91579M

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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2017

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

9.30 i te ata Rāpare 23 Whiringa-ā-rangi 2017
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō mō ō tuhinga, whakamahia ngā whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i ngā tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

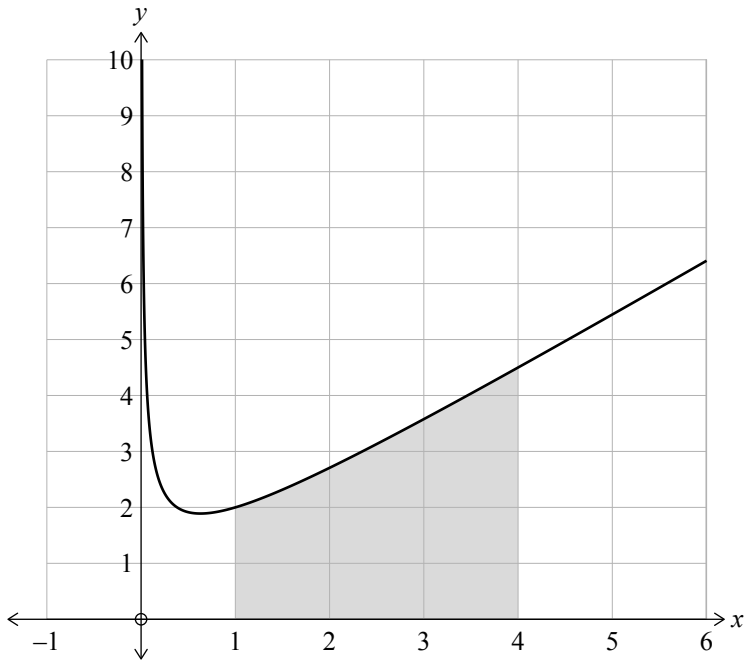
MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int 4\sec^2 2x\,\mathrm{d}x$.

(b) Whakamahia te tikanga pāwhaitua hei tātai i te horahanga e rohea ana e te ānau $y = \dfrac{x^2 + \sqrt{x}}{x}$ me ngā rārangi $y = 0$, $x = 1$ me $x = 4$ (ki te wāhi kauruku o te hoahoa i raro).



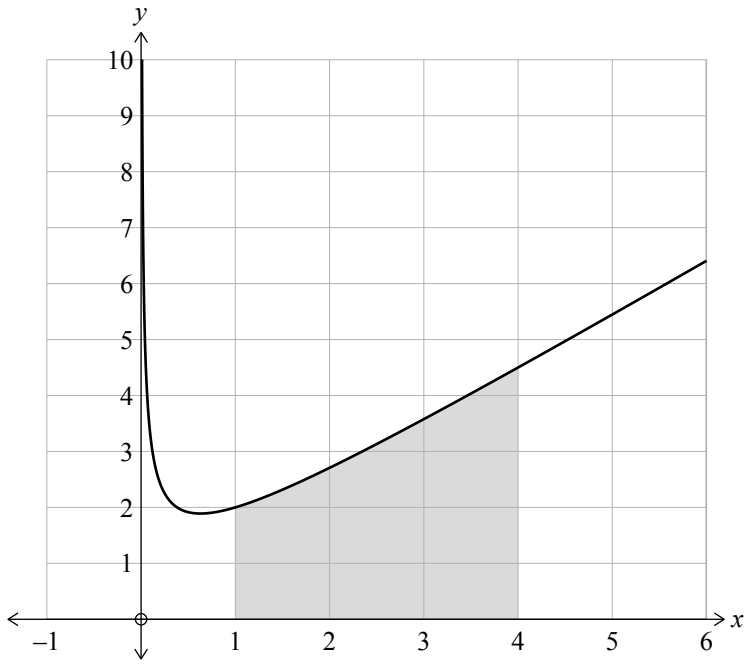
Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Find $\int 4\sec^2 2x \, dx$.

