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SUPERVISOR'S USE ONLY

Level 3 Mathematics and Statistics (Statistics), 2016

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

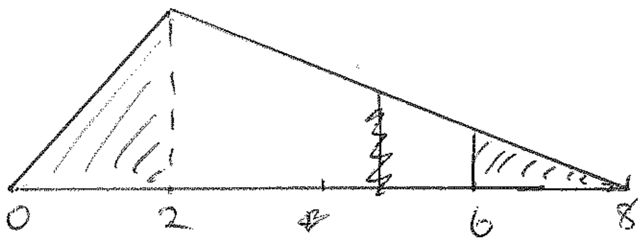
TOTAL 11

ASSESSOR'S USE ONLY

QUESTION ONE

(a) The time it takes a shopper to find a carpark at the supermarket can be modelled by a random variable that takes on values between 0 minutes and 8 minutes. The most likely time it takes a shopper to find a carpark is 2 minutes.

Using an appropriate model, calculate the probability that it will take less than two minutes OR more than six minutes for a shopper to find a carpark.



$a = \frac{1}{2}b \times h$

$1 = \frac{1}{2}b \times h$

triangular distribution

$h = 2/(b-a)$ $h = 2/8 = 0.25$

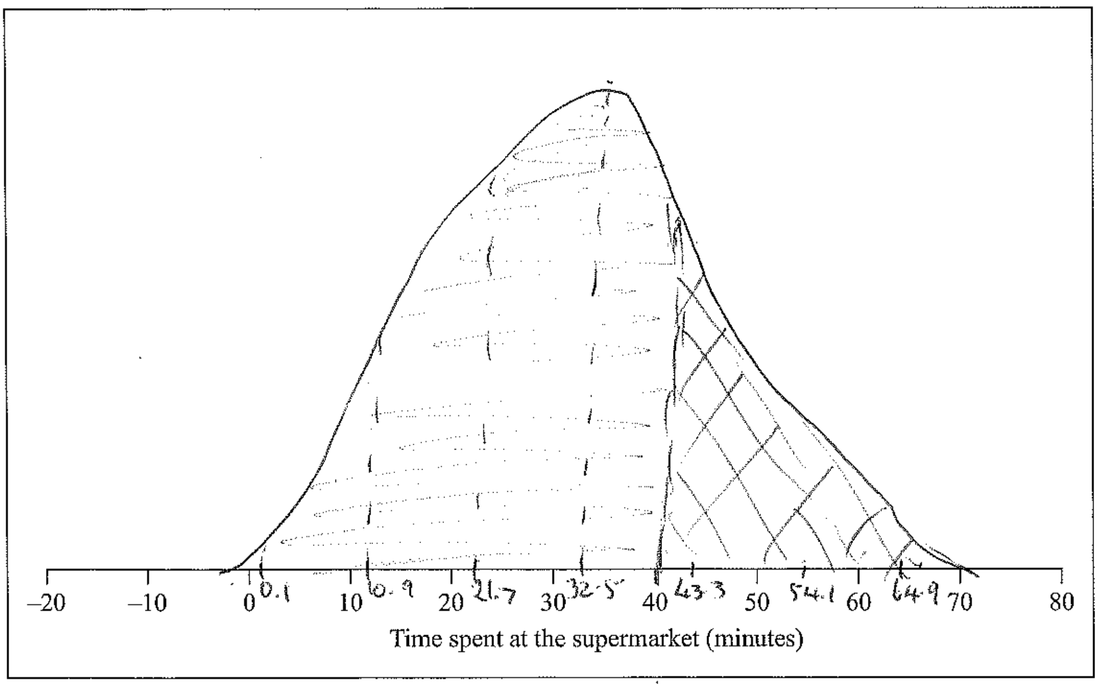
$a = \frac{1}{2}b \times h$ $a = \frac{1}{2} \times 2 \times 0.25 = 0.25$ $P(X<2) = 0.25$

$f(6) = \frac{2(b-x)}{(b-a)(b-c)} = 2(8-6)/((8-0) \times (8-2))$

$= 0.0833$

(b) A supermarket has modelled the time shoppers spend at the supermarket using a normal distribution with a mean of 32.5 minutes and a standard deviation of 10.8 minutes.

(i) Sketch this probability distribution model on the axis below.



ASSESSOR'S USE ONLY

(ii) Using this model, calculate the probability that two different randomly selected shoppers both spend more than 40 minutes at the supermarket.

Give any assumption(s) that need to be made.

Each shopper is selected independently

$P(X \leq 40) = 0.244$ (3sf)

$P = 0.244 \times 0.244 = 0.0595$ that two shoppers will both spend 40+ minutes in supermarket

(iii) Following an observational study of shoppers, the supermarket has changed its model for the time shoppers spend at the supermarket. For this new model, the supermarket has kept the mean the same as the old model, but has adjusted the standard deviation. Using this new model, the percentage of shoppers who take longer than 40 minutes at the supermarket is estimated to be 31.1%.

