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SUPERVISOR'S USE ONLY

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QUALIFY FOR THE FUTURE WORLD
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Level 3 Calculus, 2017

91579 Apply integration methods in solving problems

9.30 a.m. Thursday 23 November 2017
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 11

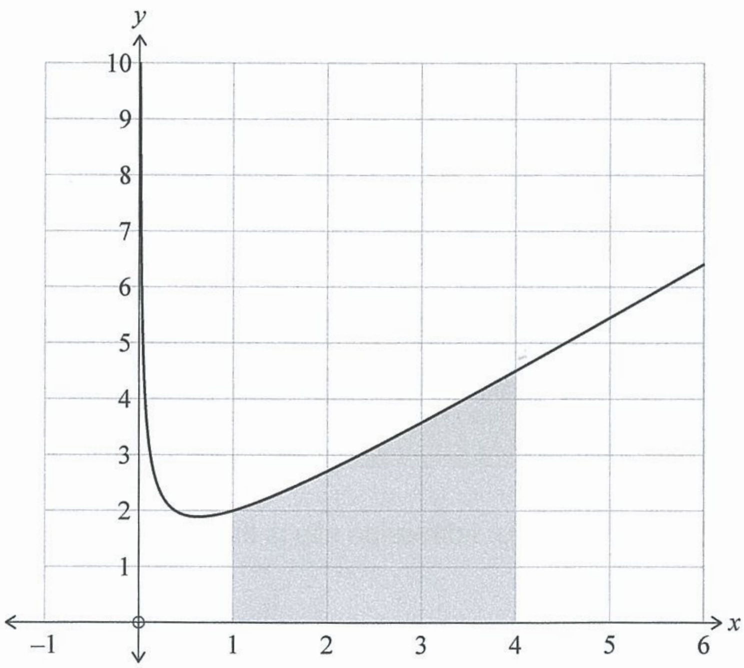
ASSESSOR'S USE ONLY

QUESTION ONE

(a) Find $\int 4\sec^2 2x \, dx$.

$2\tan 2x + C$

(b) Use integration to find the area enclosed between the curve $y = \frac{x^2 + \sqrt{x}}{x}$ and the lines $y = 0$, $x = 1$, and $x = 4$ (the area shaded in the diagram below).



You must use calculus and show the results of any integration needed to solve the problem.

$A = \int_1^4 \left(\frac{x^2 + \sqrt{x}}{x}\right) dx.$ → $x^{0.5} \times x^{-1} = x^{-0.5}$

$A = \int_1^4 \left(\frac{x^2}{x} + \frac{\sqrt{x}}{x}\right) dx$

$A = \int_1^4 x + x^{-0.5}) \, dx.$

$A = \left[0.5x^2 + 2x^{0.5}\right]_1^4$

$0.5[4^2 + 2\sqrt{4}] - 0.5[1^2 + 2\sqrt{1}]$

$A = 9.5$

ASSESSOR'S USE ONLY

(c) An object's acceleration is modelled by the function

$$a(t) = 1.2\sqrt{t}$$

where a is the acceleration of the object, in m s^{-2}
and t is the time in seconds since the start of the object's motion.

Since it is from the start assumes it starts at 0sec. $C = 0$.

If the object had a velocity of 7 m s^{-1} after 4 seconds, how far did it travel in the first 9 seconds of motion?

You must use calculus and show the results of any integration needed to solve the problem.

$V'(t) = a(t)$.
$V(t) = \int a(t)\,dt$.
$V(t) = \int 1.2 \times t^{0.5}\,dt$.
$V(t) = 0.8 \times t^{1.5} + C$
$7 = 0.8 \times 4^{1.5} + C$
$0.6 = C$
$V(t) = 0.8t^{1.5} + 0.6$.
$S(t) = \int V(t) \cdot dt$
$S = 0.32t^{2.5} + 0.6t + C$
$S = 0.32t^{2.5} + 0.6t$
$S(9) = 0.32 \times 9^{2.5} + 0.6 \times 9$
$S(9) = 83.16\text{m}$

(d) Find the value of k if $\int_0^k 3e^{2x}\,dx = 4$.

You must use calculus and show the results of any integration needed to solve the problem.

