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91028



910280



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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SUPERVISOR'S USE ONLY

Level 1 Mathematics and Statistics, 2016

91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Thursday 17 November 2016
Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|--|---|
| Investigate relationships between tables, equations and graphs. | Investigate relationships between tables, equations and graphs, using relational thinking. | Investigate relationships between tables, equations and graphs, using extended abstract thinking. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL

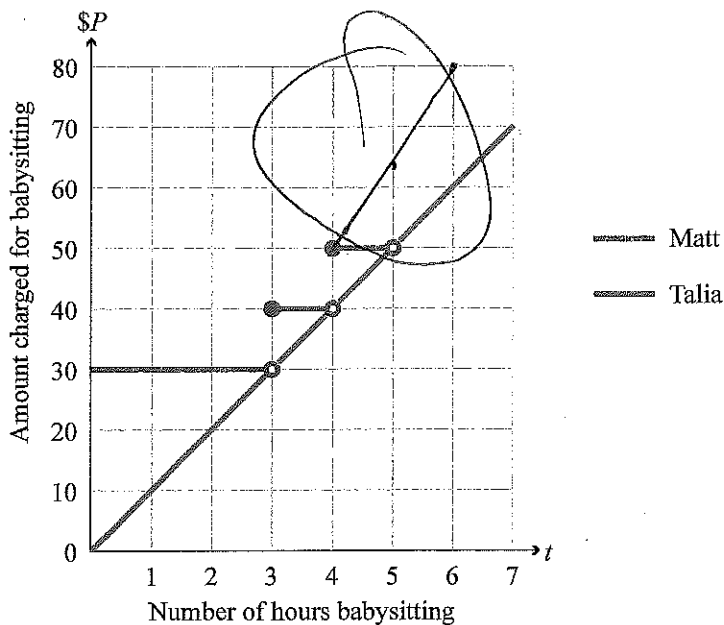
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ASSESSOR'S USE ONLY

QUESTION ONE

Tama and Pita have three different babysitters to choose from: Matt, Talia, and Sasha.

- (a) The graph of the amounts that Matt and Talia charge is shown below.



- (i) How much would Matt be paid if he babysits for 4.5 hours?

\$50.00

- (ii) Once Matt has babysat for 5 or more hours, he increases his charge for the additional hours to \$15 an hour or part of an hour that he babysits.

On the grid above, show the amount Matt would charge if he babysits for 5 or more hours.

- (iii) Find the average amount Matt charges per hour if he babysits for 6 hours.

$$50 + 15 = 65$$

$$P = 15t + 50$$

$$15 \times 5 = 90$$

$$90 - 50 = 40$$

- (iv) Talia charges an **average** of \$10 per hour for any amount of time that she works. This is shown on the graph above with the red line.

Give the equation of the graph.

$$P = 10t$$

- (v) Sasha will babysit for up to 7 hours for \$55.

Make recommendations on who Tama and Pita should have as their babysitter, based on the amount that each babysitter charges.

Sasha as it is cheaper to have her as their babysitter, because at 7 hours Mark will charge \$80.00 and Talia will charge \$70.00, but Sasha only charges \$55.

- (b) (i) Give the equation of the graph shown on the right.