Discuss how the standard deviation of the new model for the time shoppers spend at the supermarket compares with the standard deviation of the old model for the time shoppers spend at the supermarket.

You may wish to refer to your answers in parts (i) and (ii) to support your explanation.

The old model with a standard deviation of 10.55 minutes and a mean of 32.5 minutes had a P(40+) of 0.244 or 24.4%. By keeping the same mean, but changing the standard deviation the new percentage for spending 40+ mins is 31.1%. This increases the standard deviation.

(iv) Discuss ONE potential limitation with using a normal distribution to model the time spent at this supermarket.

It statistically becomes very small for a person to spend a greater amount of time in the supermarket, say over 80 minutes, which for some people who have big families, or shop 1 time a month this could be a reality for.

A4

QUESTION TWO

ASSESSOR'S USE ONLY

(a) A supermarket has eight employees who are "on call" to help out during busy periods. Based on the supermarket's records, the probability of one of these employees being unavailable when called is estimated to be 0.14.

The supermarket needs to call all eight employees during one particularly busy period.

(i) Using an appropriate model, calculate the probability that fewer than three of these employees will be unavailable when called.

not including 3

Binomial: $n = 8$ $\pi = 0.14$

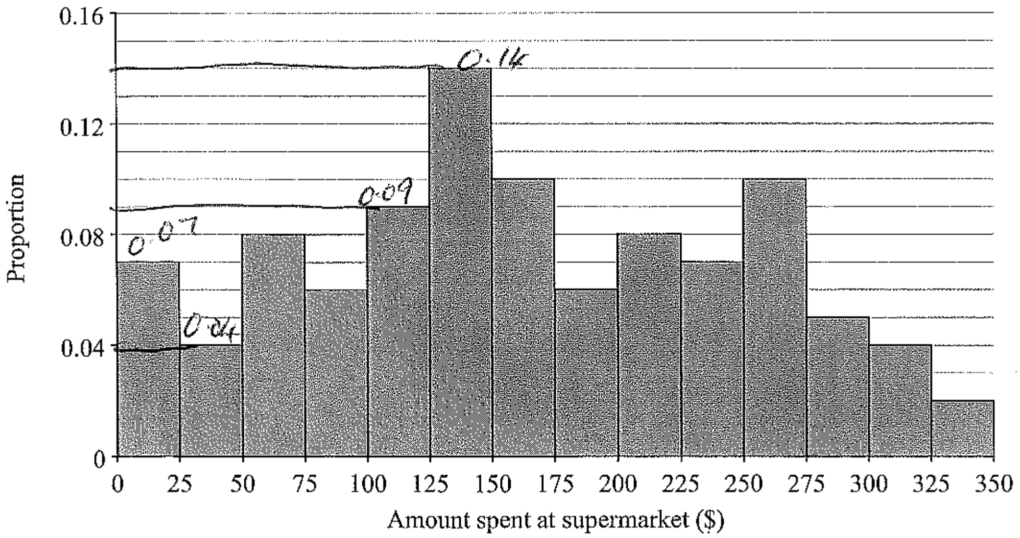
$P(X < 3) = 0.91$

(ii) Justify the use of the probability distribution for your answer in (i).

Binomial distribution fits this model as there are a set number of people/trials (8); each person is independent from the other person, there are only two possible outcomes

(b) A supermarket is running a promotion where shoppers get one collectable item for every \$50 they spend at the supermarket in one purchase.

Using a very large amount of electronic sales data, the supermarket has produced the following graph:



(i) Use this data to complete the table below, which shows a probability distribution model for the random variable N, the number of collectable items gained in one purchase.

n	0	1	2	3	4	5	6
P($N = n$)	0.11	0.14	0.23	0.16	0.15	0.15	0.06

ASSESSOR'S USE ONLY

(ii) Using the model formed in (b)(i), calculate the mean number of collectable items gained by shoppers per purchase.

Give any assumption(s) that need to be made.

$E(X) = \mu$

$E(X) = (0 \times 0.11) + (1 \times 0.14) + (2 \times 0.23) + (3 \times 0.16)$
$+ (4 \times 0.15) + (5 \times 0.15) + (6 \times 0.06)$

$= 2.79$

$\mu = 2.79.$

(iii) The supermarket is considering changing the promotion so that shoppers get one collectable item for every \$25 they spend at the supermarket in one purchase.

Without performing additional calculations, discuss whether this will result in a doubling of the mean number of collectable items gained by shoppers per purchase.

By halving the amount needed to pay for groceries to receive a collectable item the mean number of collectables received would increase to double what it is now This is because shoppers will receive twice the number of collectables as they did previously, whilst spending the same amount of money

A4

QUESTION THREE

ASSESSOR'S USE ONLY

(a) A small supermarket located in the city centre is open 24 hours per day.

