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Level 3 Mathematics and Statistics (Statistics), 2016

91586 Apply probability distributions in solving problems

2.00 p.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 16

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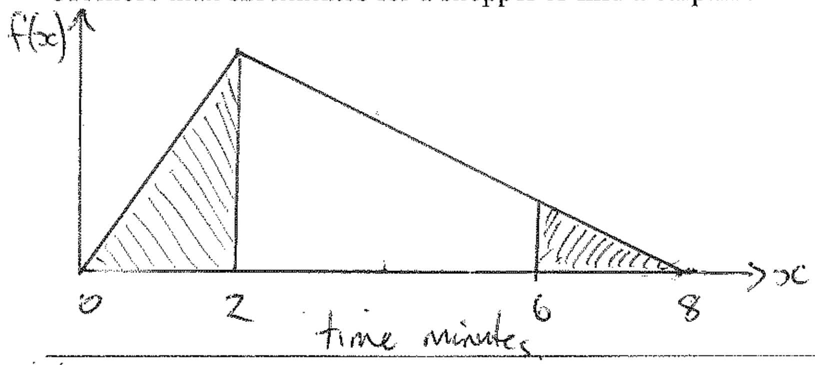
QUESTION ONE

(a) The time it takes a shopper to find a carpark at the supermarket can be modelled by a random variable that takes on values between 0 minutes and 8 minutes. The most likely time it takes a shopper to find a carpark is 2 minutes.

Using an appropriate model, calculate the probability that it will take less than two minutes OR more than six minutes for a shopper to find a carpark.



$\frac{1}{2}\times 8\times f(2) = 1$

$f(2) = \frac{1}{4}$

$\frac{f(2)}{6} = \frac{f(6)}{2}$

$f(6) = \frac{1}{12}$

$P(x<2) = \frac{1}{2}\times 2\times \frac{1}{4}$

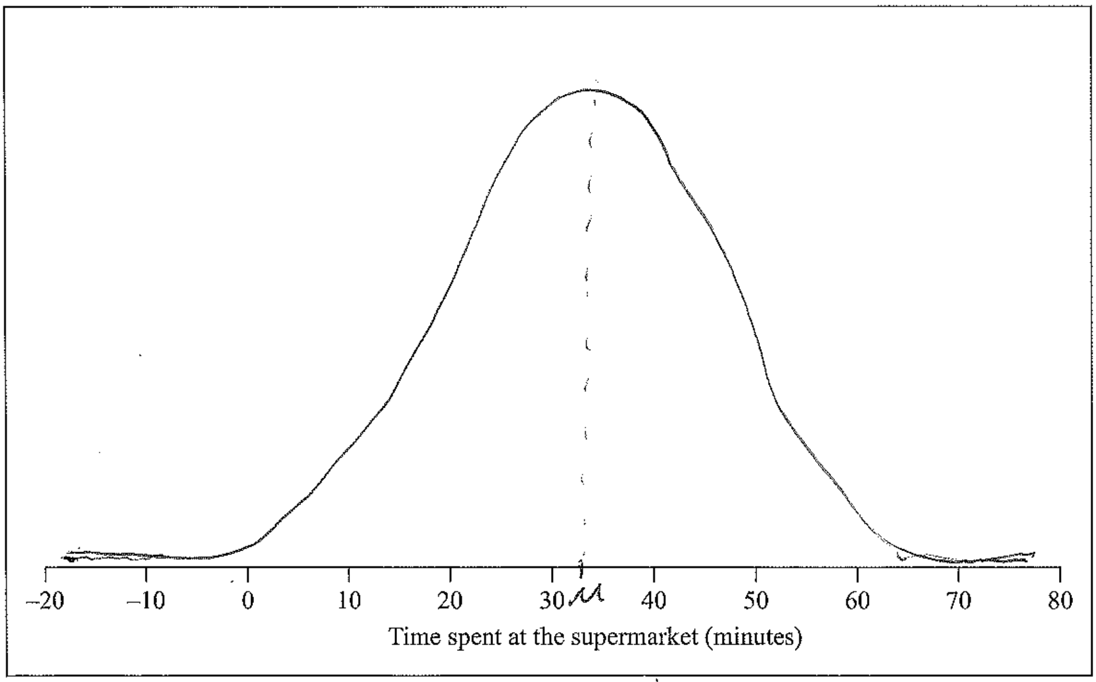
$= \frac{1}{4}$

$P(x>6) = \frac{1}{2}\times 2\times \frac{1}{12}$

$= \frac{1}{12}$

(b) A supermarket has modelled the time shoppers spend at the supermarket using a normal distribution with a mean of 32.5 minutes and a standard deviation of 10.8 minutes.

(i) Sketch this probability distribution model on the axis below.



(ii) Using this model, calculate the probability that two different randomly selected shoppers both spend more than 40 minutes at the supermarket.

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Give any assumption(s) that need to be made.

One shopper = $P(X>40)$ = so $(p(X>40))^2$

probability = $(0.2437017 6)^2$

= 0.059

Assumptions made are that the two different randomly selected shoppers are independent of each other.

