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91579



Draw a cross through the box (X) if you have NOT written in this booklet

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Mana Tohu Mātauranga o Aotearoa  
New Zealand Qualifications Authority

## Level 3 Calculus 2024

### 91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (X/X). This area will be cut off when the booklet is marked.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

**Achievement**

**TOTAL 10**



## QUESTION ONE

- (a) Find
- $\int 6 \sec(2x) \tan(2x) dx$
- .

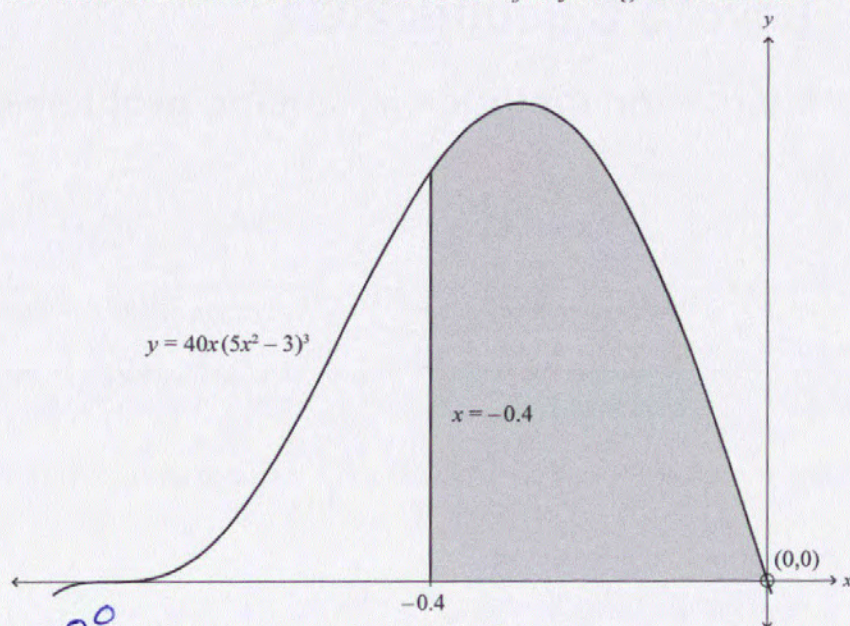
$$= 6 \sec(2x)$$

$$= 3 \sec(2x) + C$$

- (b) The graph below shows the function
- $y = 40x(5x^2 - 3)^3$
- .

Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.



$$\int_{-0.4}^0 40x(5x^2 - 3)^3$$

$$\int 40x(5x^2 - 3)^3$$

$$= 10x(5x^2 - 3)^4 \div 100x$$

$$\left[ \frac{1}{10}(5x^2 - 3)^4 \right]_{-0.4}^0$$

$$8.1 - 62.5$$

$$-54.4$$

as cannot be -ve

$$\text{Area} = 54.4 \text{ m}$$



- (c) An object's velocity can be modelled by the equation  $v(t) = 26.4\sqrt[3]{t}$ , where  $v$  is the velocity of the object in  $\text{m s}^{-1}$ , and  $t$  is the time in seconds since the start of timing.

Initially, the object was 360 metres from a point P.

Calculate the distance that this object has travelled from the point P when it has reached a velocity of  $264 \text{ m s}^{-1}$ .

*You must use calculus and show the results of any integration needed to solve the problem.*

$$264 = 26.4\sqrt[3]{t}$$

$$3\sqrt[3]{t} = 10$$

$$t = 10^3 = 1000$$

$$\int 26.4 \times t^{1/3} = t^{-2/3} \times \frac{3}{2} = -39.6t^{-2/3} + 360$$

$$t = 360$$

$$= 386.359.6$$

$$\text{or } 360\text{m}$$



- (d) Consider the differential equation  $\frac{dy}{dx} = 24\cos(3x)\sin(x)$ .

Given that  $y = 6$  when  $x = \frac{\pi}{3}$ , find the value(s) of  $y$  when  $x = \frac{\pi}{2}$ .

