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Draw a cross through the box (☒) if you have NOT written in this booklet

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Physics 2023

91524 Demonstrate understanding of mechanical systems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of mechanical systems.	Demonstrate in-depth understanding of mechanical systems.	Demonstrate comprehensive understanding of mechanical systems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Booklet L3–PHYSR.

In your answers use clear numerical working, words, and/or diagrams as required.

Numerical answers should be given with an appropriate SI unit.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

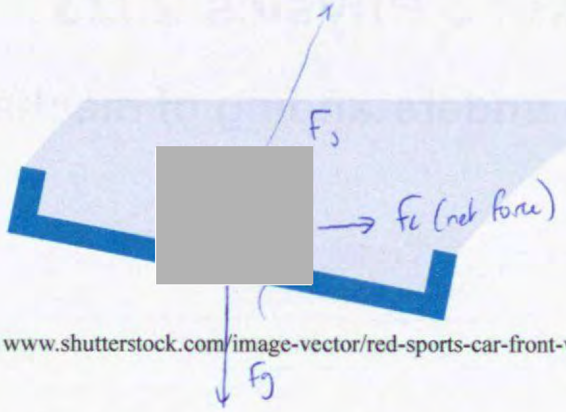
Excellence 23

QUESTION ONE: HORIZONTAL AND VERTICAL CIRCLES

Tane has a toy car track set. Part of the track is a horizontal banked curve and part of it has a vertical loop. For this question, assume that sideways friction on the tyres is negligible. The toy car has a mass of 0.120 kg.

Source: www.walmart.ca/en/ip/hot-wheels-massive-loop-mayhem-track-set-multi/6000203404407

(a) On the diagram below, draw a vector diagram to identify the net force that is responsible for the car going in a horizontal circle along the banked curve. Label the net force.



Source: www.shutterstock.com/image-vector/red-sports-car-front-view-156275714

If you need to redraw your response, use the diagram on page 9.

(b) The banked curve of the car track has a radius of 0.750 m.

Calculate the angle of banking when there is no sideways friction on the wheels of the car as it goes around the banked curve at 1.55 m s^{-1}.

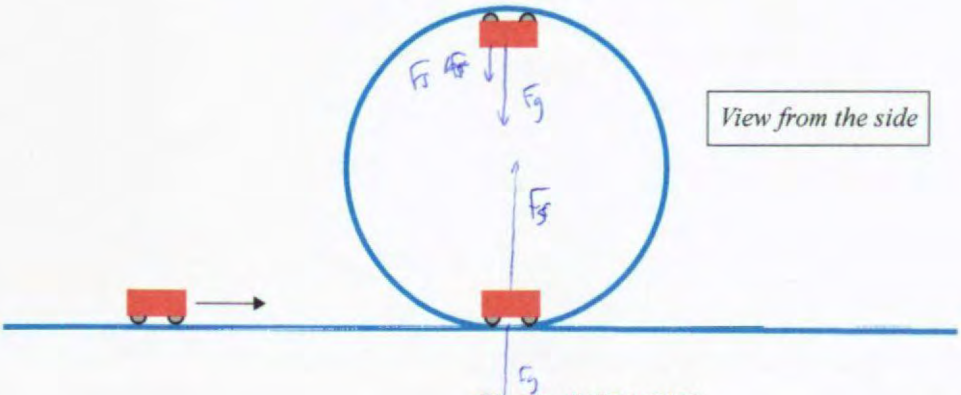
$F_c = \frac{mv^2}{r}$ $\quad \frac{0.12 \times 1.55^2}{0.75} = 0.3844$ N

$F_g = mg$ $\quad 9.81 \times 0.12 = 1.18$ N

~~sin θ~~ $\tan\theta = \frac{F_c}{F_g}$ $\quad \theta = \tan^{-1}\left(\frac{0.3844}{1.18}\right) = 18.08°$

$18.1°$

The diagram below is a simplified version of the vertical circular loop that makes up part of the car track.



If you need to redraw your response to part (c), use the diagram on page 9.

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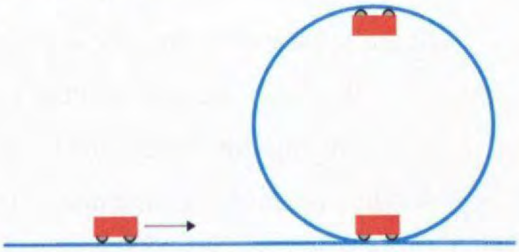
(c) Explain why the person sitting in a car on an actual roller coaster would feel heavier at the bottom of the loop compared to the top of the loop.

