b) The time taken to travel along a stretch of motorway in Dunedin by car can be modelled by a random variable that takes on values between 3 and 8 minutes. The most likely time taken is 4 minutes.

Using a triangular probability distribution model:

(i) Calculate the probability that it will take more than 4 minutes to travel along this stretch of motorway by car.

$a = 3$
$b = 8$
$c = 4$

$\frac{2}{b-a}$
$= \frac{2}{8-3}$
$= \frac{2}{5}$



$4 < x < 8$

$\frac{2(b-x)}{(b-a)(b-c)} = \frac{2(8-x)}{(8-3)(8-4)} = \frac{2(8-x)}{20} = \frac{16-2x}{20}$

(ii) Explain why the median time taken to travel this stretch of motorway by car is not 6 minutes.

because the majority shows smaller times to be more likely than larger

A3

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Achievement

TOTAL 11

ASSESSOR'S USE ONLY

Achieved Exemplar # 2
Paper total 11

ASSESSOR USE ONLY

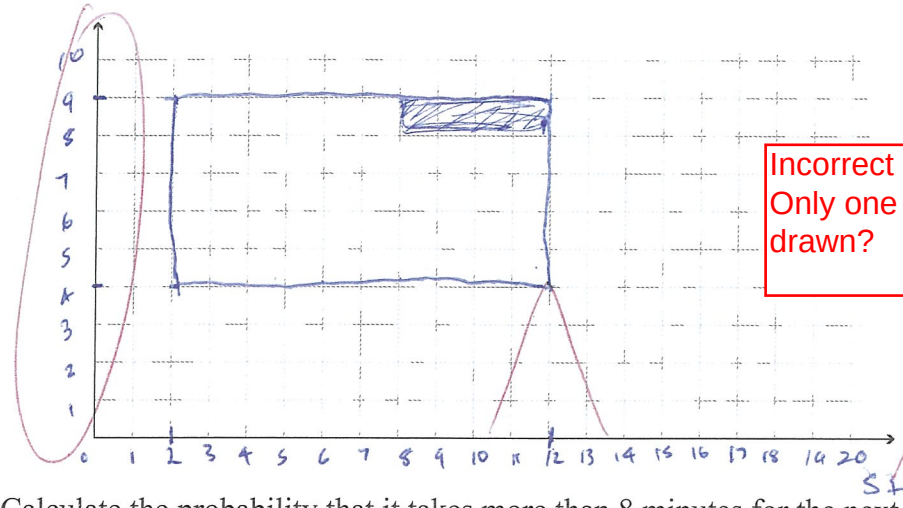
QUESTION ONE

(a) At train station A, the time it will take for the next train to arrive is between 2 and 12 minutes, with all times in between equally likely.

At train station B, the time it will take for the next train to arrive is between 4 and 9 minutes, with all times in between equally likely.

(i) Using an appropriate probability distribution model, sketch each distribution on the same axes below.

Add as much relevant information as possible and clearly label each distribution.



Incorrect scale Y axis
Only one rectangle drawn?

(ii) Calculate the probability that it takes more than 8 minutes for the next train to arrive at train station A and more than 8 minutes for the next train to arrive at train station B.

Give any assumption(s) that needs to be made.

$A = P(x > 8)$
$B = P(x > 8)$

$P(x > 8 \cap x > 8) = P(A) \cap P(B)$

$A = Bh$

$A = 4/10 \times 1/5$

$= 3/5$

$= 0.6$

Incorrect probability

ASSESSOR'S USE ONLY

(b) It is estimated that 13% of the cars driven on New Zealand roads are red.

Suppose that the colours of the next seven cars that pass in an opposite lane to a driver are observed.

(i) Using an appropriate probability distribution model, calculate the probability that at least two of the seven cars are red.

Binomial $p = 0.13$ $n = 7$ $x = 2$

$P(x \geq 2) = 1 - P(x \leq 1)$

$= 1 - 0.77165$

$= 0.22835$ (5 dp) ✓

(ii) Justify your selection of a probability distribution model.

a fixed probability is given ✓ (0.13)
fixed number of trials (7) (n) ✓
the event of a car being red is independant meaning if one car was ~~independant~~ red it would not affect the other car being red.

only one condition given in context

(iii) For n number of cars that pass in an opposite lane to a driver, the probability that at least one of the cars is red is 0.965 (rounded to 3 decimal places).

Determine the value of n.

Support your answer with appropriate statistical statements and calculations.

