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Level 1 Mathematics and Statistics, 2016

91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Thursday 17 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 23

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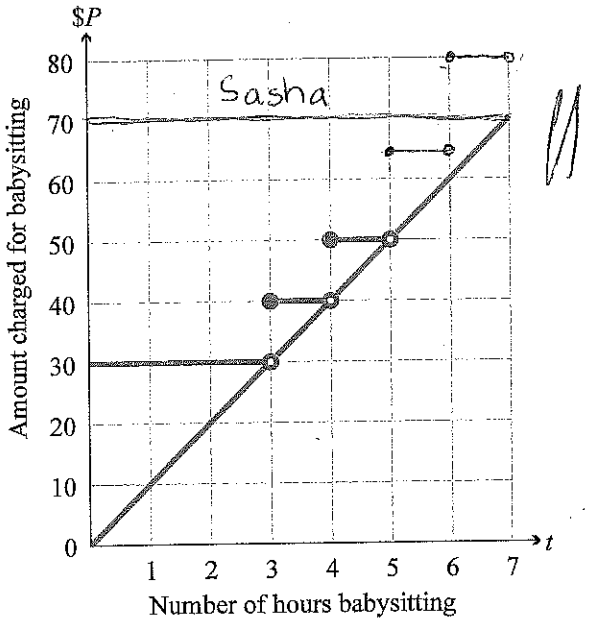
QUESTION ONE

Tama and Pita have three different babysitters to choose from: Matt, Talia, and Sasha.

(a) The graph of the amounts that Matt and Talia charge is shown below.



If you need to redraw your answer, use the graph on page 13.

(i) How much would Matt be paid if he babysits for 4.5 hours?

$50 ✓

(ii) Once Matt has babysat for 5 or more hours, he increases his charge for the additional hours to \$15 an hour or part of an hour that he babysits.

On the grid above, show the amount Matt would charge if he babysits for 5 or more hours.

(iii) Find the average amount Matt charges per hour if he babysits for 6 hours.

$\frac{80}{6} = 13.33$ ✓

Matt charges an average of \$13.3 per hour.

(iv) Talia charges an **average** of \$10 per hour for any amount of time that she works.

This is shown on the graph above with the red line.

Give the equation of the graph.

$P = 10t$ ✓

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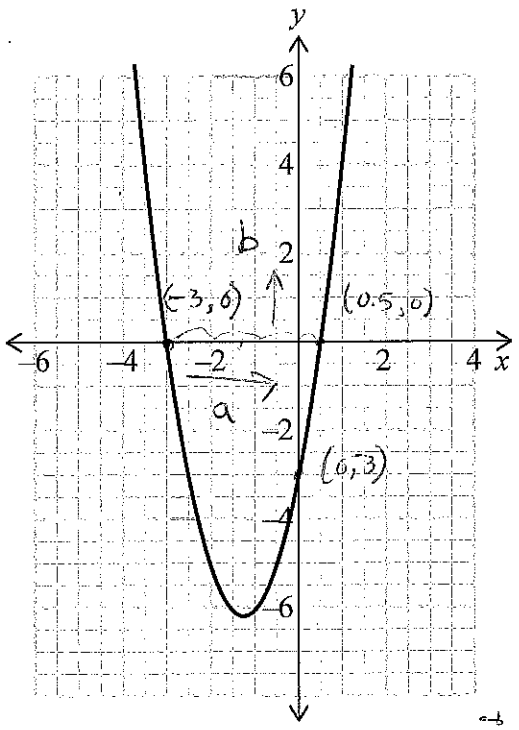
(v) Sasha will babysit for up to 7 hours for $55.

Make recommendations on who Tama and Pita should have as their babysitter, based on the amount that each babysitter charges.

Tama and Pita should choose Talia, because she has the best price, up to 7 hours. Sasha should not be picked because, even if Tama and Pita are out for any time under 7 hours, she will charge $70. Matt should not be chosen because after the 5th Hour his prices increase drastically.

(b) (i) Give the equation of the graph shown on the right.



$y = k(x+3)(x-0.5)$

$-3 = k(0+3)(0-0.5)$

$-3 = k(-1.5)$

$\frac{-3}{-1.5} = k$

$k = 2$

$y = 2(x+3)(x-0.5)$

(ii) The graph is then translated a units to the right and up b units.

