91579

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SUPERVISOR'S USE ONLY

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NZQA
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QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Calculus, 2016

91579 Apply integration methods in solving problems

9.30 a.m. Wednesday 23 November 2016
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 19

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QUESTION ONE

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For parts (a) and (b) find each integral.

Remember the constant of integration.

(a) $\int \frac{2x^4 - x^2}{x^3} \mathrm{d}x$

$\int \frac{2x^4}{x^3} - \frac{x^2}{x^3} \, dx \rightarrow \int 2x - \frac{1}{x} \, dx$

$= x^2 - \ln|x| + c$

(b) $\int \sec(3x)\tan(3x)\mathrm{d}x$

$\frac{1}{3}\sec(3x) + c$

(c) If $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos x}{3y}$ and $y = 1$ when $x = \frac{\pi}{6}$, find the value of y when $x = \frac{7\pi}{6}$.

You must use calculus and give the results of any integration needed to solve this problem.

$dy\,(3y) = \cos x \, dx.$

$\frac{3}{2}y^2 = \sin x + c$

$\frac{3}{2} = \sin\left(\frac{\pi}{6}\right) + c$

$\frac{3}{2} = \frac{1}{2} + c$

$\frac{3}{2} - \frac{1}{2} = c$

$\frac{2}{2} = c$

$c = 1$

$\frac{3}{2}y^2 = \sin x + 1$

$\frac{3}{2}y^2 = \sin\left(\frac{7\pi}{6}\right) + 1$

$\frac{3}{2}y^2 = -0.5 + 1$

$\frac{3}{2}y^2 = 0.5$

$3y^2 = 1$

$y^2 = \frac{1}{3}$

$y = \sqrt{\frac{1}{3}}$ or 0.577

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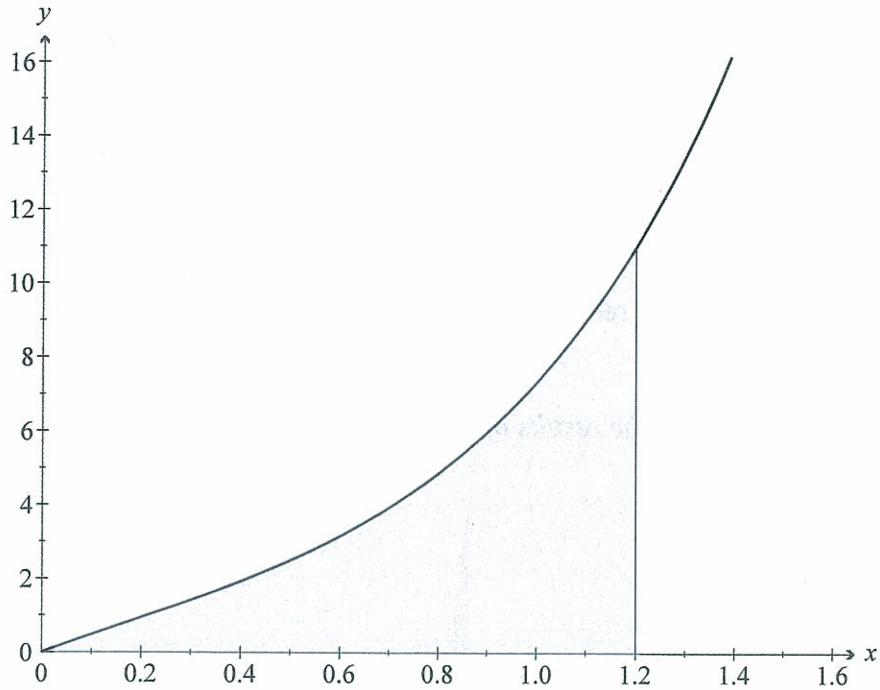
(d) Use integration to find the area enclosed between the curve $y = e^{2x} - \frac{1}{e^{3x}}$ and the lines $y = 0$, $x = 0$, and $x = 1.2$ (the area shaded in the diagram below).



You must use calculus and give the results of any integration needed to solve this problem.

$\int_0^{1.2} e^{2x} - \frac{1}{e^{3x}} \, dx \rightarrow \int e^{2x} - e^{-3x}$

$= \left[\frac{1}{2}e^{2x} + \frac{1}{3}e^{-3x}\right]_0^{1.2}$

$\left[\frac{1}{2}e^{2(1.2)} + \frac{1}{3}e^{-3(1.2)}\right] - \left[\frac{1}{2}e^{2(0)} + \frac{1}{3}e^{-3(0)}\right]$

$\left[\frac{1}{2}e^{2(1.2)} + \frac{1}{3}e^{-3(1.2)}\right] - \left[\frac{1}{2} + \frac{1}{3}\right]$

$= 4.69 \text{ unit}^2$

(e) Mr Newton has a container of oil and places it in the garage. Unfortunately, he puts the container on top of a sharp nail and it begins to leak.