$P(x \geq 1) = 0.965$ $\mu =$ $n = ?$

$^nC_x\, p^q\, q^z$

4

A3

QUESTION TWO

ASSESSOR'S USE ONLY

(a) A car-driving training company prepares customers for the restricted licence test. The company has recorded information about customers who were successful at passing the test.

The table below shows the probability distribution of the random variable N, the number of attempts at the restricted licence test by customers of this company. The number of attempts includes the attempt where the customer was successful at passing the test.

n	1	2	3	4
$P(N = n)$	0.82	0.14	0.03	0.01

(i) Calculate the mean number of attempts at the restricted licence test by customers of this company who were successful at passing the test.

$E(x = x) = (1 \times 0.82) + (2 \times 0.14) + (3 \times 0.03) + (4 \times 0.01)$

$= 0.82 + 0.28 + 0.09 + 0.04$

$= 1.23$

Mean is 1.23 number of attempts

(ii) The cost of the restricted license test is $137 per attempt.

In addition to this, the company charges a fixed price for customers to prepare them for the test, regardless of how many attempts the customer takes.

For customers of this company that were successful at passing the test, the mean amount paid for tests and driving training was $468.51.

Calculate the fixed price charged by the company.

part (i) correct

cost = 137x + y

468.51

(b) A local transport authority has been monitoring the number of bus breakdowns over a long period of time. Based on the data collected, the mean number of breakdowns per hour is 0.3.

The operations manager for the transport authority uses a Poisson distribution to model the number of bus breakdowns during a set period of time.

(i) Using this model, calculate the probability that there are no more than two bus breakdowns during any four-hour period.

Give any assumption(s) that needs to be made.

Needed to give two assumptions for r

$\mu = 0.3$ per hour

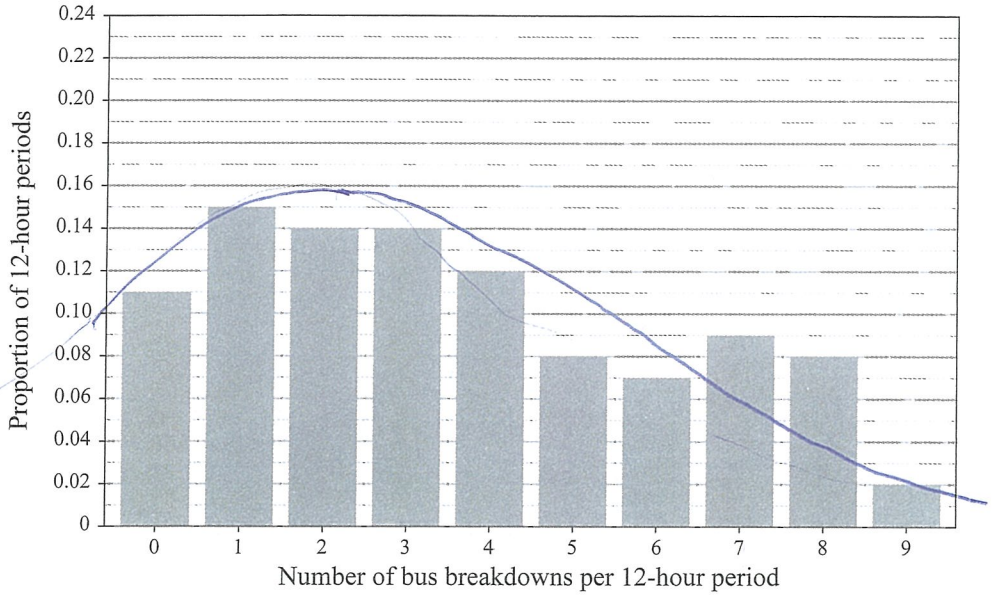
$P(x \leq 2) = 0.9964$ (4dp)

$0.3 \times 4 = \mu$ per 4 hours

$\mu = 1.2$ 4 hour $P(x \leq 2) = 0.8795$ (4dp)

ASSESSOR'S USE ONLY

(ii) The operations manager has produced a graph of the data collected on the number of bus breakdowns during 12-hour periods (the experimental distribution).



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As part of your discussion, you should compare the features of the experimental and model (theoretical) distributions. You may wish to draw on the graph above, but should include any probabilities related to these drawings in your working below.