(iii) Following an observational study of shoppers, the supermarket has changed its model for the time shoppers spend at the supermarket. For this new model, the supermarket has kept the mean the same as the old model, but has adjusted the standard deviation. Using this new model, the percentage of shoppers who take longer than 40 minutes at the supermarket is estimated to be 31.1%.

Discuss how the standard deviation of the new model for the time shoppers spend at the supermarket compares with the standard deviation of the old model for the time shoppers spend at the supermarket.

You may wish to refer to your answers in parts (i) and (ii) to support your explanation.

$\mu = 32.5$ $\quad p(x>40) = 31.1\% = 0.311$ $(\sigma = 10.8) =$ old σ

inverse normal $\quad z = \frac{x-\mu}{\sigma}$ $\quad z = 0.4930178 1$

$0.4930178 1 = \frac{40-32.5}{\sigma}$

$0.49301781\sigma = 7.5$

$\sigma = 15.21$ The new standard deviation has a value of 15.21 which is greater than sd of old model.

This shows the data in the new model is more spread out

(iv) Discuss ONE potential limitation with using a normal distribution to model the time spent at this supermarket.

The data may not be centred around the mean value of 32.5, hence the model of a normal distribution may limit the time spent at this supermarket. More people may spend less time in the supermarket than 32.5 minutes but a handful of customers may stay for excess of 60 minutes which raises the mean.

M5

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QUESTION TWO

(a) A supermarket has eight employees who are "on call" to help out during busy periods. Based on the supermarket's records, the probability of one of these employees being unavailable when called is estimated to be 0.14.

The supermarket needs to call all eight employees during one particularly busy period.

(i) Using an appropriate model, calculate the probability that fewer than three of these employees will be unavailable when called.

$n = 8$ $\quad p = 0.14$ $\quad P(x < 3) = P(X \leq 2)$

$= 0.9109$

(ii) Justify the use of the probability distribution for your answer in (i).

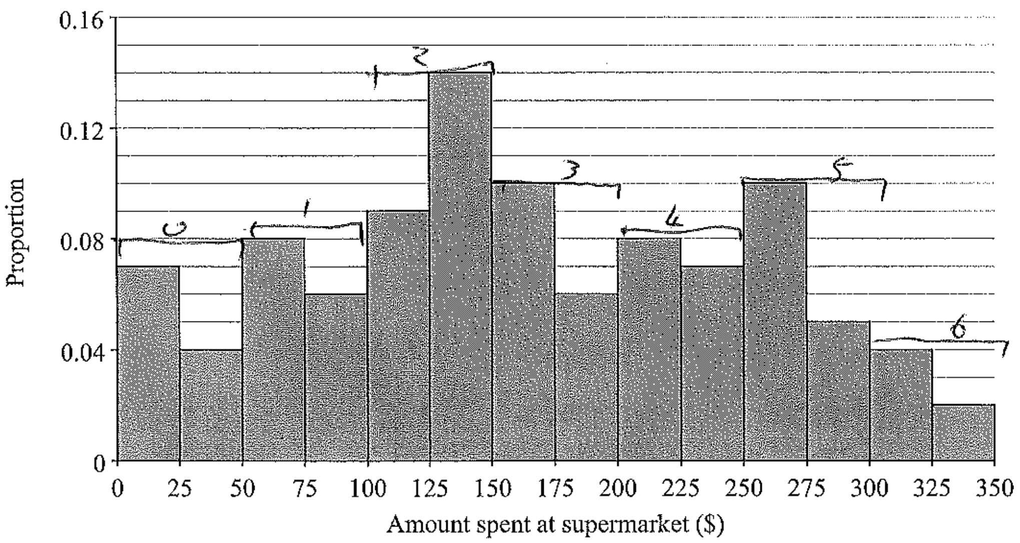
Binomial distribution

There is a fixed number of trials (8)

There are only two possible outcomes

(b) A supermarket is running a promotion where shoppers get one collectable item for every \$50 they spend at the supermarket in one purchase.

Using a very large amount of electronic sales data, the supermarket has produced the following graph:



(i) Use this data to complete the table below, which shows a probability distribution model for the random variable N, the number of collectable items gained in one purchase.

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n	0	1	2	3	4	5	6
P($N = n$)	0.11	0.14	0.23	0.16	0.15	0.15	0.06

(ii) Using the model formed in (b)(i), calculate the mean number of collectable items gained by shoppers per purchase.

Give any assumption(s) that need to be made.

$E(N) = 0.14 + 2(0.23) + 3(0.16) + 4(0.15) + 5(0.15) + 6(0.06)$

$= 0.14 + 0.46 + 0.48 + 0.6 + 0.75 + 0.36$

$= 2.79$

$\mu = 2.79.$

This assumes that the electronic sales data collected is representative of all sales, including non-electronic cash transactions.

(iii) The supermarket is considering changing the promotion so that shoppers get one collectable item for every \$25 they spend at the supermarket in one purchase.

Without performing additional calculations, discuss whether this will result in a doubling of the mean number of collectable items gained by shoppers per purchase.