The rate of decrease of the volume of oil in the container is given by the differential equation

$$\frac{dV}{dt} = -kVt$$

where V is the volume of oil remaining in the container t hours after the container was put in the garage.

The volume of oil in the container when it was placed in the garage was 3000 mL.

After 20 hours, the volume of oil in the container was 2400 mL.

How much, if any, of the oil will remain in the container 96 hours after it was placed in the garage?

You must use calculus and give the results of any integration needed to solve this problem.

$dV \cdot \frac{1}{V} = kt \cdot dt$

$\ln|V| = \frac{k}{2}t^2 + c$

$V = e^{k/2 t^2 + c}$

$V = V_0 e^{\frac{k}{2}t^2}$ $\quad t = 0 \quad V = 3000$.

$3000 = V_0 e^0$

$3000 = V_0$.

$V = 3000 e^{\frac{k}{2}t^2}$

$2400 = 3000 e^{\frac{k}{2}(20^2)}$

$\frac{24}{30} = e^{\frac{k}{2}400}$

$\frac{24}{30} = e^{200k}$

$0.8 = e^{200k}$

$\ln|0.8| = 200k$

$-1.12 \times 10^{-3} = k$

$V = 3000 e^{-5.6 \times 10^{-4} t^2}$

$V = 3000 e^{-5.6 \times 10^{-4} (96)^2}$

$V = 17.55 \text{ mL}$

after 96 hours.

ASSESSOR'S USE ONLY

E8

QUESTION TWO

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(a) Find $\int (5x^2 - 1)^2 \, dx$.

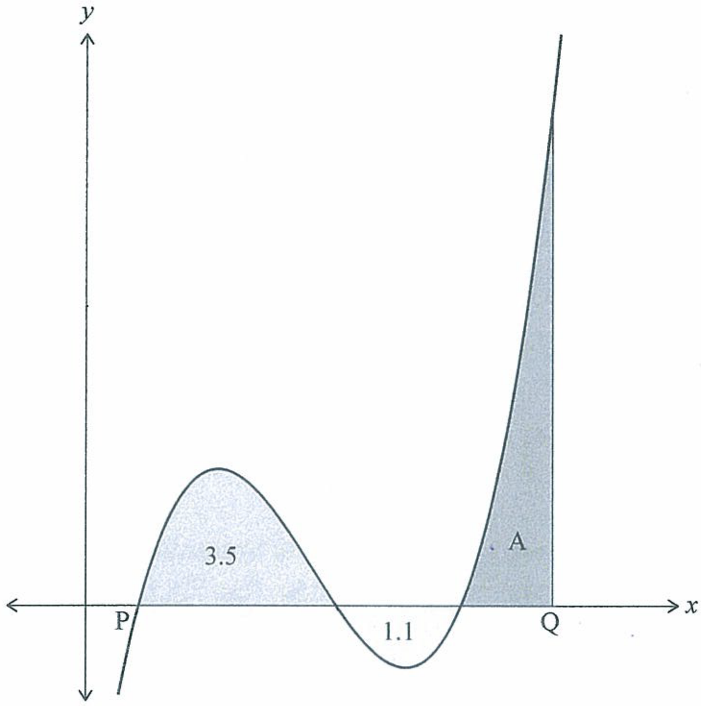
$\int (5x^2 - 1)^2 \, dx$

$\frac{1}{30x}(5x^2-1)^3 \quad + C$

$30x(5x^2-1)^3$

$\frac{3}{30x}(5x^2-1)^2 \cdot 10x$

(b) The graph of a function $y = f(x)$ is shown below.



The areas of two of the shaded regions are given.

If $\int_P^Q f(x)\,dx = 9.4$, what is the area of shaded region A?

$3.5 - 1.1 + A = 9.4$

$2.4 + A = 9.4$

$A = 7 \text{ unit}^2$

$\nearrow 0\cdot2t + 0\cdot3t^{\frac{1}{2}}$

ASSESSOR'S USE ONLY

(c) The acceleration of an object is given by $a(t) = 0.2t + 0.3\sqrt{t}$ for $0 \leq t \leq 10$.
where a is the acceleration of the object in m s^{-2}
and t is the time in seconds from when the object started to move.

The object was moving with a velocity of 5 m s^{-1} when $t = 4$.