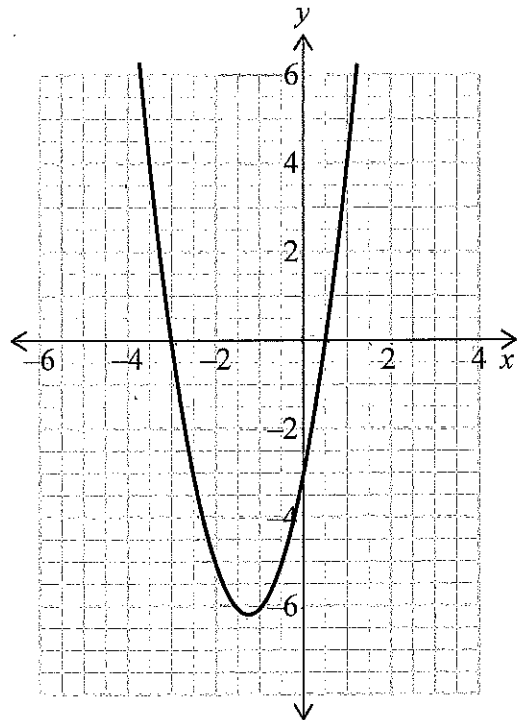
$$y = (-3, 0) \quad (0.5, 0)$$

$$x = 3 \text{ or } -0.5$$

$$y = -1.25x^2 + 3.5x - 3$$

$$\text{Vertex} = \frac{3.5 - 0.5}{2} = 1.25$$

$$(-1.25, -6)$$



- (ii) The graph is then translated a units to the right and up b units.

Give:

- the equation of the translated graph
- the x -value at the vertex.

~~Handwritten scribbles~~

$$y = c(x^2 - a) + b$$

~~$$y = c(x^2 - a) + b$$~~

$$y = x^2 - a + b$$

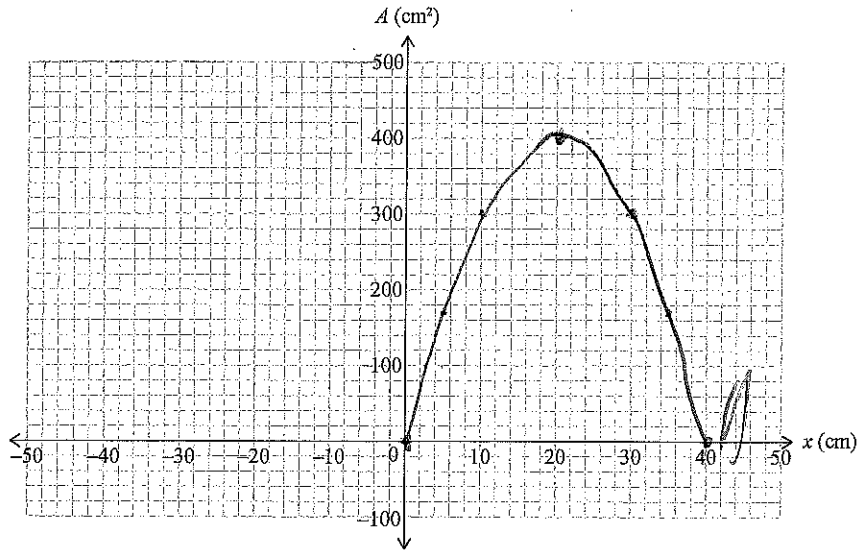
A4

QUESTION TWO

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- (a) (i) Maria is investigating a set of rectangles that have an area modelled by $A = -(x^2 - 40x)$.

Sketch the graph of the possible range of areas of the rectangles as the value of x changes.



| x | y |
|-----|-----|
| 10 | 300 |
| 20 | 400 |
| 30 | 300 |

If you need to redraw this graph, use the grid on page 13.

$$5 \mid 175$$

- (ii) What is the maximum possible area of the rectangles?

~~(20 x 400)~~ ~~2000m~~ ~~(20 / 400)~~
 length = 200 cm
 width = 20 cm **800cm²**

- (iii) For what values of x are the areas less than 300, cm²?

10 < x < 30

- (iv) What is the maximum area of another set of rectangles that have an area, $A = -(x^2 - mx)$?

~~2000~~

Handwritten scribbles at the top of the page.

(b) The points listed in the table below lie on a parabola.

| x | y |
|----|----|
| -2 | -6 |
| -1 | 0 |
| 3 | 4 |
| 5 | -6 |

| | |
|----|----|
| -2 | -6 |
| -1 | 0 |
| 0 | 4 |
| 1 | 4 |
| 2 | 4 |
| 3 | 4 |

$(3, 4)$

$x^2 \pm \dots$

(i) Sketch the parabola represented by these points, and give the coordinates of the intercepts and the vertex.