(b) Use integration to find the area enclosed between the curve $y = \dfrac{x^2 + \sqrt{x}}{x}$ and the lines $y = 0$, $x = 1$, and $x = 4$ (the area shaded in the diagram below).



You must use calculus and show the results of any integration needed to solve the problem.

MĀ TE KAIMĀKA ANAKE

(c) Ka whakatauiratia te whakaterenga o tētahi ahanoa mā te pānga

$$a(t) = 1.2\sqrt{t}$$

ina ko a te whakaterenga o te ahanoa, i te m s^{-2}
ko t te wā ā-hēkona mai i te tīmatanga o te neke o te ahanoa.

Mēnā he 7 m s^{-1} te tere o te ahanoa i muri i te 4 hēkona, e hia te tawhiti o te haere i ngā hēkona tuatahi e 9 o te nekehanga?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) Whiriwhiria te uara o k ina ko $\int_0^k 3e^{2x}\,dx = 4$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) An object's acceleration is modelled by the function

$$a(t) = 1.2\sqrt{t}$$

where a is the acceleration of the object, in m s^{-2}
and t is the time in seconds since the start of the object's motion.

If the object had a velocity of 7 m s^{-1} after 4 seconds, how far did it travel in the first 9 seconds of motion?

You must use calculus and show the results of any integration needed to solve the problem.

(d) Find the value of k if $\int_0^k 3e^{2x}\,dx = 4$.

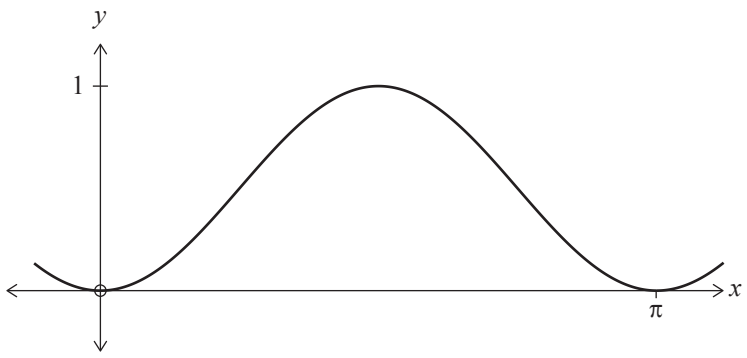
You must use calculus and show the results of any integration needed to solve the problem.

(e) Ko te uara toharite o tētahi pānga $y = f(x)$ mai i $x = a$ ki $x = b$ ko te

$$\text{Uara toharite} = \frac{\int_a^b f(x)\,\mathrm{d}x}{b-a}$$

Whiriwhiria te uara toharite o $y = \sin^2 x$ i waenga i a $x = 0$ me $x = \pi$.

E whakaatuhia ana he wāhanga o te kauwhata $y = \sin^2 x$ i raro nei.



Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

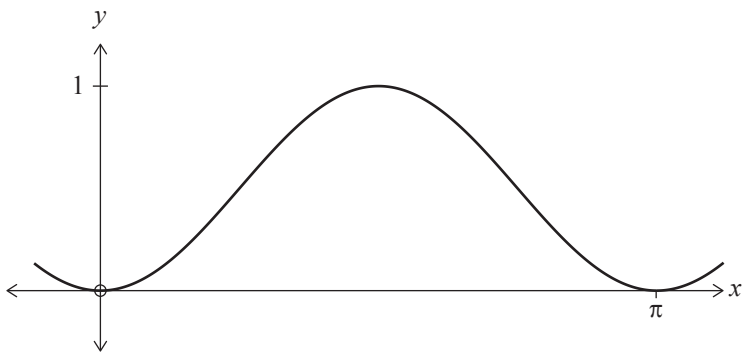
ASSESSOR'S USE ONLY

(e) The mean value of a function $y = f(x)$ from $x = a$ to $x = b$ is given by

$$\text{Mean value} = \frac{\int_a^b f(x)\,\mathrm{d}x}{b-a}$$

Find the mean value of $y = \sin^2 x$ between $x = 0$ and $x = \pi$.

