2

SUPERVISOR'S USE ONLY

91262

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NEW ZEALAND QUALIFICATIONS AUTHORITY
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QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2017

91262 Apply calculus methods in solving problems

2.00 p.m. Friday 24 November 2017
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 19

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QUESTION ONE

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(a) A function f is given by $f(x) = x^5 + 3x^2 - 7x + 2$.

Find the gradient of the graph of the function at the point where $x = 1$.

$f'(x) = 5 \times x^4 + 6 \times x^2 - 7$

$x = 1$ $\quad f'(1) = 5 \times 1^4 + 6 \times 1^2 - 7$

$= 5 + 6 - 7$

$= 3$

(b) Find the equation of the tangent to the graph of the function

$f(x) = 6 + 14x - 2x^3$

at the point (2,18) on the graph.

$f'(x) = 6x^2 + 14$

$18 = 6(2)^2 + 14 \rightarrow 38$

$y = mx + c$

$18 = 38(2) + c.$

$18 = 76 + c$ $\qquad f'(2) = 6(2)^2 + 14$

$c = 18 - 76$ $\qquad m = 38.$

$= -58$ $\qquad y = 38x - 58$

(c) The movement of an object is recorded from the time it passes a fixed point. After t seconds it has a speed v m s^{-1}, which can be modelled by the function

$v(t) = 0.5t^2 - 2t + 1$

Use calculus to find how long it takes to reach an acceleration of 2.8 m s^{-2}.

$v(t) = 0.5t^2 - 2t + 1$

$a(t) = t - 2$

$2.8 = t - 2$

$t = 4.8s.$

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(d) A tangent to the graph of the function $f(x) = 3x^2 - 4x$ has a gradient of 2, and passes through the point $(5,a)$, where a is a constant.

Find the value of a.

$f(x) = 3x^2 - 4x$ $\quad M = 2.$

$6x - 4 = 2$

$x = 1$

$f(x) = 3(1)^2 - 4(1)$

$= 3 - 4$

$= -1$

$a = -1$

(e) The function $f(x) = x^3 + ax^2 + bx + 2$ has turning points when $x = -1$ and $x = 3$.

Find the values of a and b.

$f'(x) = 3x^2 + 2ax + b$ $\quad x = -1$ $\quad x = 3$

$x + 1 = 0$ $\quad x - 3 = 0$

Roots $x = -1$ and $x = 3$ give $(x+1)(x-3) = 0$

$x^2 - 2x - 3 = 0$ $\quad) \times 3$

$3x^2 - 6x - 9 = 0$

$f'(x) = 3x^2 + 2ax + b$

$2a = -6$ $\quad b = -9$

$a = -3$

If $f(x) = x^3 - 3x^2 - 9x + 2.$

$f'(x) = 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3, \; x = -1$

E7

QUESTION TWO

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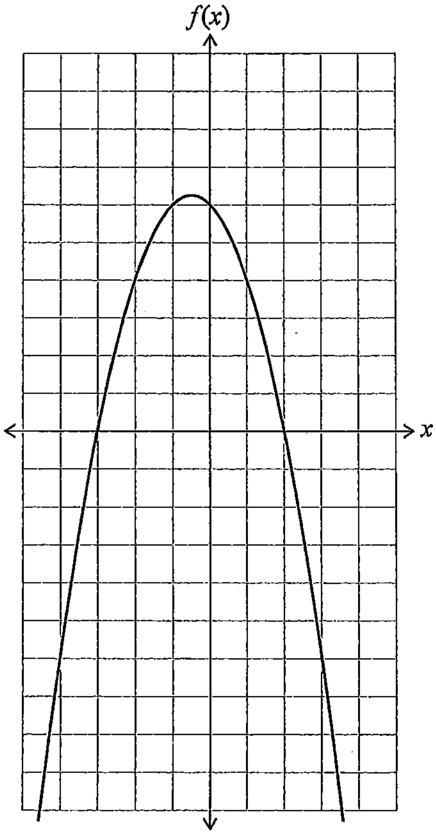
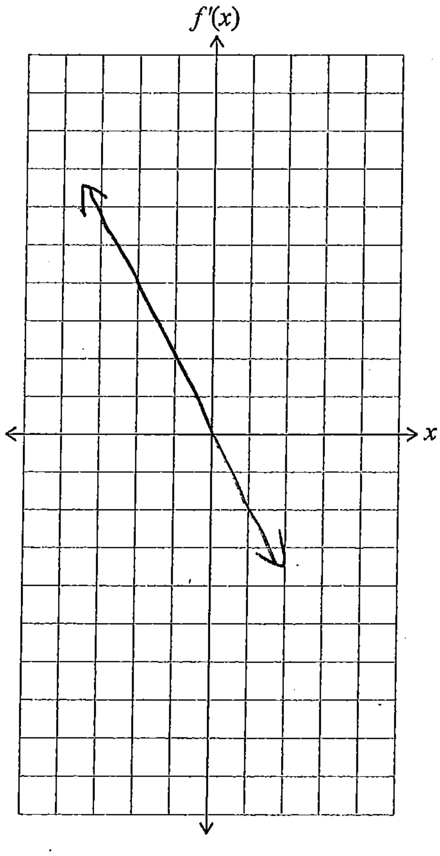
(a) The diagram below shows the graph of the function $y = f(x)$