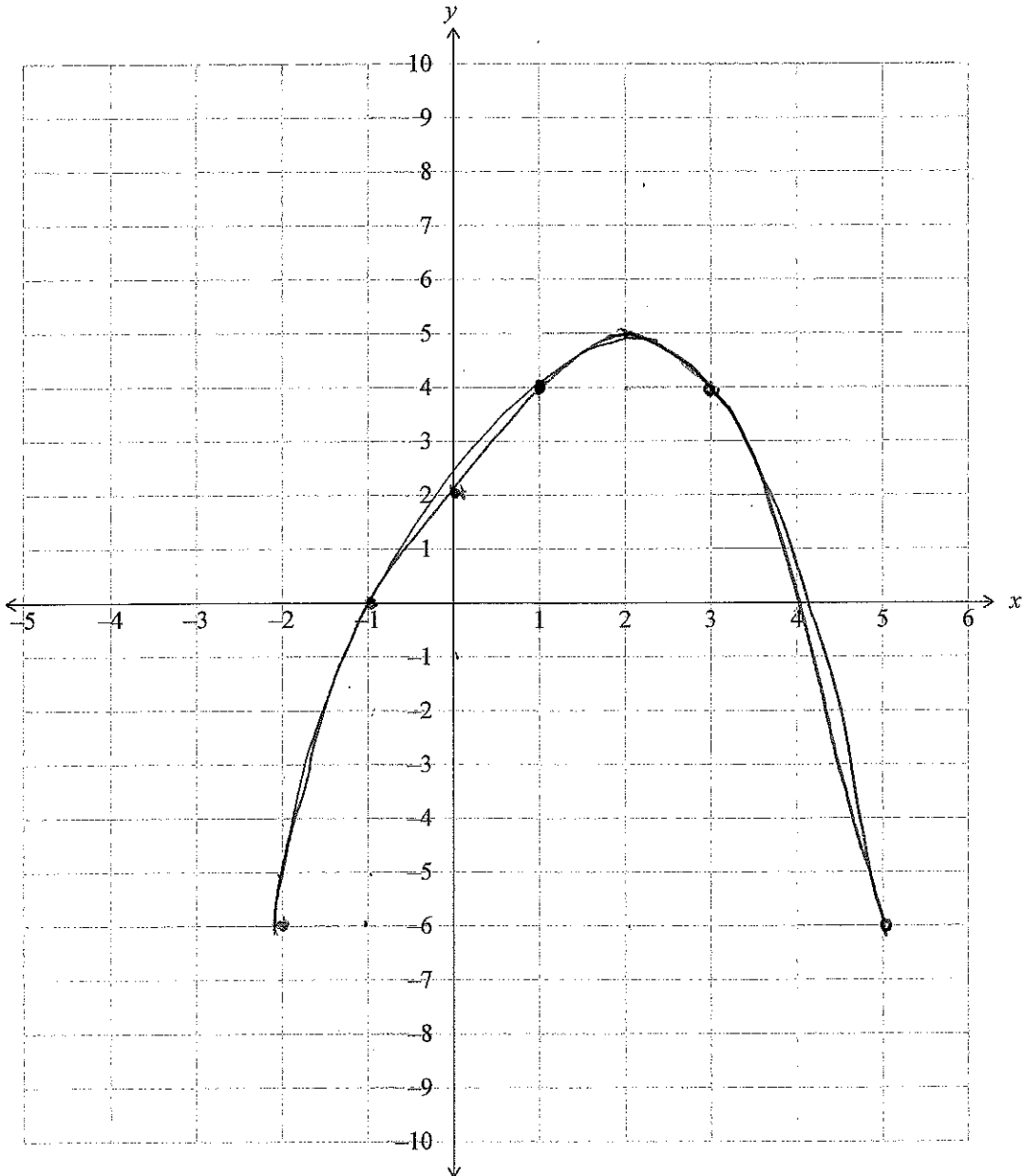
Handwritten notes: x intercepts $(-1, 0)$ $(4, 0)$ $x^2 \pm 10x + 4$

Handwritten notes: y intercept $(0, 2)$

Handwritten sketch of a parabola opening downwards.