How far was the object from its starting point after 9 seconds?

You must use calculus and give the results of any integration needed to solve this problem.

$v(t) = \frac{0\cdot2}{2}t^2 + \cancel{3}\ \frac{2}{3} \times 0\cdot3t^{\frac{3}{2}} + c$

$v(t) = 0\cdot1t^2 + 0\cdot2t^{\frac{3}{2}} + c$

$5 = 0\cdot1(4)^2 + 0\cdot2(4)^{\frac{3}{2}} + c$

$5 = 1\cdot6 + 1\cdot6 + c$

$5 = 3\cdot2 + c$

$1\cdot8 = c$

$v(t) = 0\cdot1t^2 + 0\cdot2t^{\frac{3}{2}} + 1\cdot8$

$v(9) = 0\cdot1(9)^2 + 0\cdot2(9)^{\frac{3}{2}} + 1\cdot8$

$= 8\cdot1 + 5\cdot4 + 1\cdot8$

$= 15\cdot3$ m/s

(d) Find the value of the constant m such that $\int_m^{2m} (2x - m)^2 \, dx = 117$.

You must use calculus and give the results of any integration needed to solve this problem.

$\left[\frac{1}{6}(2x-m)^3\right]_m^{2m} = 117$

$\frac{1}{6}(2(2m)-m)^3 - \frac{1}{6}(2(m)-m)^3 = 117$

$\frac{1}{6}(4m-m)^3 - \frac{1}{6}(2m-m)^3 = 117$

$\frac{1}{6}(3m)^3 - \frac{1}{6}(m)^3 = 117$

$\frac{1}{6}(9m^3 - m^3) = 117$

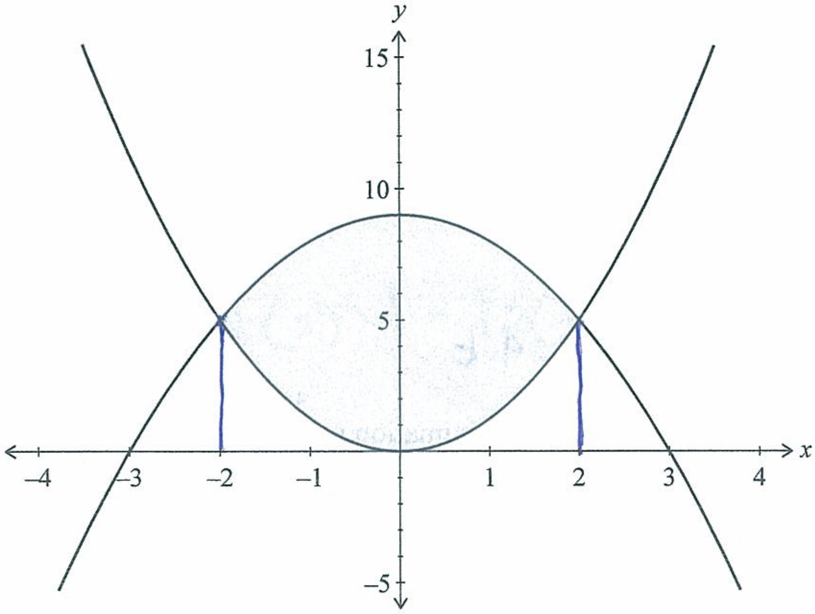
$8m^3 = 702$

$m^3 = 87\cdot75$

$m = 4\cdot44$

ASSESSOR'S USE ONLY

(e) The graphs of $y = (k-1)x^2$, $k > 1$ and $y = 9 - x^2$ are shown in the diagram below.



The shaded region has an area of 24.

Find the value of k.

You must use calculus and give the results of any integration needed to solve this problem.

$\int_{-2}^{2} 9 - x^2 - (k-1)x^2 \, dx$

$= \int_{-2}^{2} 9 - x^2 - (x^2k - x^2) \, dx$

$= \int_{-2}^{2} 9 - \cancel{x^2} - x^2k + \cancel{x^2} \, dx$

$= \int_{-2}^{2} 9 - x^2k \, dx$

$\left[9x - \frac{k}{3}x^3\right]_{-2}^{2}$

$\left[9(2) - \frac{k}{3}(2)^3\right] - \left[9(-2) - \frac{k}{3}(-2)^3\right] = 24.$

$18 - \frac{8}{3}k - \left(-18 + \frac{8k}{3}\right) = 24.$

$18 - \frac{8}{3}k + 18 - \frac{8k}{3} = 24.$

$-\frac{16}{3}k = 24 - \cancel{36}.$

$k = \frac{1}{4}$ or 0.25

M5

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find the value of k if $\int_1^4 \left(4+\frac{k}{x^2}\right)\mathrm{d}x = 0$.