$[e^{3x}]_0^k = 4$
$[e^{3 \times k}] - [e^{3 \times 0}] = 4$
$e^{3k} - 1 = 4$
$e^{3k} = 5$
$\ln e^{3k} = \ln 5$.
$3k \ln e = \ln 5$
$3k = \ln 5$
$k = 0.536$

(e) The mean value of a function $y = f(x)$ from $x = a$ to $x = b$ is given by

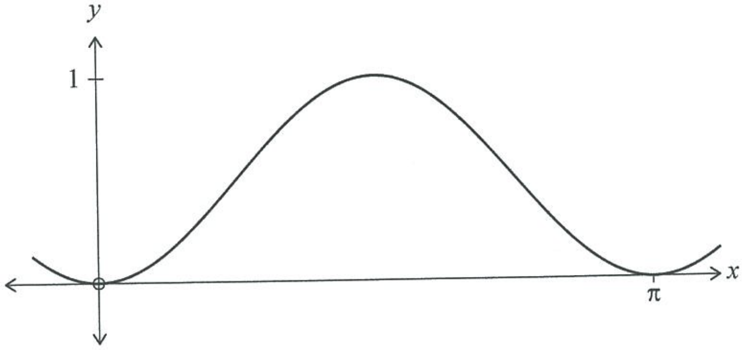
$$\text{Mean value} = \frac{\int_a^b f(x)\,dx}{b-a}$$

$f(x)$

Find the mean value of $y = \sin^2 x$ between $x = 0$ and $x = \pi$.

a b

Part of the graph of $y = \sin^2 x$ is shown below.



You must use calculus and show the results of any integration needed to solve the problem.

$$\text{Mean Value} = \frac{\int_0^\pi \sin^2 x\,dx}{\pi - 0}.$$

1st find definite integral

$(\sin x)^2$. $\int u^2\,dx$.

$u = \sin x$

$\frac{du}{dx} = -\cos x$

$\frac{du}{-\cos x} = dx$.

$x = \sin^{-1} u$.

$\int u^2 \frac{du}{-\cos x}$

$\int (\sin x)^2 \frac{du}{-\cos x}$.

$\int u^2 \frac{du}{-\cos(\sin^{-1} u)}$

ASSESSOR'S USE ONLY

M5

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int \frac{6}{2x-1}\,\mathrm{d}x$.

$3 \ln |2x-1| + c$

(b) Find $\int (2x-5)^4\,\mathrm{d}x$.