You must use calculus and show the results of any integration needed to solve the problem.

$$24\cos(3x)\sin(x)$$

$$= 12[\sin(4x) - \sin(2x)]$$

$$= 12\sin(4x) - 12\sin(2x)$$

$$= 6\cos(2x) - 6\cos(4x) + C$$

$$x = \frac{\pi}{3}$$

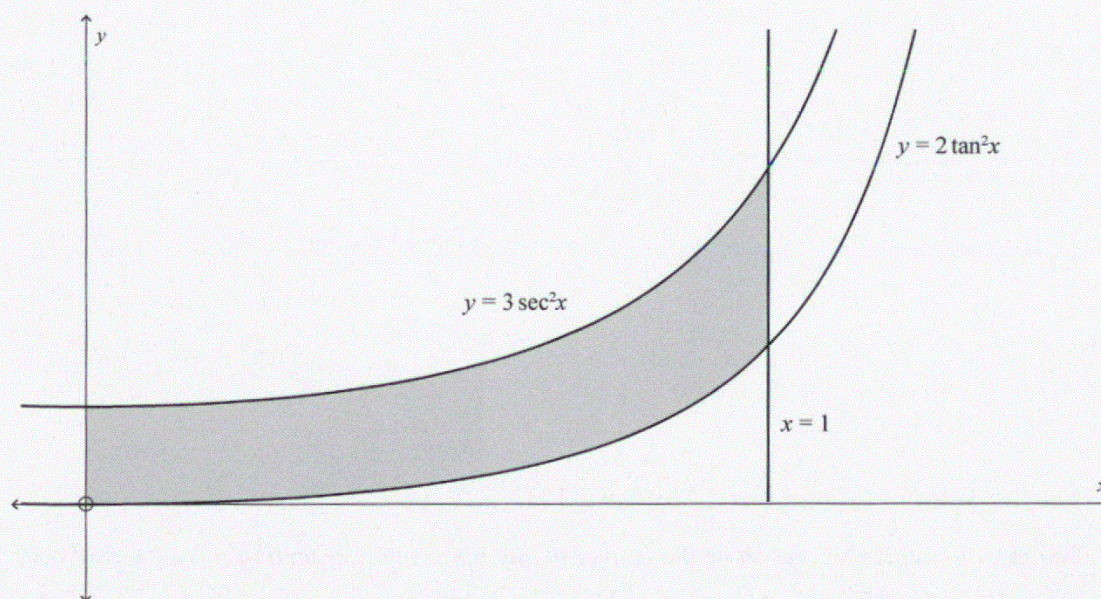
$$6\cos\left(\frac{2\pi}{3}\right) - 6\cos\left(\frac{4\pi}{3}\right) + C = 6$$

$$C = 6$$

$$6\cos\left(\frac{2\pi}{2}\right) - 6\cos\left(\frac{4\pi}{2}\right) + 6 = -6$$



- (e) The graph below shows the curves  $y = 3 \sec^2 x$  and  $y = 2 \tan^2 x$ .



Find the area of the shaded region enclosed by the two curves,  $x = 1$ , and the  $y$ -axis.

You must use calculus and show the results of any integration needed to solve the problem.

$$3 \sec^2 x$$

$$\left[ 3 \tan x - \int 2 \tan^2 x \right]_0^1$$

$$\int 2 \tan^2 x = 2 \sec^2 x - 1$$

$$\int \text{Ans} = 2 \tan x - x$$

$$\left[ 3 \tan x - 2 \tan x + x \right]_0^1$$

$$= [\tan x + x]_0^1$$

$$= 2.557 \text{ u}^2$$

$$\text{or } 2.56 \text{ u}^2$$



## QUESTION TWO

- (a) Find
- $\int (3x^4 + 4)^2 dx$
- .

$$\frac{1}{12x^3} (3x^4 + 4)^3$$

- (b) Find the value of
- $k$
- , given that
- $\int_k^{16} 3\sqrt{x} dx = 112$
- .

You must use calculus and show the results of any integration needed to solve the problem.

$$3\sqrt{16} = 12 - 3\sqrt{k} = 112$$

$$3\sqrt{k} = 100$$

$$9k = 10000$$

$$k = 1111.1$$

$$\int 3\sqrt{x}$$

$$= 3(x)^{1/2}$$

$$2x^{1.5}$$

$$x = 16$$

$$= 128 - 2k^{1.5} = 112$$

$$- 2k^{1.5} = -16$$

$$2k^{1.5} = 16$$

$$k^{1.5} = 8$$

$$\sqrt[1.5]{8} = 4$$

$$k = 4$$

$$k = 4$$



- (c) Consider the differential equation  $\frac{dy}{dx} = 12y^2e^{3x}$ .

Given that  $y = 0.5$  when  $x = 0$ , find the value of  $y$  when  $x = \frac{1}{3}$ .

