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Level 3 Mathematics and Statistics (Statistics), 2016

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 21

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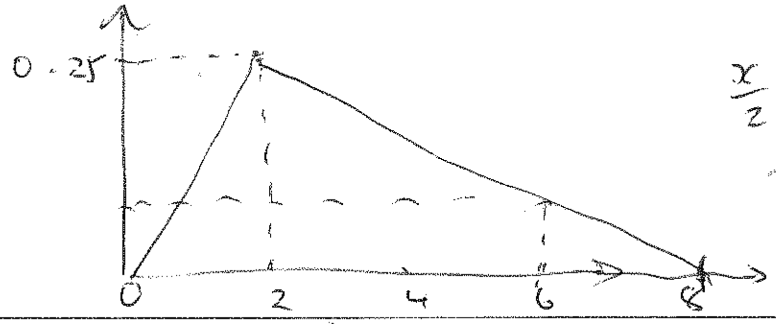
QUESTION ONE

(a) The time it takes a shopper to find a carpark at the supermarket can be modelled by a random variable that takes on values between 0 minutes and 8 minutes. The most likely time it takes a shopper to find a carpark is 2 minutes.

Using an appropriate model, calculate the probability that it will take less than two minutes OR more than six minutes for a shopper to find a carpark.



$\frac{x}{2} = \frac{0.25}{6}$

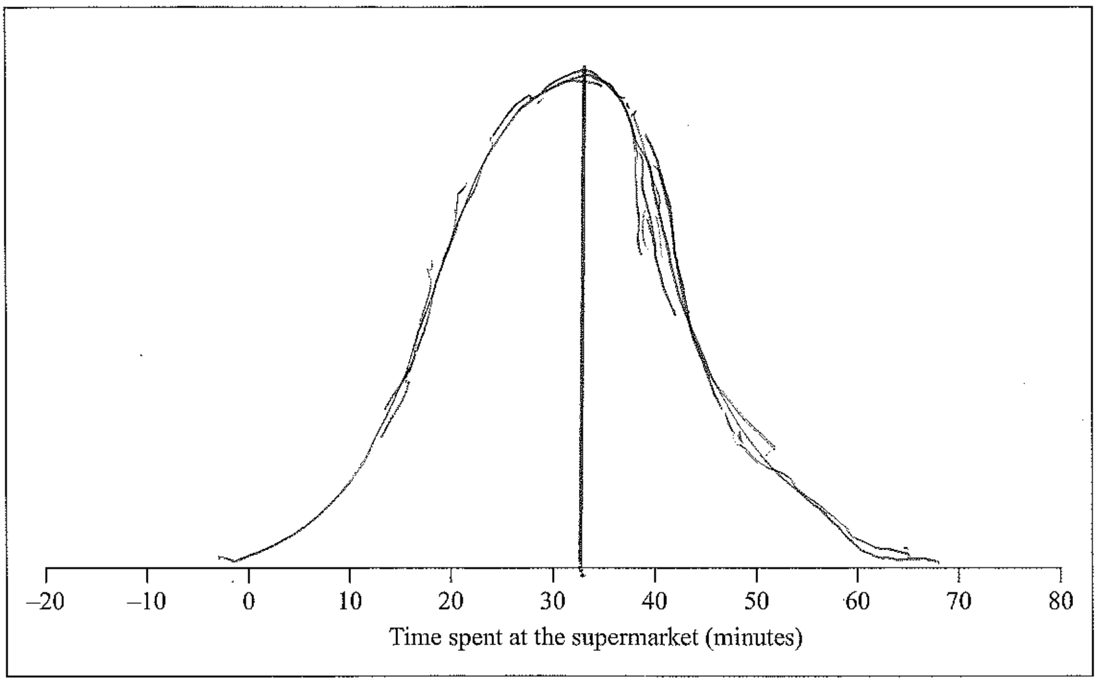
$x = \frac{1}{12}$

$P(X<2) + P(X>6) = (\frac{1}{2} \times 2 \times 0.25) + (\frac{1}{2} \times 2 \times \frac{1}{12})$

$= \frac{1}{3}$

(b) A supermarket has modelled the time shoppers spend at the supermarket using a normal distribution with a mean of 32.5 minutes and a standard deviation of 10.8 minutes.

(i) Sketch this probability distribution model on the axis below.



(ii) Using this model, calculate the probability that two different randomly selected shoppers both spend more than 40 minutes at the supermarket.

Give any assumption(s) that need to be made.

$P(X>40) = 0.2437$

$P(\text{both shoppers}) = (0.2437)^2$

$= 0.0594$

Assuming that the times taken for both shoppers are independent.

(iii) Following an observational study of shoppers, the supermarket has changed its model for the time shoppers spend at the supermarket. For this new model, the supermarket has kept the mean the same as the old model, but has adjusted the standard deviation. Using this new model, the percentage of shoppers who take longer than 40 minutes at the supermarket is estimated to be 31.1%.