Begin your answer by drawing labelled vectors in the diagram at the bottom of the opposite page, to represent the forces acting on the car when it is at the top of the loop AND when it is at the bottom of the loop.

The feeling of weight comes from the support/normal/reaction force opposing the gravitational force. At the bottom and top, there must be a net force towards the radius that acts as the centripetal force giving the car ~~rotatio~~ circular motion around the loop. ~~A~~ F_g, the weight force is equal and acts down in all situations. At the bottom, to create a vertical net force towards the centre, the reaction force must be greater than the weight force, therefore creating a sense of heaviness ~~of~~ or weight on the person. At the top, the reaction force due to its ~~up~~ velocity acts in the same direction as the weight force, therefore the person feels a smaller reaction force that makes them feel lighter/less heavy, because F_r does not have to overcome F_g to produce a centripetal force, which it does at the bottom.

(d) The toy car of mass 0.120 kg approaches the vertical circular loop of radius 0.250 m.



Calculate the speed with which the car must approach the bottom of the loop to be able to go around the vertical circular loop, such that the car seems weightless at the top of the loop.

Minimum speed when $F_c = F_g$ $\quad \frac{mv^2}{r} = mg \quad g = \frac{v^2}{r} \quad v^2 = gr \quad v = \sqrt{gr}$

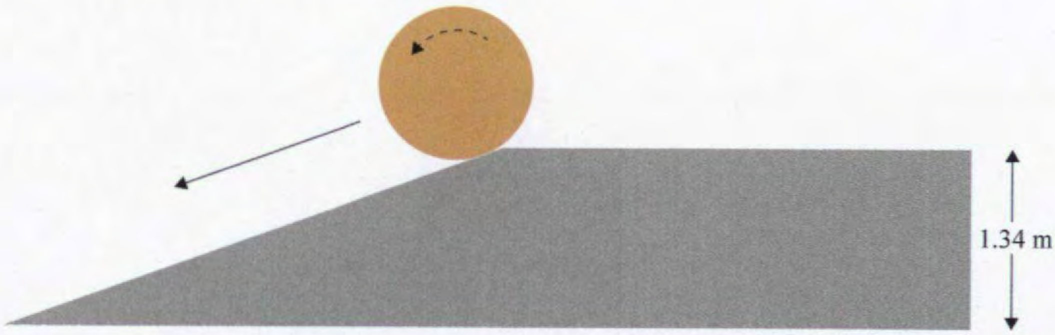
$\sqrt{9.81 \times 0.25} = 1.57\ ms^{-1}$

$1.6\ ms^{-1}$ (2.s.f)

Will appear weightless ~~k~~ when there is no reaction force, which occurs when $F_c = F_g$

QUESTION TWO: ROTATIONAL MOTION

Tane works weekends unloading barrels. In one instance he rolls an empty barrel of mass 5.50 kg and radius 0.280 m, down a ramp that is 1.34 m high. The linear speed of the barrel when it reaches the bottom of the ramp is 3.40 m s^{-1}.



(a) Describe the energy changes that take place as the barrel rolls down the ramp.

Gravitational potential energy is converted into linear and rotational kinetic energy.

(b) Calculate the rotational inertia of the barrel.

Begin your answer by calculating:

- the gravitational potential energy at the top
- the angular velocity of the barrel as it reaches the bottom of the ramp.

Assume no energy is lost due to friction.

$E_p = mgh$ $\quad v = r\omega$ $\quad E_p = 5.5 \times 9.81 \times 1.34 = 72.3$ J