$u = 2x-5$

$\frac{du}{dx} = 2.$

$du = 2dx.$

$dx = 0.5du.$

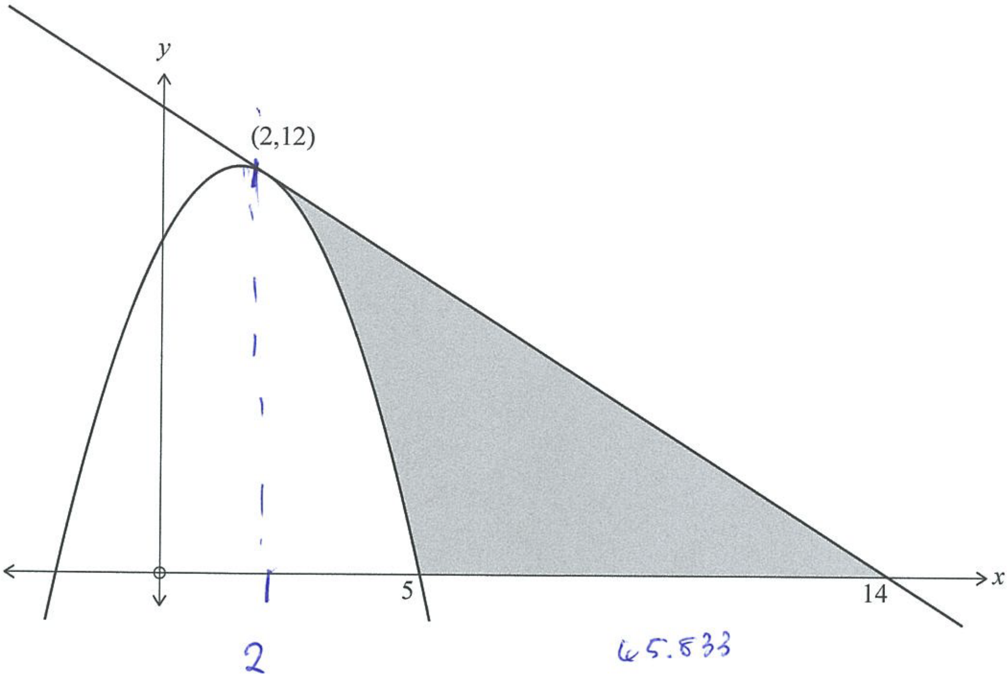
$\int u^4 dx.$

$\int 0.5u^4 du.$

$= 0.1u^5$

$= 0.1(2x-5)^5 + c$

(c) The diagram below shows the curve $y = -x^2 + 3x + 10$, and the line $y = -x + 14$, which is the tangent to the curve at the point (2, 12).



65.833

Calculate the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

① (2 − 14) − ② (2 − 5).

Area $= \int_2^{14} -x + 14 \, dx$

$\left[-2x^2 + 14x\right]_2^{14}$

$\left[-2 \times 14^2 + 14 \times 14\right] - \left[-2 \times 4 + 28\right]$

$= 216$

A of parabola $= \int_2^5 -x^2 + 3x + 10$

$\left[\frac{-x^3}{3} + \frac{3}{2}x^2 + 10x\right]_2^5$

$\left[\frac{-5^3}{3} + \frac{3 \times 5^2}{2} + 50\right] - \left[\frac{-2^3}{3} + \frac{3 \times 2^2}{2} + 20\right]$

$= 22.5$

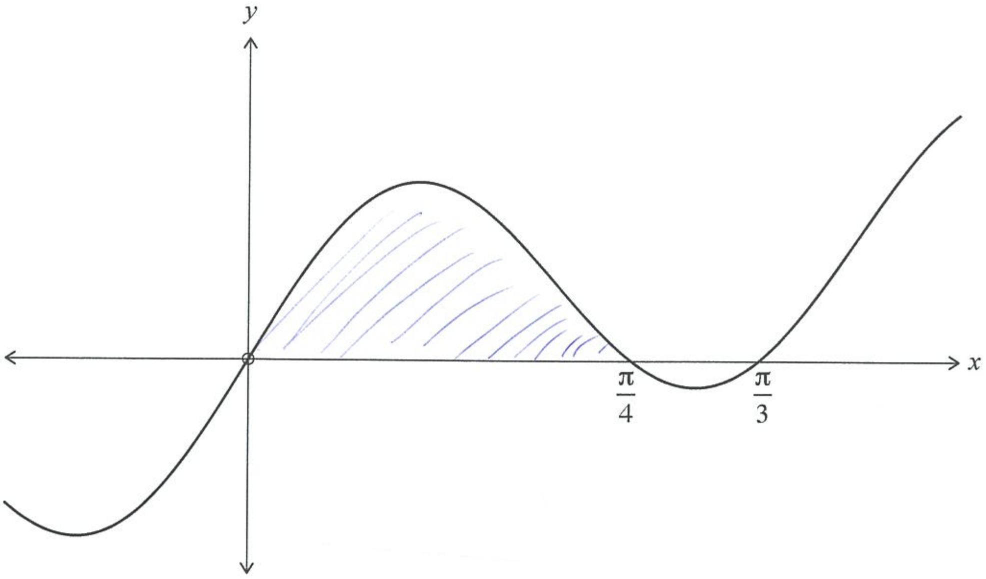
$216 - 22.5$

$= 193.5$

integrate outer
divide derivative of
inner.

(d) Part of the graph of $y = \sin 3x \cos 2x$ is shown below.

ASSESSOR'S USE ONLY



Find the area enclosed between the curve $y = \sin 3x \cos 2x$ and the lines $y = 0$, $x = 0$, and $x = \frac{\pi}{4}$.

You must use calculus and show the results of any integration needed to solve the problem.

$$A = \int_0^{\frac{\pi}{4}} \sin 3x(\cos 2x)\, dx$$

$$\frac{-\frac{1}{3}\cos 3x}{2\sin 2x} \cancel{dx}$$

$$= \left[\frac{\cos 3x}{6\sin 2x}\right]_0^{\frac{\pi}{4}}$$

$$\left[\frac{\cos \times 3\frac{\pi}{4}}{6\sin 2\times\frac{\pi}{4}}\right] - \left[\frac{\cos 0}{6\sin 0}\right]$$

$$= 0.117851$$

(e) The acceleration of an object is modelled by the function $a(t) = \frac{20 \ln t}{t}$.

where a is the acceleration of the object in m s^{-2}
and t is the time in seconds since the start of the object's motion.