Handwritten notes: ~~vertex~~

Handwritten notes: vertex: $(2, 5)$

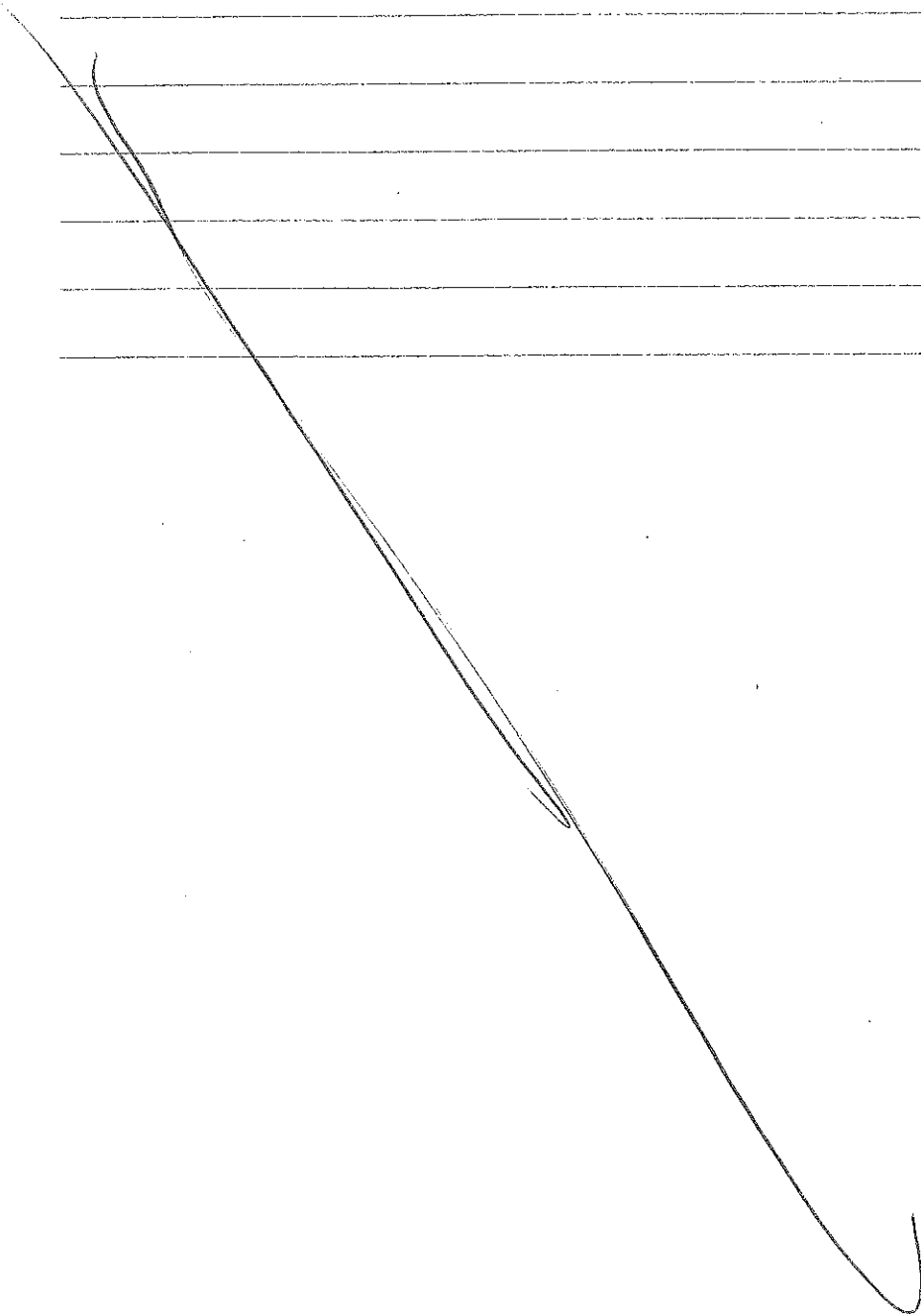


If you need to redraw this graph, use the grid on page 14.

(ii)

Give the equation of the graph.

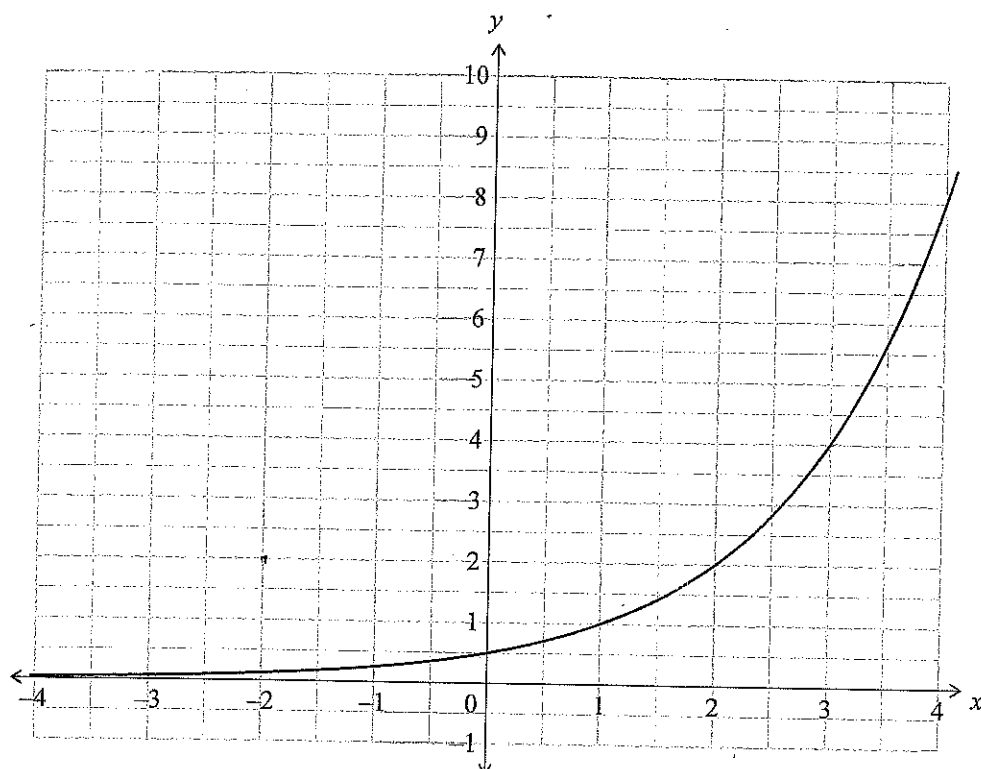
$$y = -1.5x^2 + 3x$$

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QUESTION THREE

- (a) (i) Give the equation of the graph below.



$$\begin{aligned}
 2^4 &= 16 \\
 2^3 &= 8 \\
 2^2 &= 4 \\
 2^1 &= 2 \\
 2^0 &= 1 \\
 2^{-1} &= \frac{1}{2} \\
 2^{-2} &= \frac{1}{4} \\
 2^{-3} &= \frac{1}{8}
 \end{aligned}$$

$$y = p^x$$

$$y = 2^x$$

asymptote ~~at~~

$$y = 2^x$$

- (ii) Give the equation of the resulting graph if the graph above is reflected in the y axis.

$$(0.5, 0)$$

$$y = 0.5x^2 \pm \underline{bx} \pm \underline{c}$$

Positive parabola

up right way

$$\text{Vertex} = (0.5, 0)$$

$$y = 1$$

$$0.2 = (2 \times 4) = 8$$

$$3.2 = (2 \times 3) = 3$$

- (b) A new fun park was very popular when it opened. In the first three months, an average of 4000 people visited the park each month.