Assuming the probability of the amount spent at the supermarket will not change in response to the new promotion, this change would be expected to result in a doubling of the mean number of collectables. For every dollar a customer spends, where before they earned $\frac{1}{50}$ of a collectable, they will now earn $\frac{1}{25}$ of a collectable, effectively doubling the 'amount' of collectables for every dollar spent.
When they reach the \$50 threshhold they will now have earned 2 collectables as opposed to 1.

M5

QUESTION THREE

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(a) A small supermarket located in the city centre is open 24 hours per day.

(i) Between 10 pm and 6 am each day, the mean number of shoppers who arrive at the supermarket per 5 minutes is 1.3.

Using a suitable probability distribution model, calculate the probability that more than two shoppers arrive at the supermarket during a 5-minute period between 10 pm and 6 am.

$\lambda = 1.3$

$P(X > 2) = 1 - P(X \leq 2)$

$= 1 - 0.857$

$= 0.143.$

(ii) Between 6 am and 10 pm each day, using footage from its security cameras, the supermarket found that in 94% of 5-minute periods, there was at least one shopper arriving at the supermarket.

Discuss how the mean number of shoppers who arrive at the supermarket per 5 minutes between 6 am and 10 pm compares to the mean number of shoppers who arrive at the supermarket per 5 minutes between 10 pm and 6 am.

$P(X \geq 1) = 0.94$

$P(X = 0) = 0.06$

$0.06 = e^{-\lambda}$

$\lambda = 2.81$

The mean number of shoppers between 6am and 10pm is more than double (2.81) the mean number of shoppers between 10pm and 6am (1.3) for every 5 minute interval.

(iii) Discuss ONE other factor (in addition to the time of day) the supermarket should consider when modelling the number of shoppers who arrive at their supermarket per 5 minutes.

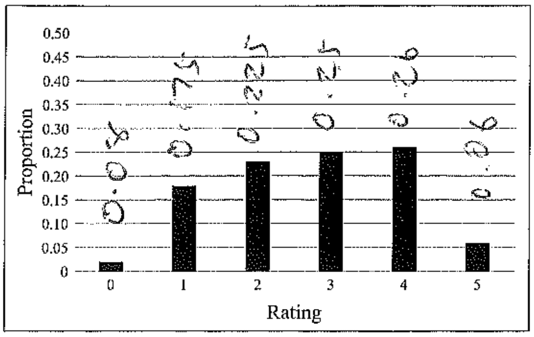
The maximum capacity of the store should be taken into account. It is only a small supermarket and so may not be able to hold many customers at once.

(b) A large supermarket has re-designed its checkout area, including installing more self-service checkouts and changing the layout of checkouts. Before and after the re-design, the supermarket conducted two different surveys of shoppers.

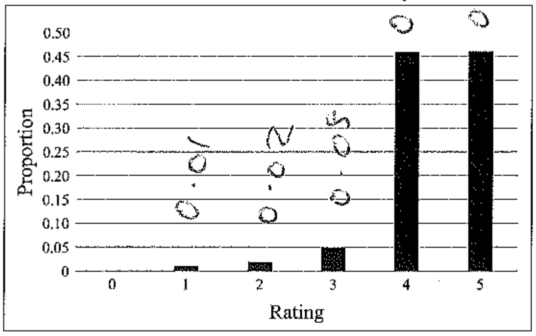
In each survey (before, after), shoppers were asked to rate their experience with checking out of the supermarket as a score on a scale of 0 (very unhappy) to 5 (very satisfied).

The results for each survey are shown below:

Before



After



(i) Identify which set of data has less variation in rating scores.

Support your answer with statistical reasoning.

$Var(x) = E(X^2) - [E(X)]^2$ $\quad$ $Var(X)_{after} = 19.22 - 18.836$

$Var(X)_{before} = 8.988 - 7.371$ $\quad$ $\cancel{= 0.384}$

$= 1.614$ $\quad$ The 'before' distribution has the least variance and hence the least variation in the scores.

(ii) Discuss if it would be appropriate to use a Poisson distribution to model the ratings for the 'before' survey.

Support your answer with statistical reasoning.

A poisson distribution can be used if an event happens independently and randomly over time at a constant rate and two instances of the event cannot occur simultaneously. A poisson distribution could not be used to model the ratings for the 'before' survey as the ratings are not a count of events occurring over a fixed interval of time.

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M6

Merit exemplar 2016

Subject:	Mathematics	Standard:	91586	Total score:	16

Q	Grade score	Annotation
1	M5	1(a) The two probabilities have not been added, so 'u' not 'r' 1(b)(ii) Assumption needs to be about the amounts of time the shoppers spend being independent so 'u' not 'r' 1(b)(iii), (iv) A good answer in (iii) but (iv) is not sufficient for E level so 'r'
2	M5	2(a) Part (i) is correct but no context is given with conditions 2(b)(ii) This interpretation of "electronic sales data was accepted as an assumption. Plus correct calculation gives 'r'
3	M6	3(a)(ii) New mean calculated correctly and compared to other mean, so 'r' 3(a)(iii) Not a valid factor 3(b) A good E level answer in (ii) but unfortunately in (i) there is an error in the variance calculation and wrong conclusion made of 'Before' so only 'r'