$\omega = \frac{v}{r}$ $\quad \frac{3.4}{0.28} = 12.14$ rads^{-1}

assuming energy is conserved then $E_p = E_k$ (rot) final + E_k (lin) final

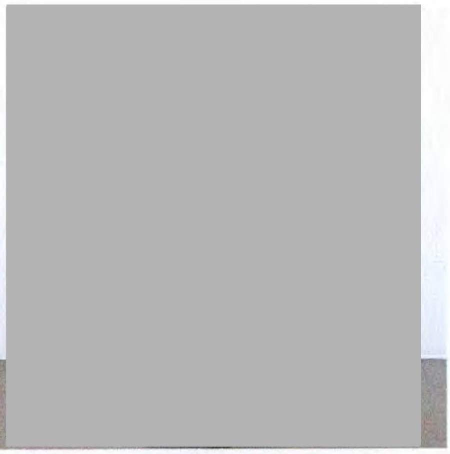
$72.3 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$72.3 = (0.5 \times 5.5 \times 3.4^2) + (\frac{1}{2} \times 12.14^2)I$

$40.51 = 73.7I$

$I = 0.55$ kgm^2

Tane then sits on a swivelling stool, holding a full bottle of water in each hand. He notices that when he holds the bottles with his arms outstretched, he tends to spin more slowly, as compared to when he brings his arms inwards, close to his body.



Source: www.exploratorium.edu/snacks/momentum-machine

(c) Explain the reason for this observation.

When he stretches his arms out, the average distribution of mass about the axis of rotation increases thus increasing the effective radius and therefore increasing his rotational inertia. $L = I\omega$. Because he exerts an internal force to hold the bottles out, there is no external torque therefore angular momentum is conserved. Because $L = I\omega$, where L is constant, if he stretches his arms out, increasing rotational inertia this causes a proportional decrease in angular velocity, therefore he spins slower with his arms out (Could be explained vice versa with bringing arms in, with opposite effect).

(d) Tane spins with an angular velocity of 3.00 rad s^{-1} when his arms are outstretched. When he brings his arms in, he reaches an angular velocity of 7.00 rad s^{-1} in a time of 4.50 s.

$\Delta\omega = 4$

Calculate:

- his angular acceleration
- the number of revolutions made in this time.

$\omega_i = 3 \quad \omega_f = 7 \quad t = 4.5 \quad \omega_f = \omega_i + at$

$4 = 4.5a$

$a = 0.89 \text{ rad s}^{-2}$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\omega_f^2 = \omega_i^2 + 2a\theta$

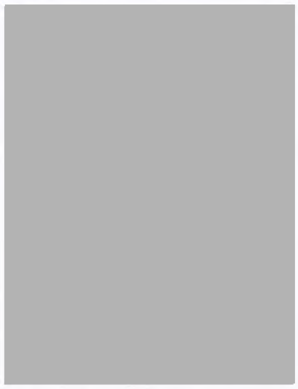
$2a\theta = 40$

$\theta = \frac{40}{2 \times 0.89} = 22.47 \text{ rads}$

$\frac{22.47}{2\pi} = 3.58$ revolutions

3.6 (2 s.f.)