After the first three months, the attendance began to drop by approximately 15% each month for the next nine months.

After the first three months, the approximate number of visitors to the park can be modelled by:

$$P = 4000 \times 0.85^{n-3}, \text{ where } n \text{ is the number of months since the park opened.}$$

- (i) Complete the table below showing the approximate number of people who visited the fun park during each month for the first year.

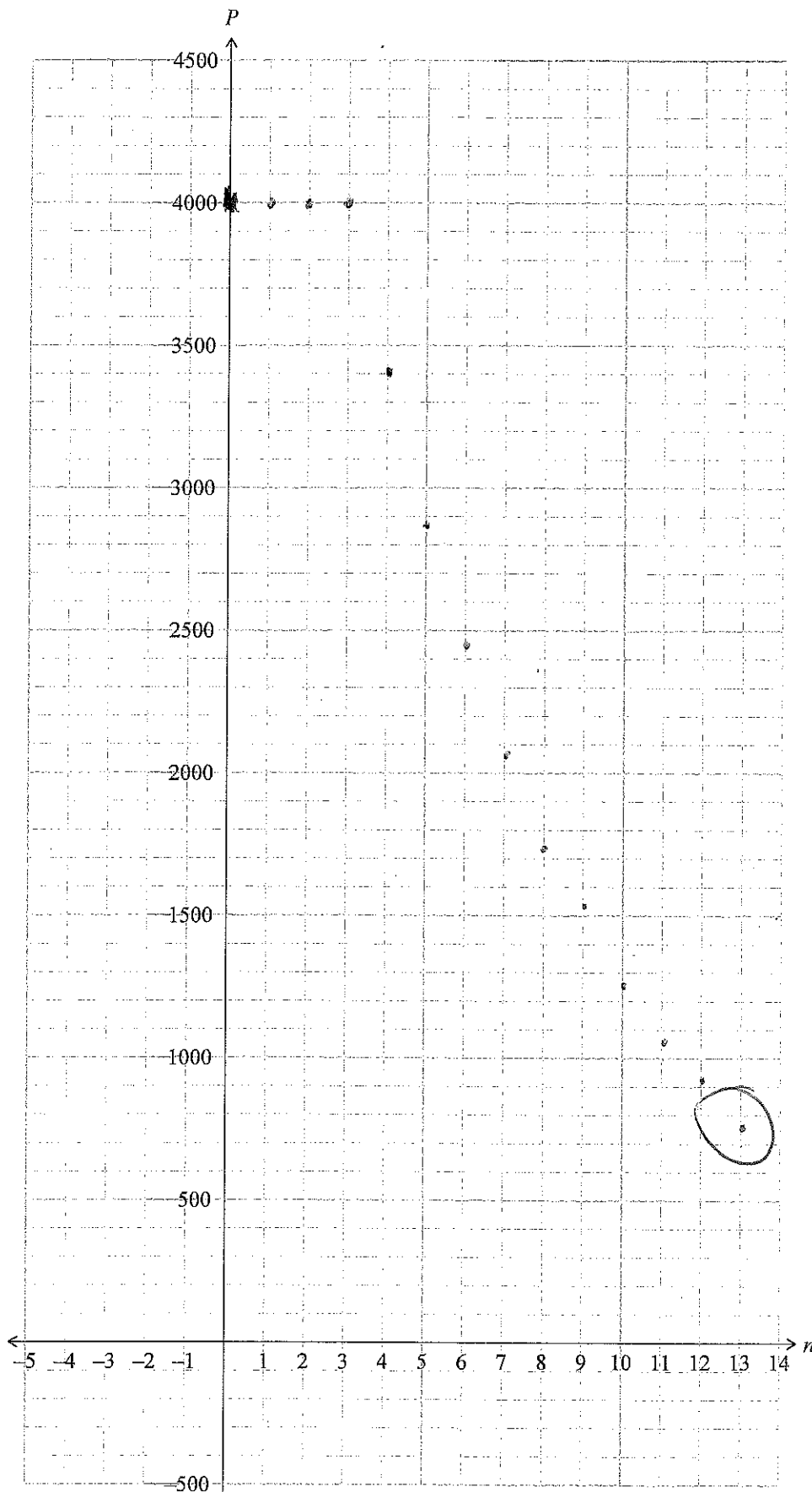
| Month (n) | Approximate number of people visiting park this month (P) |
|------------------|--|
| 1 | 4000 |
| 2 | 4000 |
| 3 | 4000 |
| 4 | 3400 |
| 5 | 2890 |
| 6 | 2457 |
| 7 | 2088 |
| 8 | 1775 |
| 9 | 1509 |
| 10 | 1282 |
| 11 | 1090 |
| 12 | 926 |