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same scale.



If you need to redraw this graph, use the grid on page 11.

ASSESSOR'S USE ONLY

(b) The graph of a function $f(x) = 2x^3 + bx^2 - 2$ has a turning point when $x = -1$.

Find the value of b.

$f'(x) = 6x^2 + 2b$

$f'(-1) = 6(-1)^2 + 2b = 0$

$6 + 2b = 0$

$b = 3$

(c) Use calculus to show that the line $y = 15x - 12$
is a tangent to the graph of the function $f(x) = 4x^2 - x + 4$.

$f'(x) = 8x - 1$

$15 = 8x - 1$ $\quad y = 15(2) - 12$

$16 = 8x$ $\quad y = 30 - 12$ $\quad f'(2) = 8(2) - 1$

$x = 2$ $\quad y = 18$ $\quad M = 15$

$y - y_1 = m(x - x_1)$

$y - 18 = 15(x - 2)$

$y = 15x - 30 + 18$

$y = 15x - 12.$

(d) Use calculus to find the value of k if the line $y = 6x + k$ is a tangent to the graph of the function $f(x) = x^2 + 2x - 1$.

$f'(x) = 2x + 2$ $\quad y = (-1)^2 + 2(-1) - 1$

$0 = 2x + 2$ $\quad y = -2$

$\frac{-2}{2} = \frac{2x}{2}$ $\quad$ points $= (-1, -2)$

$x = -1$

There is more space for your answer on the following page.

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(e) Use calculus to prove that the graph of the function

$$y = x^3(3 - x)$$

has a local maximum when $x = \frac{9}{4}$.

Justify that the turning point is a local maximum.

$y = 3x^3 - x^4$

$y' = 9x^2 - 4x^3$

when $x = 9/4$

$y' = 9(9/4)^2 - 4(9/4)^3$

$y' = \frac{9^3}{4^2} - \frac{9^3}{4^2}$

$= 0$ Since $y' = 0$ ~~...~~ T.P at $x = 9/4$

M5

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The gradient graph of a function $f(x)$ is given by

$$f'(x) = 6x^2 - 2x + 4$$

The point (1,3) lies on the graph.

Find the equation of the function $f(x)$.