$4 + kx^{-2}$

$\left[4x - \frac{k}{x}\right]_1^4 = 0$.

$\left(4\times 4 - \frac{k}{4}\right) - \left(4\times 1 - \frac{k}{1}\right) = 0$.

$16 - \frac{k}{4} - 4 + k = 0$.

$-\frac{k}{4} - k = -12$.

$\frac{k}{4} + k = 12$. $k = 9.6$

(b) Use the values given below to find an approximation to $\int_1^4 f(x)\mathrm{d}x$, using Simpson's rule.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	1.4	2	3	3.8	2.8	2.2	1.8

$= \frac{1}{3}\, 0.5\left(1.4 + 1.8 + 4(2 + 3.8 + 2.2) + 2(3 + 2.8)\right)$

$= \frac{1}{6}\left(3.2 + 4(8) + 2(5.8)\right)$

$\frac{1}{6}\left(3.2 + 32 + 11.6\right.$

$\frac{46.8}{6}$

$= 7.8 \text{ unit}^2$

(c) Use integration to find the area enclosed between the graphs of the functions $y = 2 - x^2$ and $y = -x$.

You must use calculus and give the results of any integration needed to solve this problem.

$2 - x^2 = -x$

$0 = x^2 - x - 2$ $\quad x = 2$ and -1

$\int_{-1}^{2} 2 - x^2 - (-x)\, dx \rightarrow \int_{-1}^{2} (2 - x^2 + x)\, dx$

$\left[2x - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-1}^{2}$

$\left[2(2) - \frac{1}{3}(2)^3 + \frac{1}{2}(2)^2\right] - \left[2(-1) - \frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2\right]$

$\left(4 - \frac{8}{3} + 2\right) - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$

$4 - \frac{8}{3} + 2 + 2 - \frac{1}{3} - \frac{1}{2} = 4.5$ unit^2

(d) Find $\int \left(\frac{e^{3x} - x^2}{e^{3x} - x^3}\right) dx$.

~~top is the derivative of bottom~~

$\frac{1}{3} \ln |e^{3x} - x^3| + C$

numerator is the derivative of denomenator so the coeffient $\frac{1}{3}$ would due the wanted derivative.

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(e) If $\sec x \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{y+\sin x}$ and $y = -1$ when $x = 0$, find the value of y when $x = \frac{\pi}{2}$.

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$\sec x \, dy = e^{y+\sin x} dx.$

$\sec x \, dy = e^{y} \times e^{\sin x} dx.$

$\frac{1}{e^{y}} dy = \frac{e^{\sin x}}{\sec x} dx.$

$\cancel{e^{y}} \, dy = \cos e^{\sin x} dx.$

$\ln|e^{y}| = e^{\sin x} + c.$

$y = e^{\sin x} + c.$

$-1 = e^{\sin 0} + c$

$-1 = 1 + c$

$-2 = c$

$y = e^{\sin x} - 2$

$y = e^{\sin \frac{\pi}{2}} - 2.$

$y = e^{1} - 2$

$y = 0.718.$

M6

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91579

Merit Exemplar: Total Score = 19

Question 1 = This question provides evidence for E8 because the candidate has gained 1 t grade for their efforts in part e)

a) They have separated the variables and integrated the differential equation correctly

b) They have used the initial variables correctly to find A

c) They have used the other variables provided to find k

d) They have substituted t = 96 in to correctly find V = 17.55

Question 2= = This question provides evidence for M5 because the candidate has gained 1 r grade for their efforts in part e)

a) They have correctly recognised that the x values of the points of intersection of the two curves are -2 and 2, and recognised these are the limits for the integral

b) They have set up a correct integral involving the equation of the top curve minus the equation of the bottom curve

c) They have integrated this integral correctly

d) They have not gained a grade of t for this question because their final answer of 0.25 is incorrect

Question 3 = = This question provides evidence for M6 because the candidate has gained 2 r grades for their efforts in part c) and d)

in c)

a) They have correctly found the x values of the points of intersection of the two curves and recognised these are the limits for the integral

b) They have set up a correct integral involving the equation of the top curve minus the equation of the bottom curve

c) They have integrated this integral correctly

d) They have substituted the limits in and used algebra to correctly come up with an answer of 4.5

in d)

a) They have correctly integrated the function recognising that the integral of f'(x)/f(x) is ln(f(x))

in e)

a) They only got a grade of u because they only integrated one part of the integral correctly