} 0
 } 0
 } 0
 } 600
 } 50 } 90
 } 433 } 71
 } 369 } 64

//

- (ii) Draw the graph showing the approximate number of people visiting the fun park each month.

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If you
need to
redraw this
graph, use
the grid on
page 15.

- (iii) At the end of the month that the number of visitors dropped below 2000 for the first time, the management decided to open only on weekends.

Find how many months of the year the park was open only on the weekends, and explain by using the features of the graph, how this information can be found.

For 5 months. This is shown by the smaller decrease in number of people visiting the park as it is a lot smaller than in the other months. //

- (iv) In the second year, more people visit the park during the first three months.

As the year progresses, the number of people visiting the park declines at the same rate as it did for the first year.

The managers want to limit to a maximum of 2 months, the period when the park is running just on weekends.

What is the average number of people who would need to be visiting the park each month in the first three months if this was to be achieved?

~~For 3 months same so people visiting must be higher~~
Amount ~~of~~ of people visiting must be higher as it is only open on weekends for 2 months instead of 5. //

Annotated Exemplar Template

Achieved exemplar 2016

| Subject: | Mathematics | Standard: | 91028 | Total score: | 12 |
|----------|-------------|---|-------|--------------|----|
| Q | Grade score | Annotation | | | |
| 1 | A4 | <p>a(i) Correct.</p> <p>a(ii) Candidate has not recognised that graph is a step function for Matt.</p> <p>a(iii) Incorrect.</p> <p>a(iv) Correct equation.</p> <p>a(v) Candidate has gained u for attempting to find some differentiation between the three babysitters.</p> <p>b(i) and (ii) Incorrect.</p> | | | |
| 2 | A4 | <p>a(i) Correct graph.</p> <p>a(ii) incorrect.</p> <p>a(iii) Candidate has indicated incorrect region.</p> <p>b(i) Candidate has gained only u because y-intercept not identified and vertex is incorrect. Only x-intercepts are correct on graph.</p> <p>b(ii) Incorrect.</p> | | | |
| 3 | A4 | <p>a(i) Candidate has recognised exponential equation but omitted "-1".</p> <p>a(ii) Incorrect.</p> <p>b(i) Table correctly completed.</p> <p>b(ii) Discrete points correctly plotted but grade has been dropped from r to u because of point at 13 months.</p> <p>b(iii) Candidate has gained u for "5 months" which was when visitor numbers dropped below 2000 but incorrectly interpreted the question.</p> <p>b(iv) Incorrect.</p> | | | |