Discuss how the standard deviation of the new model for the time shoppers spend at the supermarket compares with the standard deviation of the old model for the time shoppers spend at the supermarket.

You may wish to refer to your answers in parts (i) and (ii) to support your explanation.

$P(X > 40) = 0.311$

$\frac{X-\mu}{\cancel{\sigma}} = 2$

$\frac{40-32.5}{\sigma} = 0.4930$

$\sigma = 15.2$

Standard deviation of new model is 15.2 which is larger than the standard deviation of the old model 10.8.

(iv) Discuss ONE potential limitation with using a normal distribution to model the time spent at this supermarket.

A normal distribution has no upper and lower bounds, which is unrealistic as someone cannot spend a negative amount of time at the supermarket. There is therefore a lower bound and an upper bound as a shopper would have to leave after closing

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E8

QUESTION TWO

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(a) A supermarket has eight employees who are "on call" to help out during busy periods. Based on the supermarket's records, the probability of one of these employees being unavailable when called is estimated to be 0.14.

The supermarket needs to call all eight employees during one particularly busy period.

(i) Using an appropriate model, calculate the probability that fewer than three of these employees will be unavailable when called.

$P(X<3) = P(X \leq 2)$

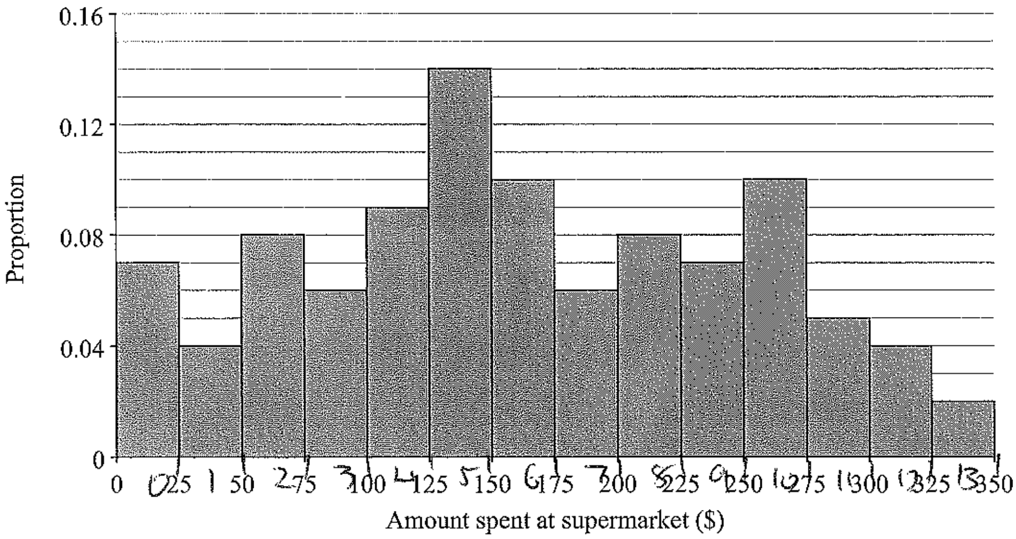
$= 0.9109$

(ii) Justify the use of the probability distribution for your answer in (i).

Using a binomial distribution because there is a fixed number of trials of 8 employees and each employee being unavailable is independent of each other. There are only two possible outcomes - they are available or they are unavailable, and the probability of them being unavailable at 0.14 remains the same.

(b) A supermarket is running a promotion where shoppers get one collectable item for every \$50 they spend at the supermarket in one purchase.

Using a very large amount of electronic sales data, the supermarket has produced the following graph:



(i) Use this data to complete the table below, which shows a probability distribution model for the random variable N, the number of collectable items gained in one purchase.

n	0	1	2	3	4	5	6
$\mathbf{P}(N = n)$	0·11	0.14	0·23	0.16	0.15	0.15	0.06

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(ii) Using the model formed in (b)(i), calculate the mean number of collectable items gained by shoppers per purchase.

Give any assumption(s) that need to be made.

$E_{Avg}(n) = 0 \times 0 \cdot 11 + 1 \times 0 \cdot 14 + 2 \times 0 \cdot 23 + 3 \times 0 \cdot 16 + 4 \times 0 \cdot 15 + 5 \times 0 \cdot 15 + 6 \times 0 \cdot 06$

$E_{avg}(n) = 2 \cdot 79$

Must be integer so 2 items will be gained on average per purchase.
Assumed that shoppers' purchase amounts are independent of each other.

(iii) The supermarket is considering changing the promotion so that shoppers get one collectable item for every \$25 they spend at the supermarket in one purchase.

Without performing additional calculations, discuss whether this will result in a doubling of the mean number of collectable items gained by shoppers per purchase.

