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Assessment Report

Level 3 Calculus 2016

Standards 91577 91578 91579

Part A: Commentary

The 2016 papers were similar in format to those of recent years. They gave a similar overall distribution of grades as previous papers with the exception of the Algebra standard, where the Excellence rate was considerably higher than in recent years.

Comments from the Panel Leaders reinforce points made in the 2015 Assessment Report. Namely, that the ability to differentiate and integrate alone will not give success in the respective standards. The ability to solve problems is required, and this will involve algebra in both the setting up of the problem and the solving after the differentiation/integration step has taken place.

There still seem to be some areas of weakness amongst a number of candidates. The inability to understand the concept of 'signed area', unfamiliarity with De Moivre's Theorem and failure to properly use a simple formula such as Simpson's rule correctly cost a number of candidates dearly.

Part B: Report on Standards

91577: Apply the algebra of complex numbers in solving problems

Candidates who were awarded **Achievement** commonly:

- understood and manipulated complex numbers
- understood the remainder theorem and used it appropriately
- solved a quadratic by completing the square
- understood and manipulated surds
- worked in both polar and rectangular forms
- successfully interpreted an Argand diagram.

Candidates who were assessed as **Not Achieved** commonly:

- made careless errors in answering questions
- unnecessarily converted a complex number from one form to another (rectangular to polar or vice versa) before performing calculations with complex numbers
- lacked algebraic skills, in particular in dealing with expressions such as $(5 - \sqrt{x})^2$ or cancelling terms in an expression that were not factors
- failed to simplify expressions early enough (eg continued working with 4i + 4i and did not simplify it to 8i or did not substitute for i^2)
- could not ‘complete the square’ of a simple quadratic
- failed to successfully interpret an Argand diagram
- misused the Remainder Theorem.

Candidates who were awarded **Achievement with Merit** commonly:

- correctly used algebra when solving equations or manipulating expressions
- understood and applied De Moivre’s Theorem correctly
- understood and manipulated modulus expressions
- solved equations with complex solutions
- understood and manipulated conjugates for complex numbers
- manipulated powers of i successfully

Candidates who were awarded **Achievement with Excellence** commonly:

- understood how equal roots could be used as a discriminant expression and solved the equation that resulted
- used their algebra skills to accurately set up and solve equations without unnecessary or confusing statements in their working
- understood the concept of modulus and formed and solved the correct equation using x + iy
- understood the concept of “Proof” and clearly showed the necessary steps in obtaining the required result.

Standard-specific comments

Candidates need to be clear what a proof is. Substituting a particular complex number instead of using general terms does not meet this requirement. When two complex numbers are used in a proof, they should not be complex conjugates, nor should one be a linear multiple of the other. Using such complex numbers invalidates the general nature of the proof.

Knowledge of De Moivre’s theorem, and how to apply it when the equation contains a pro-numeral, is fundamental to Complex Numbers at this level.

An answer calculated using a graphic calculator does not demonstrate relational thinking and in itself is not sufficient evidence to award a merit grade.

91578: Apply differentiation methods in solving problems

Candidates who were awarded **Achievement** commonly:

- differentiated functions involving negative and fractional indices
- differentiated power, exponential, logarithmic and trigonometric functions
- utilised the chain, product and quotient rules
- recognised the properties of continuity, differentiability, concavity, and limits from a graph
- demonstrated understanding of the relationship between the gradients of tangents and normals
- demonstrated good algebra skills.
- used their calculator accurately, especially when substituting into trigonometric functions.

Candidates who were assessed as **Not Achieved** commonly:

- were unable to use the chain rule to find derivatives
- did not recognise when the product rule was required for finding the derivative
- were unable to rewrite a function expressed with surds using negative and fractional powers in order to differentiate it
- demonstrated poor algebra skills.

Candidates who were awarded **Achievement with Merit** commonly:

- found the derivatives of trigonometric functions expressed parametrically and used them to evaluate a gradient
- found the x value of a second point where the gradient of that tangent was perpendicular to the tangent at a given point on a parabola
- demonstrated understanding of the properties of continuity, differentiability, concavity and limits from a graph
- solved a straight-forward related rates of change problem involving the volume of a sphere.
- found the required quadratic function to write an equation for an area of a rectangle that could be differentiated and solved equal to zero
- differentiated exponential and trigonometric functions using the quotient rule
- manipulated fractions involving exponential and trig functions to complete a proof.

Candidates who were awarded **Achievement with Excellence** commonly:

- used the chain and product rules to find the second derivative of an exponential function
- set up an appropriate algebraic model for the volume of the cone that would then allow them to find the derivative and solve it equal to zero
- used the trigonometric compound angle formula to set up an appropriate model relating the angle and the distance of the ball from the rugby goal posts
- used the information provided in the question to model the problem with an appropriate function.

Standard-specific comments

Candidates often demonstrated good differentiation skills but lacked the number and/or algebra skills to finish solving the problem.

Many candidates did not recognise the situations where they needed to use the chain and product rules.

Some candidates wrote incorrect inequality statements for question 2c and did not understand when using language rather than symbols that they needed to be explicit about whether the end points of the intervals were included or not.

Candidates found question 1 d challenging despite the straightforward nature of the function involved

In question 3 (e), many candidates failed to recognise that maximising tan θ would also maximise θ, since $y = tan\theta$ is an increasing function over the interval involved in the problem.

91579: Apply integration methods in solving problems

Candidates who were awarded **Achievement** commonly:

- integrated trigonometric expressions
- integrated exponential expressions
- integrated an expression and used the result to find the area under a curve
- rearranged expressions into a form that could be integrated
- integrated acceleration to find an expression for velocity
- correctly used the Simpsons rule
- recognised the 'signed area' concept of definite integration.

Candidates who were assessed as **Not Achieved** commonly:

- failed to properly simplify an algebraic fraction before integrating
- incorrectly expanded brackets before integrating
- misunderstood the 'signed area' concept of definite integration
- failed to correctly use the Simpsons rule – particularly calculating h and n
- failed to show the correct integration when calculating the area under a curve.

Candidates who were awarded **Achievement with Merit** commonly:

- split variables, integrated expressions and found the constant of integration
- integrated exponential expressions and found the area below given limits
- integrated an acceleration expression to find velocity and evaluated the constant and then integrated again to find a distance expression
- integrated a definite integral and equated with the given area to find the value of a constant

- found the area between two curves by first finding where they intersect to get the limits of integration and then correctly subtracting the two expressions.
- successfully used the reverse chain rule.

Candidates who were awarded **Achievement with Excellence** commonly:

- split variables and integrated a differential equation and then correctly calculated the constants
- manipulated an exponential expression with two variables so that they could then split the variables and integrate each side
- excelled at algebraic manipulation at all stages of problems
- followed an extended chain of reasoning to a successful conclusion.

Standard-specific comments

Q2(b) which involved a shaded area under a curve was again poorly answered this year – candidates did not understand that the area under the curve below the x axis is negative.

Question 3 e proved difficult for most candidates. They were unable to express e^(*y* + sin*x*) as e^*y*.e^sin*x*, which is the step that leads to successfully splitting the variables

Losing a negative sign when integrating cosx is a remarkably common occurrence.

Being able to integrate alone will not enable candidates to pass this standard. The ability to solve problems is required and this process will involve algebra. Hence reliable algebraic skills remain the backbone of Calculus.

Mathematics and Statistics subject page