Part of the graph of $y = \sin^2 x$ is shown below.



You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int \frac{6}{2x-1}\,\mathrm{d}x$.

(b) Whiriwhiria $\int (2x-5)^4\,\mathrm{d}x$.

QUESTION TWO

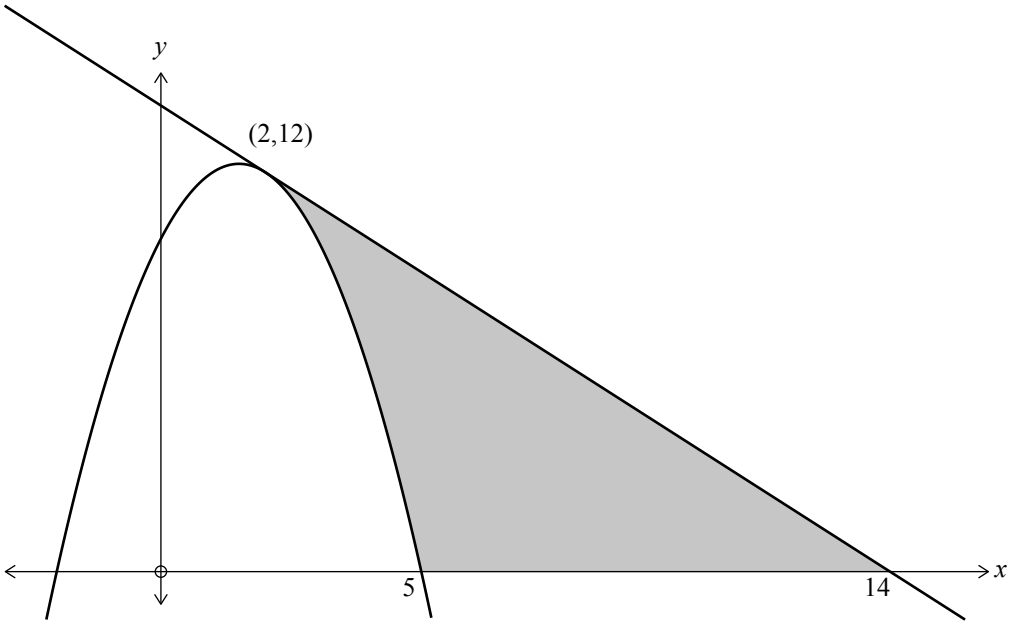
ASSESSOR'S USE ONLY

(a) Find $\int \frac{6}{2x-1}\,dx$.

(b) Find $\int (2x-5)^4\,dx$.

(c) E whakaatu ana te hoahoa i raro nei i te ānau $y = -x^2 + 3x + 10$, me te rārangi $y = -x + 14$, koia te pātapa ki te ānau i te pūwāhi (2, 12).