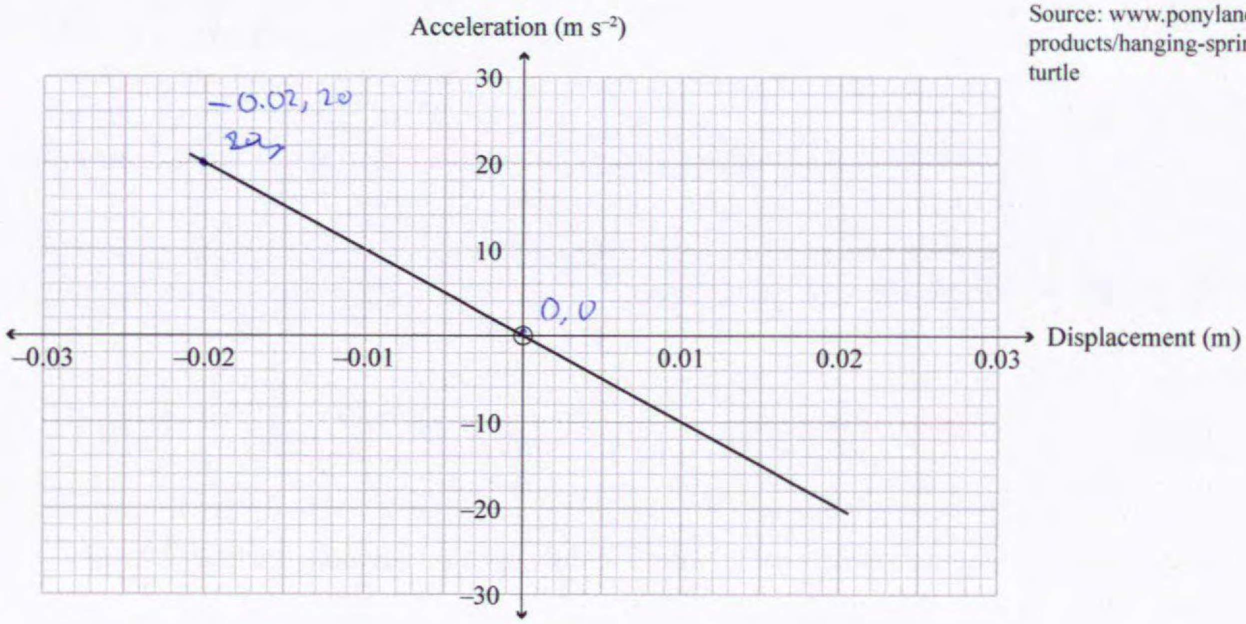
QUESTION THREE: SIMPLE HARMONIC MOTION



Source: www.ponylane.co.nz/products/hanging-spring-toy-turtle

Tanya is studying the motion of a toy bouncing up and down at the end of a spring that is hanging from the ceiling. The spring has a spring constant of 24.6 N m^{-1}.

Tanya draws an acceleration against displacement graph, as shown below, of the toy on the spring that is bouncing up and down in simple harmonic motion.



(a) Given the equation relating to simple harmonic motion as $a = -\omega^2 y$, describe how the gradient of the graph line relates to the frequency of oscillation.

The gradient of the line is equal to the negative of the spring constant ($-k$). The spring constant relates to the frequency of oscillation by $T = 2\pi\sqrt{\frac{m}{k}}$, where $T = \frac{1}{f}$. The gradient of the line is equal to $-\omega^2$, which relates to the frequency of oscillation by $\omega = 2\pi f$

(b) By calculating the gradient of the graph, show that the period of oscillation is $T = 0.199$ s, and hence determine the mass of the toy hanging on the spring.

gradient $= \frac{-20}{0.02} = -1000 = -\omega^2$ $\quad \omega = \sqrt{1000} = 31.62$ rad s^{-1}

$\omega = 2\pi f$ $\quad f = \frac{31.62}{2\pi} = 5.03$ Hz $\quad f = \frac{1}{T}$ $\quad T = \frac{1}{5.03} = 0.199$ s

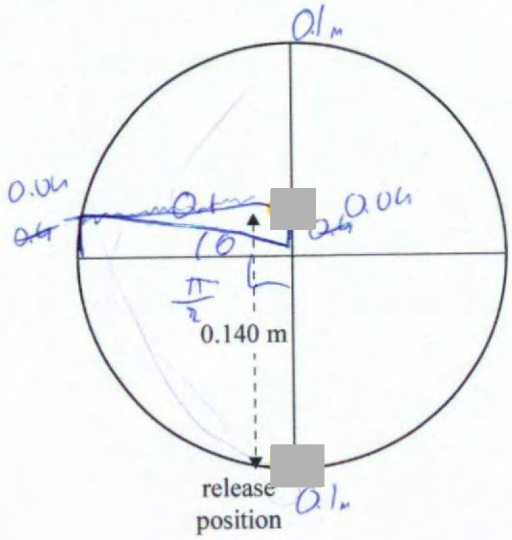
$T = 2\pi\sqrt{\frac{m}{k}}$

$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$ $\quad \frac{T^2}{4\pi^2} = \frac{m}{k}$ $\quad m = \frac{T^2 k}{4\pi^2}$

$\frac{0.199^2 \times 24.6}{4\pi^2} = 0.0247$ kg

(c) Tanya then pulls the spring of period $T = 0.199$ s **down** through a distance of 0.100 m from the equilibrium position, and then releases it so that the toy bounces up and down in simple harmonic motion.

By using a reference circle or otherwise, calculate the time the toy on the spring would take to travel a distance of 0.140 m up from its release position.



If you need to redraw your response, use the diagram on page 9.