The object was moving with a velocity of 12 m s^{-1} when $t = 4$.

Find the velocity of the object after 10 seconds.

You must use calculus and show the results of any integration needed to solve the problem.

$V(t) = \int \frac{20 \ln t}{t} \, dt$ $\quad u = t \quad t = u.$

$\frac{du}{dx} = 1$

$du = dx$

$\int \frac{\cancel{20 \ln t}}{u} \cancel{dt}$

$\int \cancel{\frac{20 \ln u}{u}} \, du.$

~~49~~

ASSESSOR'S USE ONLY

A3

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int\left(\frac{9}{x^4}+8e^{4x}\right)dx$.

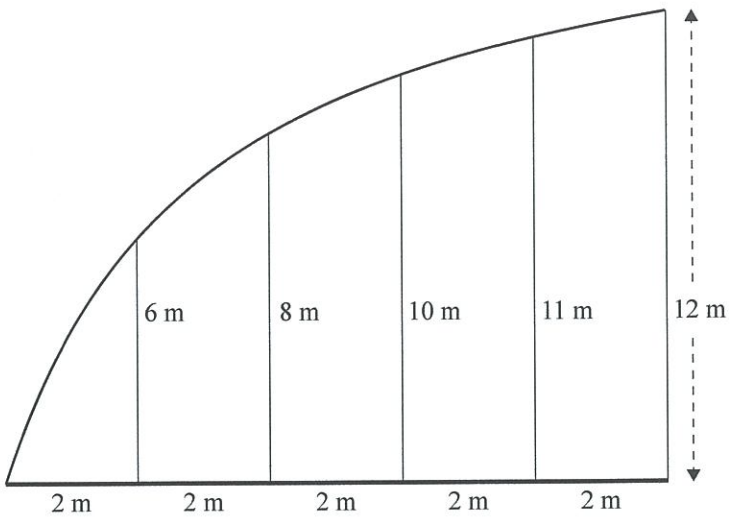
$\int \frac{9}{x^4} + 8e^{4x})dx.$

$9\ln|x^4|$

$\frac{9}{4x^3}\ln|x^4| + 2e^{4x}$

Question Three continues on the following page.

(b) Julia wants to find an approximation of the area of a paved courtyard that she wishes to construct on her property. She takes some measurements and these are shown on the diagram below.



x	0	2	4	6	8
y	0	6	8	10	11

$\frac{10}{12}$

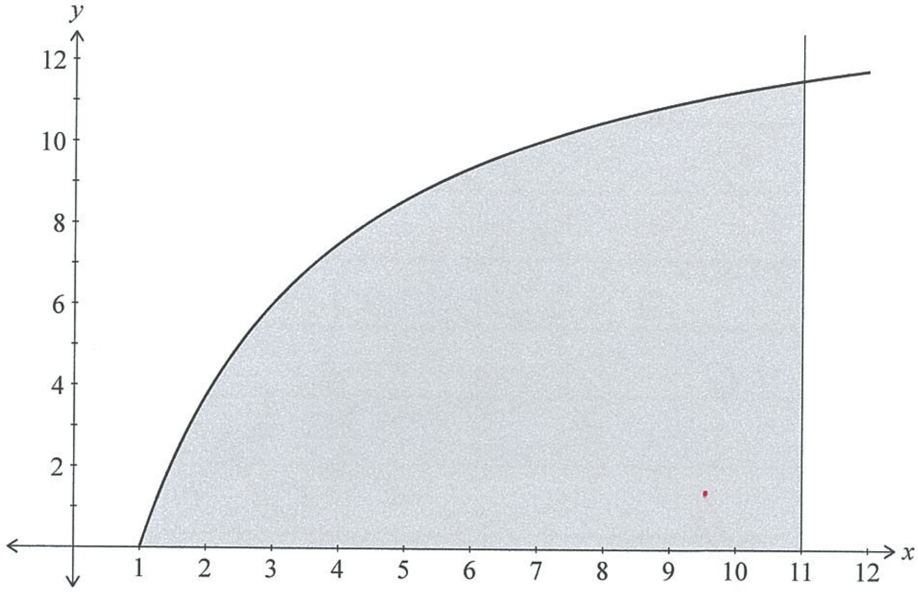
$h=2$

Using these measurements, and the Trapezium rule, find an approximation of the area of paved courtyard.

$$\int_0^{10} f(x)\,dx = \frac{1}{2} \times 2\left[0+12+2(6+8+10+11)\right]$$

$$= \underline{82\,m^2}$$

(c) Julia's friend Sarah believes that the equation of the curved border of the paved courtyard can be modelled by the function $y = \dfrac{15x-15}{x+2}$.