MĀ TE KAIMĀKA ANAKE

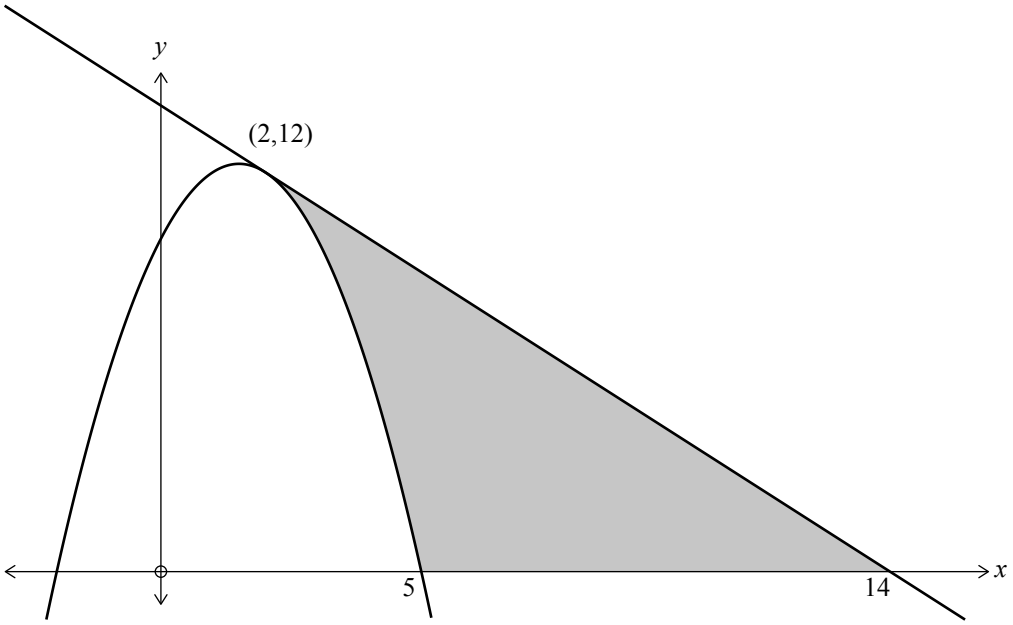


Tātaihia te horahanga o te wāhi kauruku.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) The diagram below shows the curve $y = -x^2 + 3x + 10$, and the line $y = -x + 14$, which is the tangent to the curve at the point (2, 12).

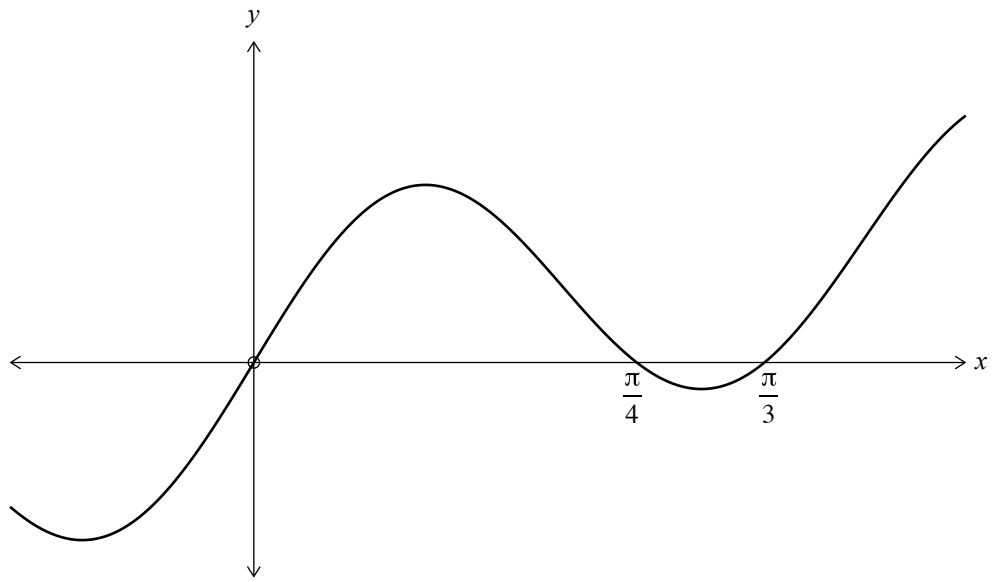


Calculate the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaatuhia ana he wāhanga o te kauwhata o $y = \sin 3x \cos 2x$ i raro nei.