$$f(x) = \frac{6x^3}{3} - \frac{2x^2}{2} + \frac{4x}{1} + c$$

$$= 2x^3 - x^2 + 4x + c$$

$$f(1) = 2(1)^3 - (1)^2 + 4(1) + c$$

$$3 = 5 + c$$

$$c = -2$$

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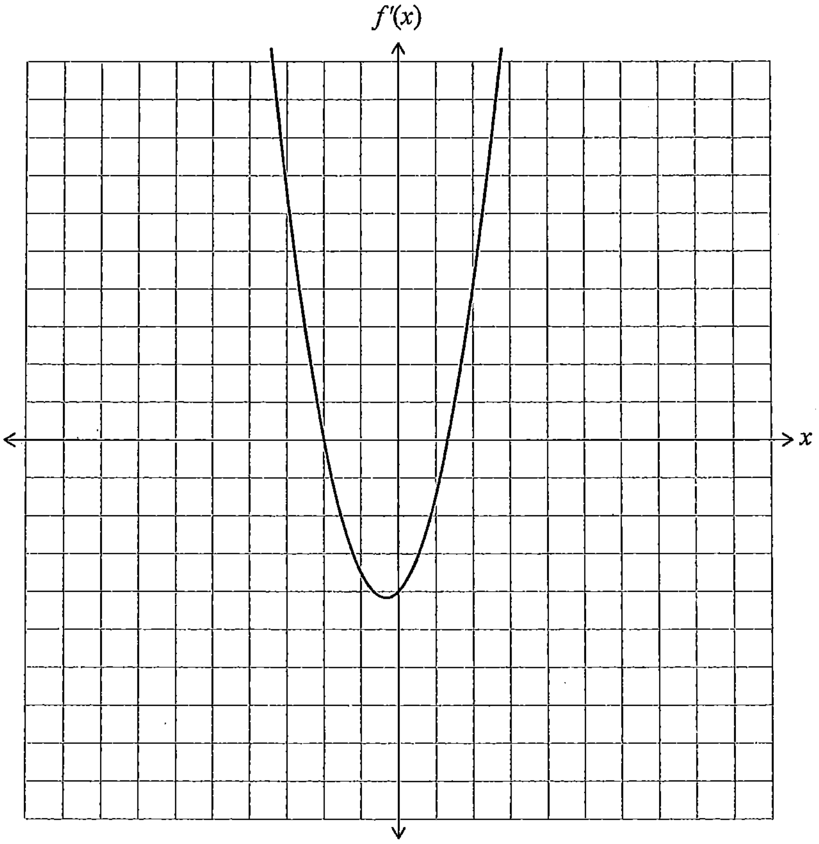
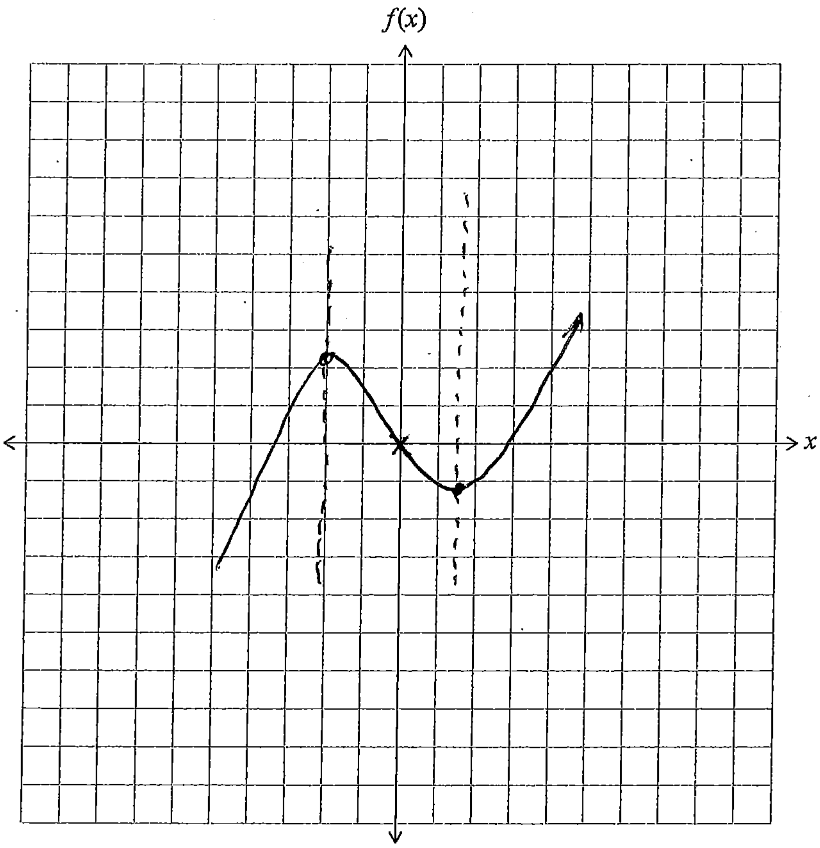
(b) The diagram below shows the graph of a gradient function $y = f'(x)$.



The point (0,0) is on the graph of the function $y = f(x)$.

On the axes below sketch the function $f(x)$.

Both sets of axes have the same scale.



If you need to redraw this graph, use the grid on page 11.

ASSESSOR'S USE ONLY

(c) An object can move in either direction on a straight track and has a constant acceleration of -4 cm s^{-2}.

A fixed point P is marked on the track.

When a recording of the object's motion begins, the object:

- is 12 cm from P
- is moving away from P, and
- has a velocity of 6 cm s^{-1}.

(i) Using calculus, find the speed of the object 5 seconds after its motion began being recorded.