Use integration to find the area of the courtyard, shown in the diagram above.

You must use calculus and show the results of any integration needed to solve the problem.

$\int_1^{12} \left(\frac{15x-15}{x+2}\right) \cdot dx$

$\int_1^{12} \frac{15(u-2)-15}{u} \cdot du$

$\frac{15u-30-15}{u} \cdot du$

$15 - \frac{45}{u} \cdot du$

$\left[15u - 45\ln|u|\right]_1^{12}$

$\left[15\times 12 - 45\times\ln|12|\right] - \left[15 - 45\times\ln|1|\right]$

$= 53.18$

$u = x+2$

$x = u-2$

$\frac{du}{dx} = 1$

$du = dx$

(d) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{\sqrt{x}}$, given that when $x = 4$, then $y = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

$\frac{dy}{dx} = \frac{y}{\sqrt{x}}$

$\frac{dy}{dx} = y\,x^{-0.5}$

$\cancel{\frac{dy}{dx} = \frac{dy}{dx}}$

$\frac{dy}{dx}\cancel{y} = x^{-0.5}$

$\frac{1}{y} \times \frac{dy}{dx} = x^{-0.5}$

$y^2 = 2x^{0.5} + C$

$1^2 = 2 \times 4^{0.5} + C$

$C = -3$

$y = \sqrt{2\sqrt{x} - 3}$.

(e) y and t satisfy the differential equation $\frac{dy}{dt} = k\cos 0.5t \times e^{\sin 0.5t}$, $0 \le t \le 5$.

ASSESSOR'S USE ONLY

Given that when $t = 0$, $y = 8$, and that when $t = 2$, $y = 12$, find the value of y when $t = 5$.

You must use calculus and show the results of any integration needed to solve the problem.

$y=8$, $t=0$.

$y=8$, must be a stationary point.

$K=8$?

$$y = \int k\cos 0.5t \times e^{\sin 0.5t} \, dt.$$

$$y = \frac{\frac{k}{0.5}\sin 0.5t}{\frac{0.5}{\cos t}\, e^{\sin 0.5t}} + C.$$

$$\tan 0.5t \times \tfrac{1}{2}k \times \frac{1}{e^{\sin 0.5t}}.$$

$$y = \tan 0.5t \times \frac{8}{e^{\sin 0.5t}} + c$$

$$12 = \tan 0.5 \times 2 \times \frac{8}{e^{\sin 0.5 \times 2}} + c$$

$$12 = 1.0926 \times 2.476555$$

$$12 = 2.7058869 + c$$

$$c = 9.29411$$

$$y = \tan 0.5t \times \frac{8}{e^{\sin 0.5t}} + 9.29411$$

A3

Subject:	Integration	Standard:	AS91579	Total score:	11

Q	Grade score	Annotation
1	M5	This question provides evidence for M5 because the candidate has gained 1**r** grades in part (c) by: In part (c) - Correctly integrating the acceleration expression to find the velocity expression and correctly using the variables given to find the constant c of 0.6 - Correctly integrating the velocity expression to find the distance expression. - Substituting in t = 9 to find d = 83.16m
2	A3	This question provides evidence for A3 because the candidate has gained 2 **u** grades in part (a) and part(b) by: In part (a) - Correctly integrating the expression In part (b) - Correctly integrating the expression
3	A3	This question provides evidence for A3 because the candidate has gained 2 **u** grades in part (b) and part(c) by: In part (b) - Correctly using the Trapezium rule to find the approximate area is 82m². In part (c) - They get a “u” grade because the expression is correctly rearranged into a form that can be integrated and this expression is then correctly integrated. They do not get an “r” grade because the limits of integration used are incorrect and thus the area calculated is incorrect.