MĀ TE KAIMĀKA ANAKE

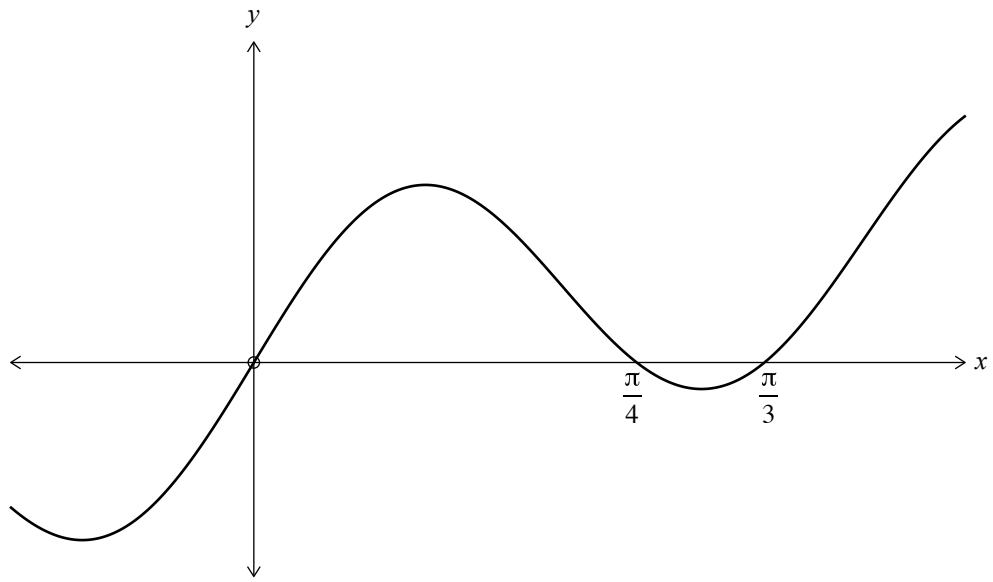


Whiriwhiria te horahanga kua rohea i waenga i te ānau $y = \sin 3x \cos 2x$ me ngā rārangi $y = 0$, $x = 0$, me te
$x = \frac{\pi}{4}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) Part of the graph of $y = \sin 3x \cos 2x$ is shown below.

ASSESSOR'S USE ONLY



Find the area enclosed between the curve $y = \sin 3x \cos 2x$ and the lines $y = 0$, $x = 0$, and $x = \frac{\pi}{4}$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Ka whakatauiratia te whakaterenga o tētahi ahanoa mā te pānga $a(t) = \dfrac{20 \ln t}{t}$.

ina ko a te whakaterenga o te ahanoa i te m s^{-2}
ko t te wā ā-hēkona mai i te tīmatanga o te neke o te ahanoa.

I te neke te ahanoa mā te 12 m s^{-1} te terenga ina ko $t = 4$.

Whiriwhiria te terenga o te ahanoa i muri i te 10 hēkona.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) The acceleration of an object is modelled by the function $a(t) = \dfrac{20 \ln t}{t}$.

where a is the acceleration of the object in m s^{-2}
and t is the time in seconds since the start of the object's motion.

The object was moving with a velocity of 12 m s^{-1} when $t = 4$.

Find the velocity of the object after 10 seconds.

You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int\left(\frac{9}{x^4}+8\mathrm{e}^{4x}\right)\mathrm{d}x$.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

QUESTION THREE

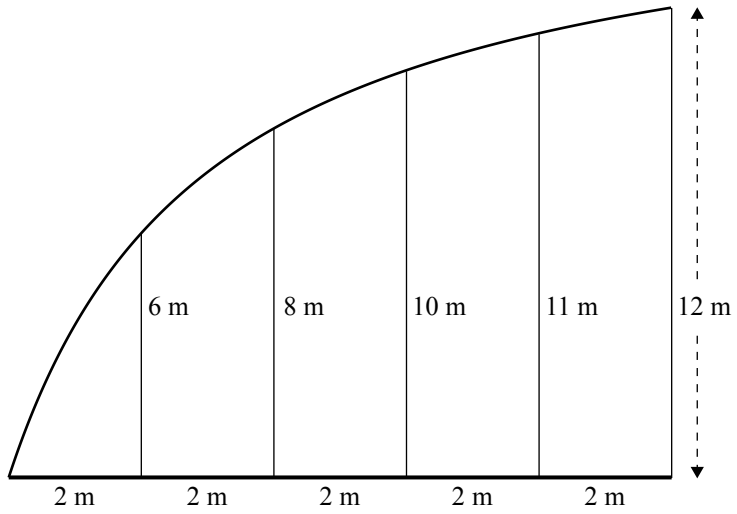
ASSESSOR'S USE ONLY

(a) Find $\int \left(\frac{9}{x^4} + 8e^{4x} \right) dx$.