*You must use calculus and show the results of any integration needed to solve the problem.*

$$4/3/23 \quad \frac{dy}{dx}$$

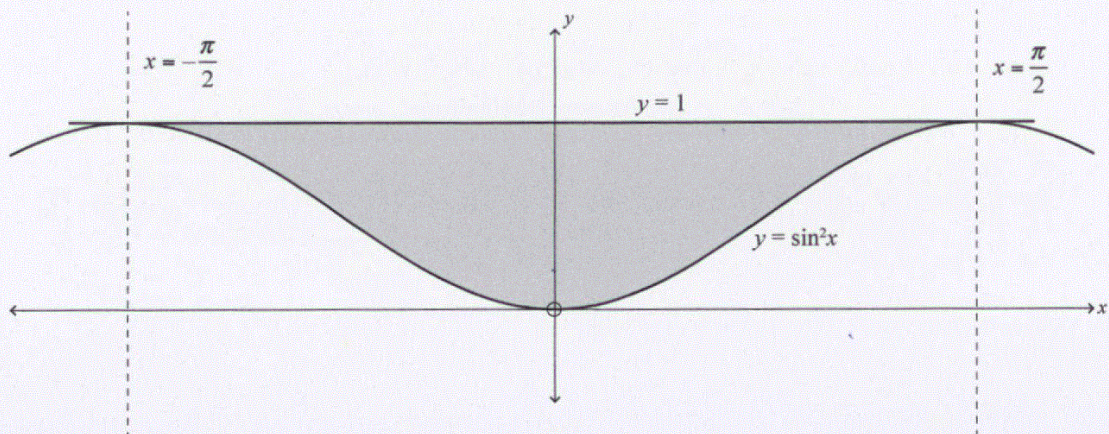
$$\int \frac{1}{12y^2} dy = \int e^{3x} dx$$

Integrate

$$\frac{1}{12} \ln|2y| = \frac{1}{3}e^{3x} + C$$



- (d) The graph below shows part of the graph of the function  $y = \sin^2 x$ .



Find the shaded area enclosed between the lines  $y = \sin^2 x$ ,  $y = 1$ ,  $x = -\frac{\pi}{2}$ ,  $x = \frac{\pi}{2}$ .

You must use calculus and show the results of any integration needed to solve the problem.

$$\begin{aligned}
 & y = \sin^2 x \quad 1 - \sin^2 x \quad \pi/2 \\
 & 1 - \sin^2 x = \cos^2 x \\
 & \int_{-\pi/2}^{\pi/2} (\cos^2 x - \sin^2 x) dx = \int_{-\pi/2}^{\pi/2} \cos 2x dx \\
 & = \left[ \frac{1}{2} \sin 2x \right]_{-\pi/2}^{\pi/2} = \frac{1}{2} (\sin \pi - \sin(-\pi)) = \frac{1}{2} (0 - (-0)) = 0 \\
 & \text{Shaded area} = \pi
 \end{aligned}$$



- (e) The mass,  $M$ , of a spherical object, with radius  $p$ , can be approximated by

$$M = \int_0^p 4\pi r^2 \frac{a}{(1 + br^3)} dr$$

where  $a$ ,  $b$ , and  $p$  are all positive constants.

Using this formula, find an expression for the mass,  $M$ , of a spherical object, giving your answer in terms of  $a$ ,  $b$ ,  $p$ , and  $\pi$ .

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## QUESTION THREE

(a) Find  $\int \left( e^{2x} + \frac{3}{e^{4x}} \right) dx$ .

$$e^{2x} + 3e^{-4x}$$

$$= \frac{1}{2}e^{2x} - \frac{3}{4}e^{-4x}$$

(b) Solve the differential equation  $\frac{dy}{dx} = \frac{5}{4x-3}$ , given that  $y = 10$  when  $x = 1$ .

*You must use calculus and show the results of any integration needed to solve the problem.*