For shoppers purchasing between (\$50–\$74.99) worth of goods, plus multiples of \$50 on top of that, they will receive doubled collectable items. For shoppers in the range \$25 – \$49.99 + multiples of \$50, they will receive one extra collectable. Assuming the scheme will not affect purchase amounts, the mean will be offset by the \$25 – \$49.99 + multiples of \$50 purchases having additional collectables given out

E7

QUESTION THREE

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(a) A small supermarket located in the city centre is open 24 hours per day.

(i) Between 10 pm and 6 am each day, the mean number of shoppers who arrive at the supermarket per 5 minutes is 1.3.

Using a suitable probability distribution model, calculate the probability that more than two shoppers arrive at the supermarket during a 5-minute period between 10 pm and 6 am.

$P(X>2) = 1 - P(X \leq 2)$

$= 1 - 0.8571248$

$= 0.1429.$

(ii) Between 6 am and 10 pm each day, using footage from its security cameras, the supermarket found that in 94% of 5-minute periods, there was at least one shopper arriving at the supermarket.

Discuss how the mean number of shoppers who arrive at the supermarket per 5 minutes between 6 am and 10 pm compares to the mean number of shoppers who arrive at the supermarket per 5 minutes between 10 pm and 6 am.

$P(X \geq 1) = 0.94$

$P(X=0) = 0.06$

$\frac{\lambda^0 e^{-\lambda}}{0} = 0.06$

$e^{-\lambda} = 0.06$

$\lambda = 2.8$

The mean number of shoppers who arrive per 5 mins between 6am + 10pm is 2.8, more than double the mean of the mean number of shoppers between 10pm + 6am

(iii) Discuss ONE other factor (in addition to the time of day) the supermarket should consider when modelling the number of shoppers who arrive at their supermarket per 5 minutes.

The supermarket should consider what day of the week it is, as some shoppers may purchase their groceries at the beginning of the week or during weekends

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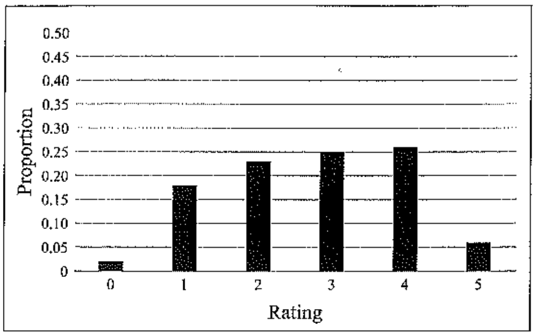
(b) A large supermarket has re-designed its checkout area, including installing more self-service checkouts and changing the layout of checkouts. Before and after the re-design, the supermarket conducted two different surveys of shoppers.

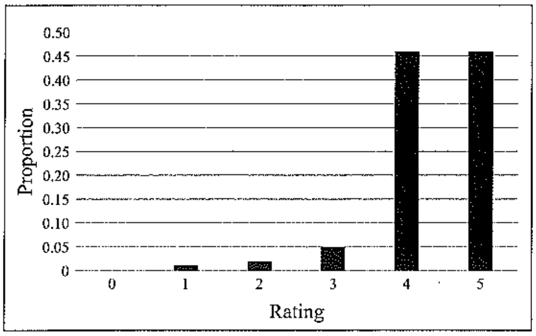
In each survey (before, after), shoppers were asked to rate their experience with checking out of the supermarket as a score on a scale of 0 (very unhappy) to 5 (very satisfied).

The results for each survey are shown below:

Before



After



(i) Identify which set of data has less variation in rating scores.

Support your answer with statistical reasoning.

There is less variation in the data after the re-design as the majority of the probability of customers rated the experience a 4 or 5 out of 5. Whereas in the before the re-design the customers ratings were varied between mainly 1+4 (more variation)

(ii) Discuss if it would be appropriate to use a Poisson distribution to model the ratings for the 'before' survey.

Support your answer with statistical reasoning.

It would ~~support~~ be appropriate to use a poisson distribution as the mean is at a fixed rate, with the ratings gained being a fixed value before the installation of self-service checkouts. Also the people who were asked in the survey are independent of each other meaning the results weren't affected by people agreeing with each other. The survey also could not occur simultaneously as only one person can be asked at any one time, and the probability of what rating that will be rated from 0 to 5 of those asked is also random. Therefore poisson distribution is an appropriate model for the rating survey before.

M6

Excellence exemplar 2016

Subject:	Mathematics	Standard:	91586	Total score:	21

Q	Grade score	Annotation
1	E8	1(b)(ii) Correct calculations and a good assumption 1(b)(iv) A good limitation given and context given for both upper and lower
2	E7	2(a)(i) Correct and all four conditions given correctly in context (only 2 needed) 2(b)(ii) Assumptions not OK, so 'u' not 'r' 2(b)(iii) A good explanation but didn't conclude whether mean doubles or not, so E7
3	M6	3(b) (ii) is not OK since says Poisson is appropriate. But (i) correctly says 'After' has less variation and gives a good description of variation for both 'Before' and 'After', so 'r' overall.