Question Three continues on page 19.

(b) Kei te rapu a Julia i tētahi āwhiwhitanga o te horahanga o tētahi papa raima e hiahia ana ia kia whakatakotoria ki tōna whenua. Ka inea e ia, ā, e whakaaturia ana ēnei i te hoahoa i raro.

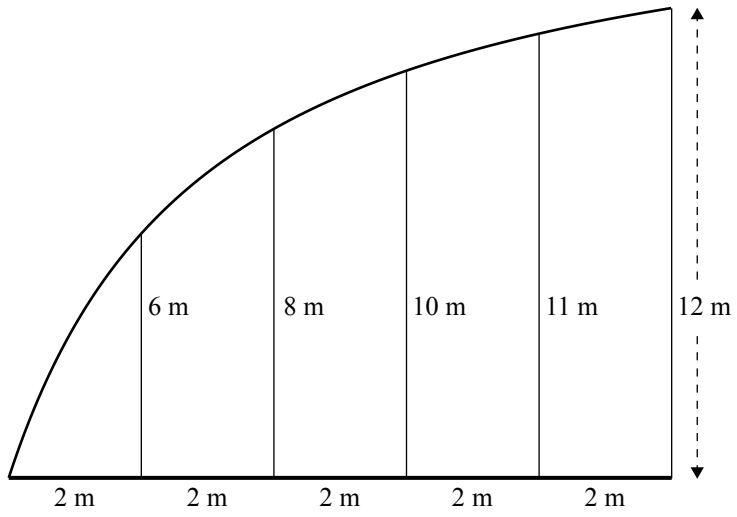
MĀ TE KAIMĀKA ANAKE



Mā te whakamahi i ēnei ine, me te Ture Taparara, whiriwhiria he āwhiwhitanga o te horahanga o te papa raima.

ASSESSOR'S USE ONLY

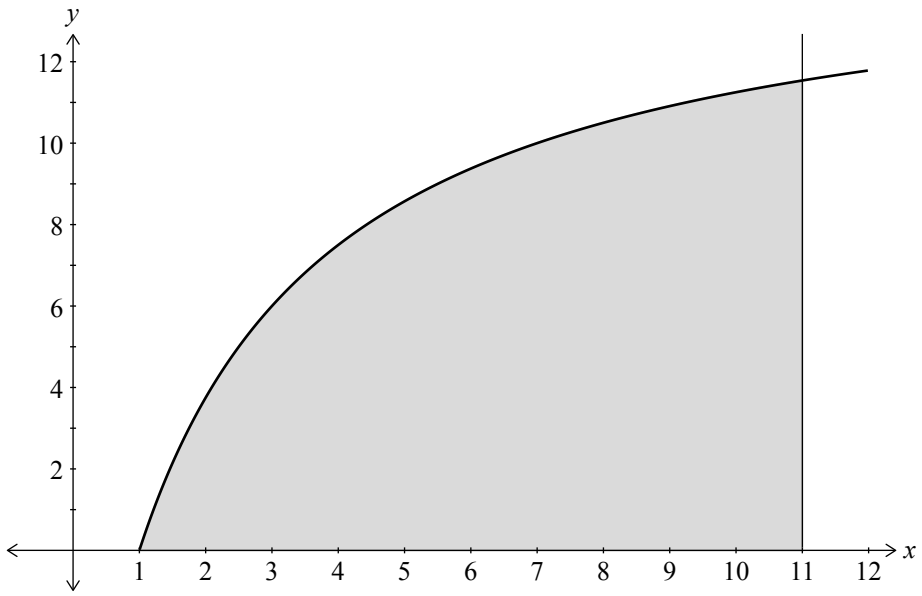
(b) Julia wants to find an approximation of the area of a paved courtyard that she wishes to construct on her property. She takes some measurements and these are shown on the diagram below.