$$\frac{5}{4} \ln(4x-3) = y$$



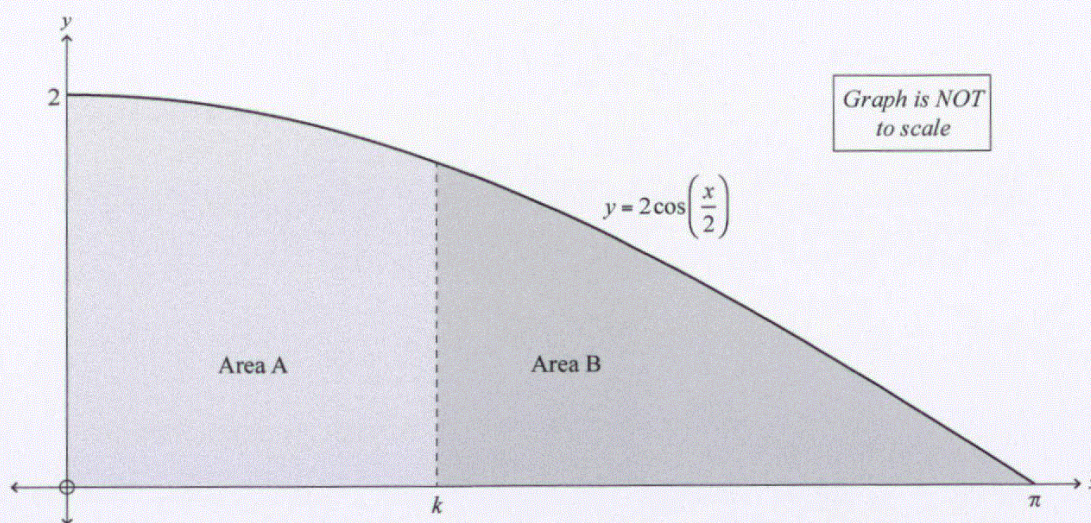
- (c) Find the value of  $m$ , given that  $\int_{-1}^m \left( \frac{4x+5}{2x+3} \right) dx = 2m$ .

$$2x \quad 2x(2+5) \quad \frac{2(2x+5)-7}{2x+3}$$

Question Three continues  
on the next page.



- (d) The graph below shows the function  $y = 2\cos\left(\frac{x}{2}\right)$ .



Find the value of  $k$  so that the shaded Area A will be equal to the shaded Area B.

You must use calculus and show the results of any integration needed to solve the problem.

$$\int_0^k 2\cos\left(\frac{x}{2}\right) dx = \int_k^\pi 2\cos\left(\frac{x}{2}\right) dx$$

$$= \left[4\sin\left(\frac{x}{2}\right)\right]_0^k = \left[4\sin\left(\frac{x}{2}\right)\right]_k^\pi$$

$$4\sin\left(\frac{\pi}{2}\right) - 4\sin\left(\frac{k}{2}\right) = 4\sin\left(\frac{k}{2}\right) - 4\sin(0)$$

$$4\sin\left(\frac{\pi}{2}\right) = 8\sin\left(\frac{k}{2}\right)$$

$$4 = 8\sin\left(\frac{k}{2}\right)$$

$$\frac{1}{2} = \sin\left(\frac{k}{2}\right)$$

$$\sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{k}{2} = 0.5234$$

$$k = 1.05$$



- (e) A teacher makes a cup of coffee at the start of interval. The teacher leaves the cup of coffee in the staff room where the temperature is  $18^\circ\text{C}$ .

After 30 minutes, the temperature of the cup of coffee is  $50^\circ\text{C}$ , but the teacher believes that the coffee is still too hot to drink.

When the teacher returns again, after a further one hour, the cup of coffee has cooled down to a temperature of  $30^\circ\text{C}$ .

The rate at which the temperature of the cup of coffee changes at any instant is proportional to the difference between the temperature of the cup of coffee,  $N$ , and the temperature of the room.

Write a differential equation that models this situation, and then solve it to calculate the temperature of the cup of coffee when it was made.

You must use calculus and show the results of any integration needed to solve the problem.

$$T - T_a = Ae^{kt} \quad T - T_a = Ae^{kt}$$

$$T_a = 18 \quad t = 30$$

$$50 - 18 = Ae^{k \times 30}$$

$$32 = Ae^{1.5k}$$

$$32 = Ae^{0.5k}$$

$$12 = Ae^{1.5k}$$

$$\ln 32 = 0.5k \quad \ln 12 = 1.5k$$

$$T - T_a = Ae^{kt}$$

$$50 - 18 = Ae^{0.5k}$$

$$30 - 18 = Ae^{1.5k}$$

$$30 + 50 + Ae^{0.5k} = Ae^{1.5k}$$

$$\frac{32}{e^{0.5k}} = A$$

$$12 = \frac{32}{e^{0.5k}} e^{1.5k}$$

$$12e^{0.5k} = 32e^{1.5k}$$

$$e^{0.5k} = 2\frac{3}{2}e^{1.5k}$$

$$0.5k = \ln \left( 2\frac{3}{2} \right) / 1.5k$$



## Achievement

**Subject:** Calculus

**Standard:** 91579

**Total score:** 10

Q	Grade score	Marker commentary
One	A3	Candidate correctly integrated trigonometric expressions.
Two	N2	Candidate correctly integrated a surd expression and used correct algebraic manipulation to solve a problem.
Three	M5	Candidate correctly calculated an area under a curve, with the correct integration of a trigonometric expression.