Give:

- the equation of the translated graph
- the x-value at the vertex.

$y = 2(x+(3-a))(x-(0.5+a)) + b$

$y = 2(x+3-a)(x-0.5-a) + b$ ← Equation of shifted graph

Original vertex x:

$y = 2(1.25+3)(-1.25-0.5)$

$y = 2(1.75)(-1.75)$

$y = -6.125$

Original Vertex $= x = -1.25$

New Vertex $= x = -1.25 + a$

E8

QUESTION TWO

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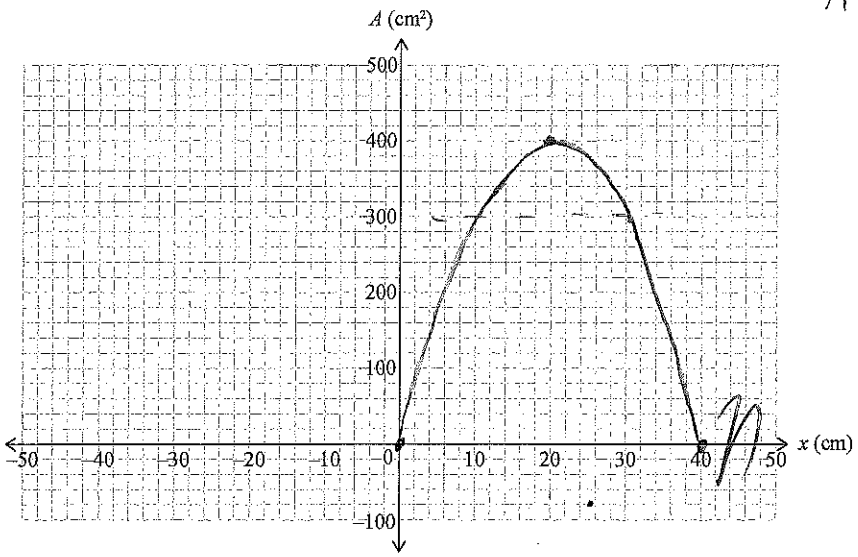
(a) (i) Maria is investigating a set of rectangles that have an area modelled by $A = -(x^2 - 40x)$.

Sketch the graph of the possible range of areas of the rectangles as the value of x changes.

$A = -x^2 + 40x$
$A = -x(x - 40)$



If you need to redraw this graph, use the grid on page 13.

(ii) What is the maximum possible area of the rectangles?

20×20
$= 400 cm^2$ //

(iii) For what values of x are the areas less than 300 cm²?

$x < 10$
$x > 30$ //

(iv) What is the maximum area of another set of rectangles that have an area, $A = -(x^2 - mx)$?

$A = -x^2 + mx$ $A = -x(x - m)$
Max Area :
$\left(\frac{1}{2}m\right)^2$ //

ASSESSOR'S USE ONLY

(b) The points listed in the table below lie on a parabola.