~~AO~~ $A = 0.1$ m

$\sin\theta = \frac{0.04}{0.1}$ $\quad \theta = \sin^{-1}\left(\frac{0.04}{0.1}\right) = 0.4115$ rads travelled past equilibrium

total θ travelled $= \frac{\pi}{2} + 0.4115 = 1.98$ rads

$\omega = \frac{\Delta\theta}{\Delta t}$ $\quad \Delta t = \frac{\Delta\theta}{\omega}$ $\quad \frac{1.98}{31.62} = 0.0627$ s

0.063 s (2 s.f)

$y = A\cos\omega t$

$\frac{0.04}{1} = \cos\omega t$

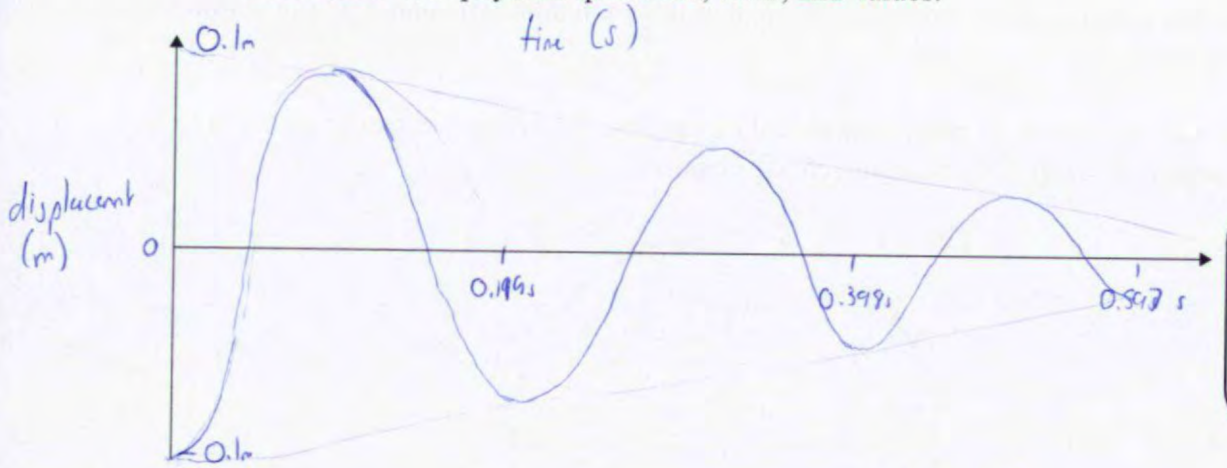
$\cos^{-1}\frac{1.4}{1}$ $\quad \omega t = 1.16$

$t = \frac{1.16}{31.62}$

Question Three continues on the following page.

(d) Tanya notices that once she has pulled ~~down~~ the toy on the spring by 0.100 m and set it oscillating in simple harmonic motion with a period of $T = 0.199$ s, the amplitude gradually decreases with time, and eventually the toy on the spring stops oscillating.

- State the name of this phenomenon, and explain what causes a decrease in amplitude.
- Using the axes below, draw a graph of amplitude against time for **three complete oscillations**.
- Label axes with physical quantities, units, and values.

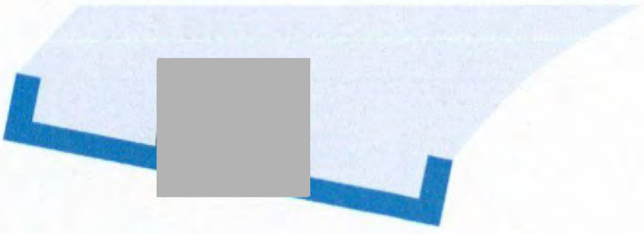


If you need to redraw your response, use the axes on page 10.

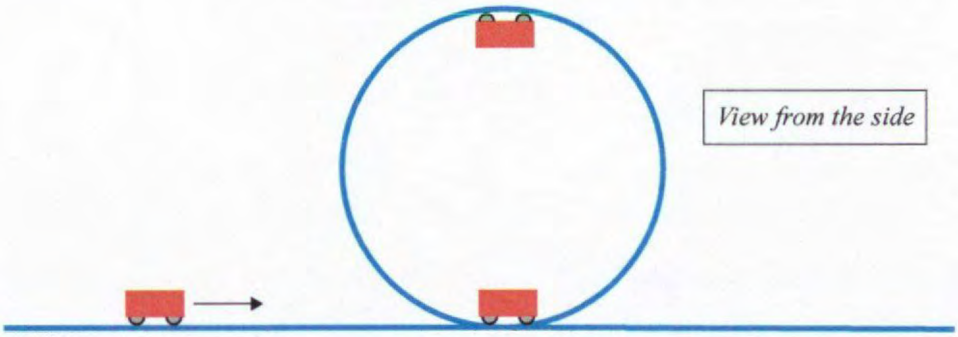
This is called damping, and occurs when a friction force causes energy to be removed from the system, thus decreasing the amplitude of oscillation exponentially over time. (Less ~~Ep~~ ~~transf~~ Ek is transferred to Ep, meaning a smaller displacement)