Poisson - independant ✓
- 2 events cannot occur simultaneously ✗
- event discrete
- interval - (continuous)
- event is proportional to interval

Binomial - fixed trials
- ~~independant~~
- ~~fixed probability~~

No theoretical probabilities calculated, no comparisons made

a poisson distribut

no it is not a good model as the 2 events can occur simultaneously. One bus can break down at the same time as another bus. A normal distribution is a beter model. the grap

- it is because the ~~type~~ buses breaking down are independant meaning if one bus breaks down, it does not affect the other.
- the ~~two~~ buses breaking down cannot happen simultaneously
- buses breaking down are discrete and time in continuous
- buses breaking down are proportional to the interval

A4

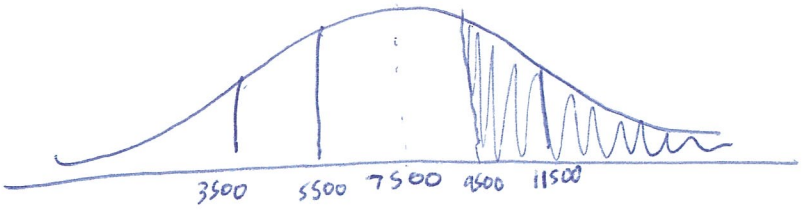
ASSESSOR'S USE ONLY

QUESTION THREE

(a) The prices of ten-year-old hatchback cars sold in New Zealand during 2014 can be modelled by a normal distribution, with mean \$7500 and standard deviation \$2000.

(i) Ten-year-old hatchback cars could be considered "over-priced" if they sold for more than \$9000.

Based on the model provided, calculate the percentage of "over-priced" cars that sold for more than \$9500.



$\mu = 7500$ $\sigma = 2000$ lower = 9500 upper = 99999

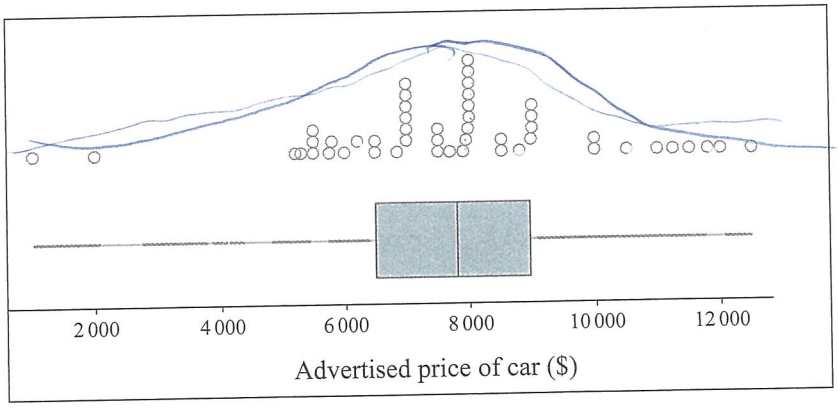
$P(x \geq 9500) = 0.15865$ (5 dp)

$\times 100$

$= 16\%$ (0 dp)

Only one probability calculated

(ii) A Year 13 student obtained data on a random sample of 49 ten-year-old hatchbacks from a New Zealand online trading website during 2014. The advertised prices of these cars are shown in the figure below.



The student claims that the prices of ten-year-old hatchback cars are not normally distributed, as the distribution of car prices in the sample is negatively skewed.

Discuss why the student may be incorrect in their reasoning.

Give at least TWO discussion points.

because the student has only obtained a sample of 49 ten year old hatchbacks from an ~~AA~~ AB

because there

ASSESSOR'S USE ONLY

(b) The time taken to travel along a stretch of motorway in Dunedin by car can be modelled by a random variable that takes on values between 3 and 8 minutes. The most likely time taken is 4 minutes.

Using a triangular probability distribution model:

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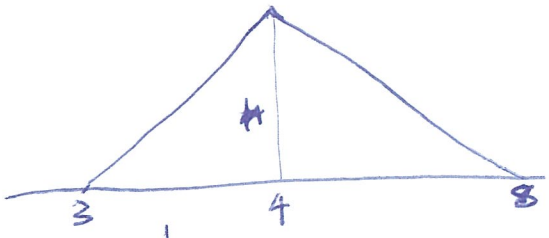

$A = \frac{1}{2} b \times h \;|\; 1 = \frac{1}{2} \times 5 \times h \;|\; 1 = 2.5h \;|\; h = \frac{1}{2.5} = 0.4$

$P(x > 4) = \frac{1}{2} b h = 0.5 \times 4 \times 0.4 = 0.8$

(ii) Explain why the median time taken to travel this stretch of motorway by car is not 6 minutes.

A4