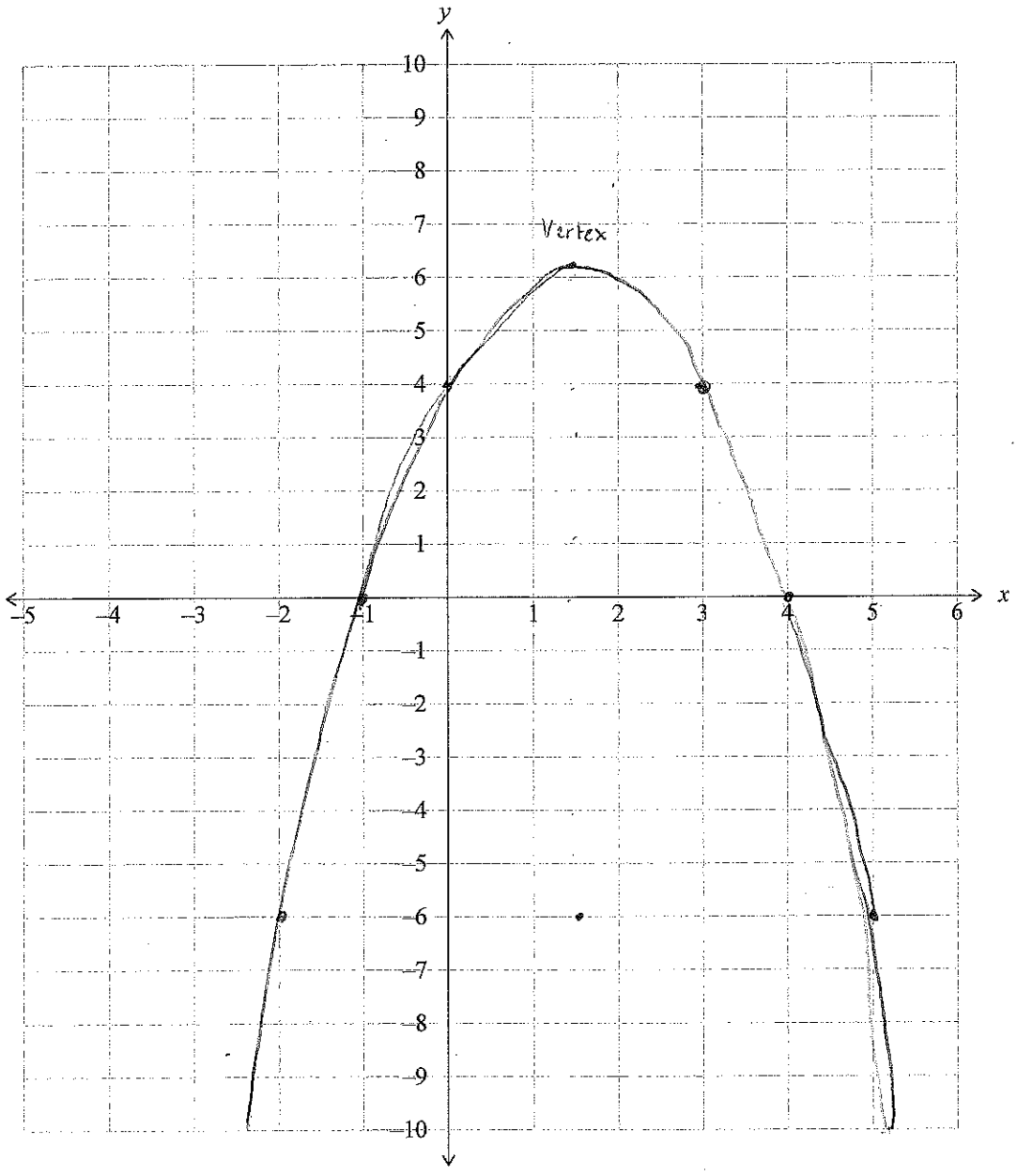
x	y
−2	−6
−1	0
3	4
5	−6

(i) Sketch the parabola represented by these points, and give the coordinates of the intercepts and the vertex.

(-1, 0) (4, 0) = x intercepts

Vertex = (1.5, 6.25)

(0, 4) = y intercept



If you need to redraw this graph, use the grid on page 14.

(ii) Give the equation of the graph.