Using these measurements, and the Trapezium rule, find an approximation of the area of paved courtyard.

(c) E whakapono ana te hoa o Julie, a Sarah, ka taea te whakatauira te whārite o te paenga ānau o te papa raima mā te pānga $y = \dfrac{15x - 15}{x + 2}$.

MĀ TE KAIMĀKA ANAKE

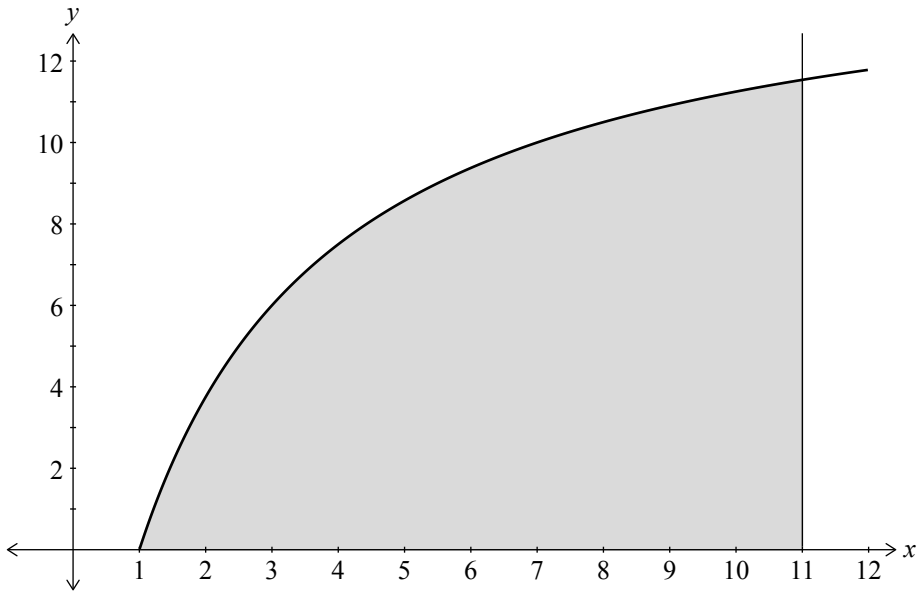


Whakamahia te pāwhaitua hei whiriwhiri i te horahanga o te papa, e ai ki te hoahoa i runga ake.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) Julia's friend Sarah believes that the equation of the curved border of the paved courtyard can be modelled by the function $y = \dfrac{15x - 15}{x + 2}$.



Use integration to find the area of the courtyard, shown in the diagram above.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Whakaotihia te whārite pārōnaki $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$, ina ko $x = 4$, kāti ko $y = 1$.

MĀ TE KAIMĀKA ANAKE

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) Solve the differential equation $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$, given that when $x = 4$, then $y = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

(e) Mā y me t e pai ai te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}t} = k\cos 0.5t \times \mathrm{e}^{\sin 0.5t}$, $0 \le t \le 5$.

MĀ TE KAIMĀKA ANAKE

Ina ko $t = 0$, $y = 8$, ā, ina ko $t = 2$, $y = 12$, whiriwhiria te uara o y ina ko $t = 5$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(e) y and t satisfy the differential equation $\frac{dy}{dt} = k\cos 0.5t \times e^{\sin 0.5t}$, $0 \le t \le 5$.

ASSESSOR'S USE ONLY

Given that when $t = 0$, $y = 8$, and that when $t = 2$, $y = 12$, find the value of y when $t = 5$.

You must use calculus and show the results of any integration needed to solve the problem.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2017

91579 Apply integration methods in solving problems

9.30 a.m. Thursday 23 November 2017
Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.