SPARE DIAGRAMS

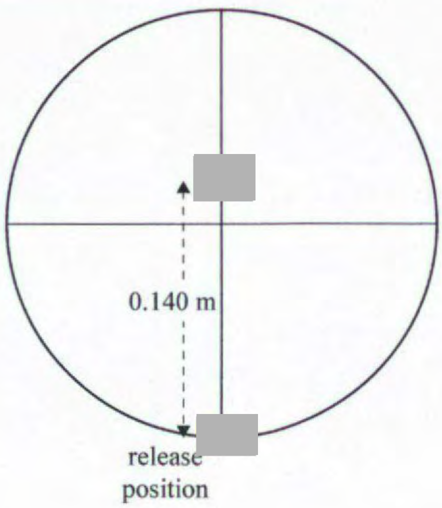
If you need to redraw your response to Question One (a), use the diagram below. Make sure it is clear which answer you want marked.



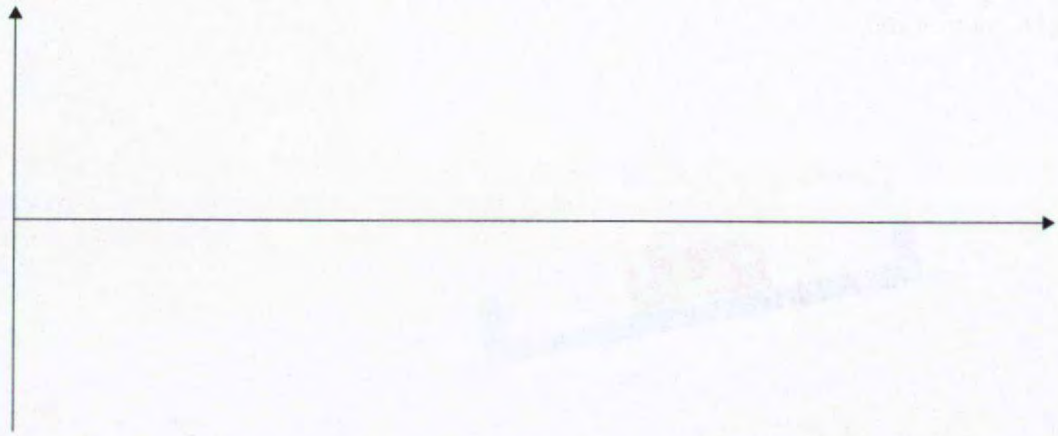
If you need to redraw your response to Question One (c), use the diagram below. Make sure it is clear which answer you want marked.



If you need to redraw your response to Question Three (c), use the axes below. Make sure it is clear which answer you want marked.



If you need to redraw your response to Question Three (d), use the axes below. Make sure it is clear which answer you want marked.



Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

91524

Standard	91524			Total score	23
Q	**Grade score**	**Marker commentary**			
1	E7	a. Labels force correctly. b. Calculates angle correctly. c. Recognises a net force is required for circular motion and correctly identifies the direction of the net force. Mistakes tension force for support force when relating forces to the sensation of weight. d. Correctly completes multi-step calculation to include velocity at the bottom of the loop.			
2	E8	a. Correctly identifies energy transformations. b. Correctly completes multi-step calculation to determine inertia. c. Can relate mass distribution to axis of rotation and so the inertia of the object. Can appropriately apply conservation of angular momentum to the situation to explain the change in angular velocity. d. Correctly completes multi-step calculation to determine number of revolutions completed.			
3	E8	a. Can interpret the graph and recognises that angular frequency and frequency are related. b. Recognises there are two parts to the question and answers both correctly. c. Uses the reference circle to determine t. d. Can explain damping in terms of energy loss due to friction. Is able to produce a labelled graph, cosine graph was too far below the line for excellence.			