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$y = k(x+1)(x-4)$

$4 = k(0+1)(0-4)$

$4 = k(-4)$

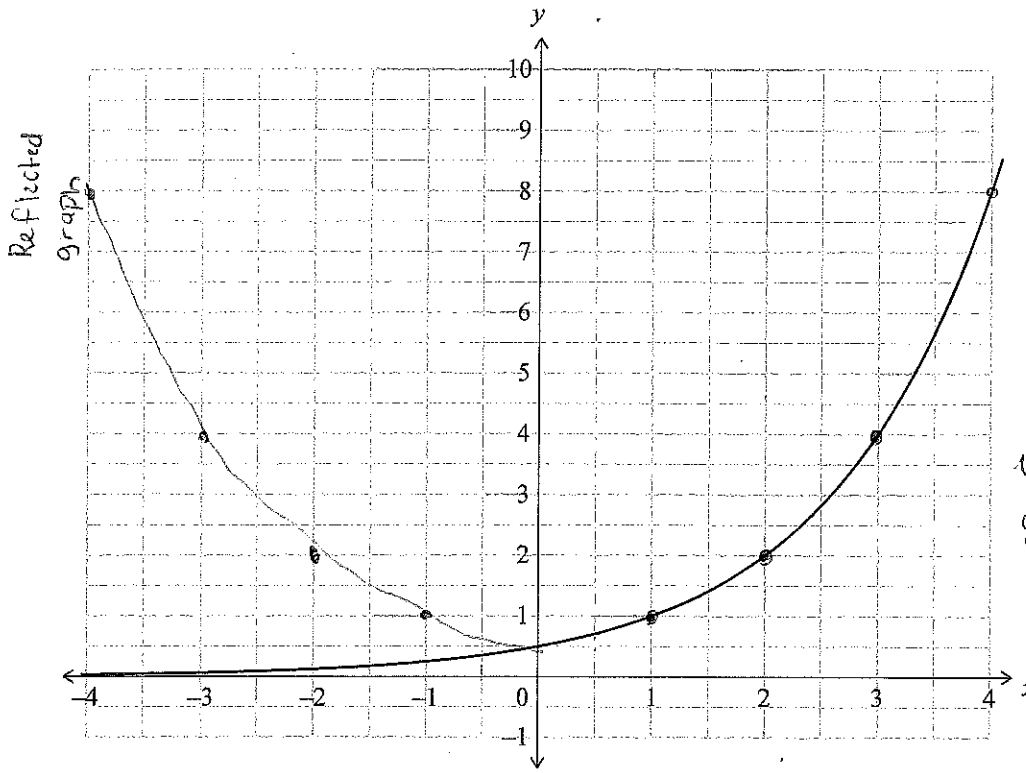
$-1 = k$

$y = -(x+1)(x-4)$ /1

QUESTION THREE

ASSESSOR'S USE ONLY

(a) (i) Give the equation of the graph below.



x	y
0	$\frac{1}{2}$
1	1
2	2
3	4
4	8

$y = 2^{(x-1)}$

$y = \frac{1}{2} \times 2^{x}$

$y = 2^{(x-1)}$ //

(ii) Give the equation of the resulting graph if the graph above is reflected in the y axis.

$y = 0.5^{(x+1)}$ //

ASSESSOR USE ONLY

(b) A new fun park was very popular when it opened. In the first three months, an average of 4000 people visited the park each month.

After the first three months, the attendance began to drop by approximately 15% each month for the next nine months.

After the first three months, the approximate number of visitors to the park can be modelled by:

$P = 4000 \times 0.85^{n-3}$, where n is the number of months since the park opened.

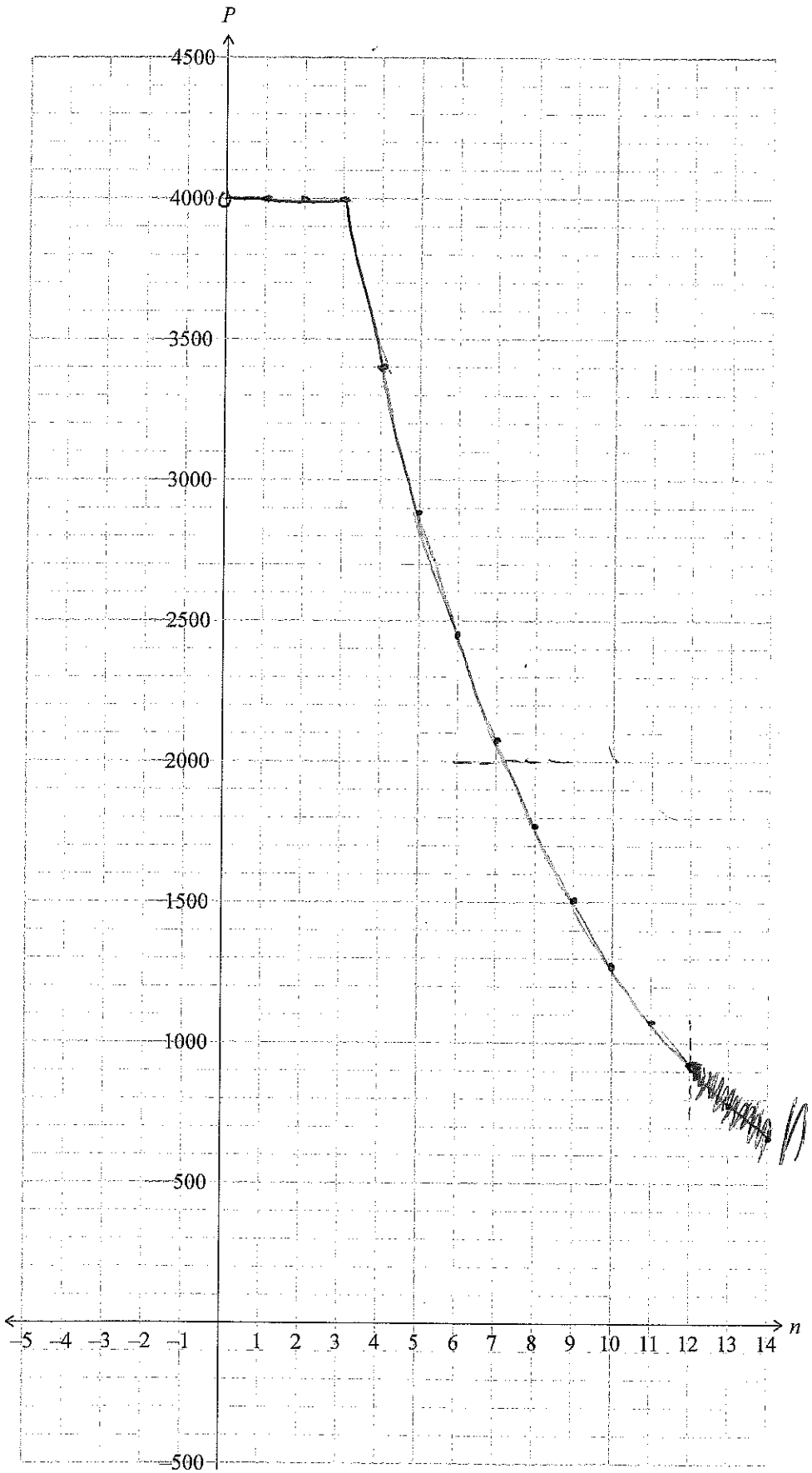
(i) Complete the table below showing the approximate number of people who visited the fun park during each month for the first year.