$a = -4$

$d' = -4t + c$

$V = -4t + c$

$6 = -4 \times 0 + c$

$c = 6$

$V = -4(5) + 6$

$= -14$ so speed is 14 cm/s.

(ii) What is the maximum distance of the object from the point P?

Justify that this is the maximum distance.

$V = -4t + 6$

$0 = -4t + 6$

$-6 = -4t \qquad t = 1.5\text{ s}$

$v' = -2t^2 + 6t + c$

$S = -2t^2 + 6t + c$

$12 = -2(0)^2 + 6(0) + c$

$c = 12$

$S = -2t^2 + 6t + 12$

$t = 1.5\text{ s}$

$S = -2(1.5)^2 + 6(1.5) + 12$

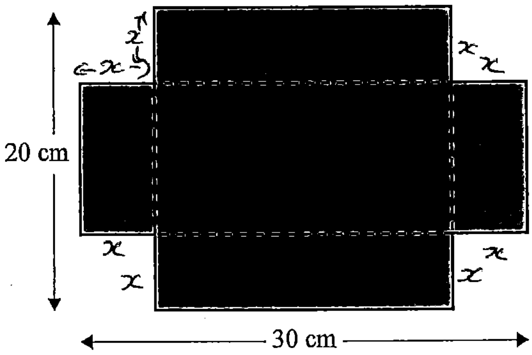
$S = 16.5\text{ cm}$

MAX AS $a = S'' = -4\text{ cm/s} < 0$ SO MAX DIST IS 16.5 cm WHEN $t = 1.5$ s

Question Three continues on the following page.

(d) Find the maximum volume of an open box (i.e. a box with a base and sides, but no lid) that can be made from a rectangular piece of cardboard measuring 20 cm by 30 cm, by removing the corner squares and folding along the dotted lines.

Justify that this is the maximum volume.



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Area $= 20 \times 30 = 600\text{cm}^2$.

length $= 20 - x - x = 20 - 2x$.

width $= 30 - x - x = 30 - 2x$

$$Vol = x(20-2x)(30-2x)$$
$$= (20x - 2x^2)(30 - 2x)$$
$$= 600x - 40x^2 - 60x^2 + 4x^3$$
$$= 4x^3 - 100x^2 + 600x.$$

$V' = 12x^2 - 200x + 600$ $\qquad x = 12.7, \quad x = 3.9$

$V'' = 24x - 200$

$x = 12.7$, $\quad 24(12.7) - 200 = 104.8 \quad (>0)$ ~~max~~ min.

$x = 3.9 \quad 24(3.9) - 200 = -106.4 \quad (<0)$ max

Max volume when $x = 3.9$cm.

E7.

Subject:	Mathematics	Standard:	91262	Total score:	19

Q	Grade score	Annotation
1	E7	1(a) Correct derivative, incorrect gradient but MEI (minor error ignored). 1(b) Incorrect derivative but consistent application of (2, 18) to find gradient and tangent equation. 1(c) Correct a(t) equation, correct substitution to find t at a = 2.8. 1(d) Correct derivative and solving to get x = 1, y = -1. Incorrect assumption that a = y at this coordinate. 1(e) Correct derivative. Relationship between roots of turning points equated to derivative demonstrated and then a comparison of equations to obtain correct values for a and b.
2	M5	2(a) Correct slope but wrong x intercept. 2(b) Incorrect derivative and f’(x) = 0 solved incorrectly so RAWW (right answer wrong working). 2(c) Correct f’(x) equated to slope of y = 15x -12 to find x and y and then the equation of line passing through these values, no evidence that (2,18) lies on f(x). 2(d) f’(x) = 0 was incorrect, consistent application of f(-1). No attempt to find k. 2(e) Correct derivative and showed that y’(9/4) = 0, then communicating this is a TP. No evidence to justify the TP was a maximum.
3	E7	3(a) Correct anti-differentiation of f’(x) and constant c calculated for (1,3). 3(b) Correct shape, (0,0) intersect with max and min points clearly shown to pass through correct x intercepts. 3(c)(i) Correct equation for v and speed at t=5s found. 3(c)(ii) Correct t= 1.5s found with correct s(t) equation and distance 16.5cm. Justification that a max using s’’ = -4 clearly stated. 3(d) Correct equation for volume and derivative, correct solutions for max/min situation, v’’ applied to determine a max at x = 3.9cm but the initial question to ‘Find the maximum volume …’ was not completed.