(i) Between 10 pm and 6 am each day, the mean number of shoppers who arrive at the supermarket per 5 minutes is 1.3.

Using a suitable probability distribution model, calculate the probability that more than two shoppers arrive at the supermarket during a 5-minute period between 10 pm and 6 am.

Using poisson distribution:
$P(X \leq 2) = 0.857$
$P(X > 2) = 0.143$ $(1 - 0.857)$

(ii) Between 6 am and 10 pm each day, using footage from its security cameras, the supermarket found that in 94% of 5-minute periods, there was at least one shopper arriving at the supermarket.

Discuss how the mean number of shoppers who arrive at the supermarket per 5 minutes between 6 am and 10 pm compares to the mean number of shoppers who arrive at the supermarket per 5 minutes between 10 pm and 6 am.

Between 6am - 10pm in 94% of 5 min periods at least 1 customer arrived. As it is not specified how many customers arrive in each of these periods, the mean number of customers during 6am - 10pm could be much greater than the 1.3 customers arriving per 5 min period between 10pm - 6am.

(iii) Discuss ONE other factor (in addition to the time of day) the supermarket should consider when modelling the number of shoppers who arrive at their supermarket per 5 minutes.

Between the hours of 9-5, many shoppers will be working in their own jobs. This could reduce the amount of customers able to shop at their supermarket during these hours.

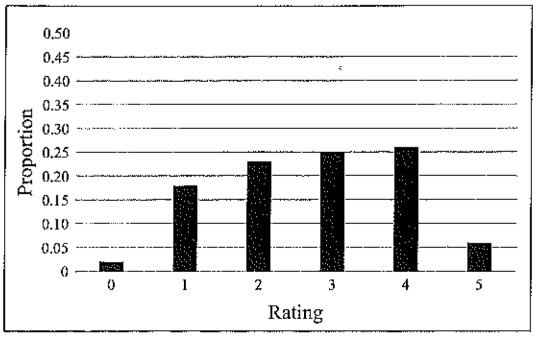
ASSESSOR'S USE ONLY

(b) A large supermarket has re-designed its checkout area, including installing more self-service checkouts and changing the layout of checkouts. Before and after the re-design, the supermarket conducted two different surveys of shoppers.

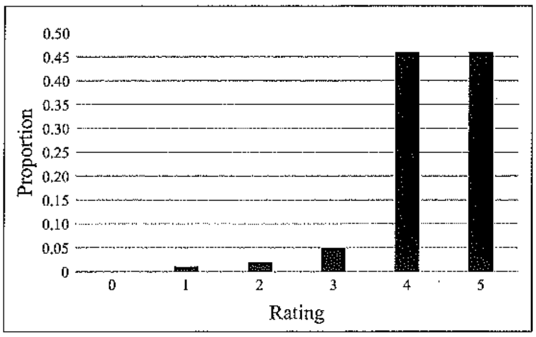
In each survey (before, after), shoppers were asked to rate their experience with checking out of the supermarket as a score on a scale of 0 (very unhappy) to 5 (very satisfied).

The results for each survey are shown below:

Before



After



(i) Identify which set of data has less variation in rating scores.

Support your answer with statistical reasoning.

The after set of data has less variation in rating scores. This is because there are more customers rating 4 and 5 than any other rating, and so the data is skewed to the left, whilst in the before data, the data is much more evenly spread.

(ii) Discuss if it would be appropriate to use a Poisson distribution to model the ratings for the 'before' survey.

Support your answer with statistical reasoning.

For a poisson to fit the before survey data, the following requirements would need to fit: each rating is independent from one another, they must be random events, the events must be proportional to the time and the events happen simultaneously. Because the before survey does not meet these requirements, it is not appropriate to use poisson.

A3

Annotated Exemplar Template

Achieved exemplar 2016

Subject:	Mathematics	Standard:	91586	Total score:	11

Q	Grade score	Annotation
1	A4	1(a) P(X<2) found but P(X>6 not completed nor probabilities added, so ‘u’ not ‘r’ 1(b)(ii) Assumption needs to be about the amounts of time shoppers spend being independent, so ‘u’ not ‘r’ 1(b)(iii) Haven’t explained why S>D> increases. (Could be done via explaining how graph changes or by calculating new value)
2	A4	2(a)(ii) Only one condition in context. Two are needed for ‘r’ 2(b)(ii) No assumption, so ‘u’ not ‘r’
3	A3	3(a)(ii) No calculations done to back up the comparison 3(a)(iii) Assumptions about the time of day are not acceptable 3(b) In (i) variation for ‘Before’ is not sufficiently explained, so ‘u’. (ii) is not E level, as no explanation given as to why this does not meet Poisson requirements