Month (n)	**Approximate number of people visiting park this month (P)**
1	4000
2	4000
3	4000
4	3400
5	2890
6	2457
7	2088
8	1775
9	1509
10	1282
11	1090
12	926

ASSESSOR'S USE ONLY

(ii) Draw the graph showing the approximate number of people visiting the fun park each month.

If you need to redraw this graph, use the grid on page 15.



(iii) At the end of the month that the number of visitors dropped below 2000 for the first time, the management decided to open only on weekends.

Find how many months of the year the park was open only on the weekends, and explain **by using the features of the graph**, how this information can be found.

The park was open on only weekends from the 8th Month onward. This means that they were open on weekends for 4 months. This can be seen when the graph passes through 2000, during the 8th Month meaning at the end they began to open only on weekends. This trend lasts to the very end of the year all the way to the 12th month //

(iv) In the second year, more people visit the park during the first three months.

As the year progresses, the number of people visiting the park declines at the same rate as it did for the first year.

The managers want to limit to a maximum of 2 months, the period when the park is running just on weekends.

What is the average number of people who would need to be visiting the park each month in the first three months if this was to be achieved?

$P > 2000 \quad t = 10$

$2000 = x \times 0.85^{(10-3)}$

$\frac{2000}{0.32} = x$

$x = 6238.75$

The average number of people that need to come in the first 3 months is 6238 people //

PJ

Annotated Exemplar Template

Excellence exemplar 2016

Subject:	Mathematics	Standard:	91028	Total score:	23

Q	Grade score	Annotation
1	E8	a(ii) Correct steps plotted with open circles. a(i), (iii),(iv). Correct. a(v) Candidate has recognised Matt most expensive to gain u but has incorrectly identified other boundaries. b(i) Correct equation. b(ii) Candidate has correctly substituted a and b and found the x-value at the vertex.
2	E7	a(i) Graph correctly drawn and candidate has recognised that area cannot be negative. a(ii) Correct. a(iii) To gain t candidate needed $0<x<10$ and $30<x<40$. Candidate didn't specify greater than 0 and less than 40. a(iv) Correct. b(i) Candidate has correctly identified the coordinates of the 3 intercepts and the vertex and drawn a correct graph which however, is a little rough. b(ii) Correct equation.
3	E8	a(i) Correct equation. a(ii) Correct variation of the equation. b(i) Table correctly completed. b(ii) Graph correctly drawn. (Discrete or continuous graph was acceptable.) b(iii) Candidate has correctly recognised 4 months with some justification. b(iv) Candidate has demonstrated understanding of situation. Correct answer should have been 6239 but Professional Judgement used to accept incorrect rounding.