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NZQA

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Calculus 2024

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area. This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Merit

TOTAL 16

QUESTION ONE

(a) Find $\int 6\sec(2x)\tan(2x)\,dx$.

$3\sec 2x + C$.

(b) The graph below shows the function $y = 40x(5x^2-3)^3$.

Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

$\frac{1}{4}(5x^2-3)^4$

$\downarrow$

$\frac{1}{4}\cdot 4(5x^2-3)^3(10x)$

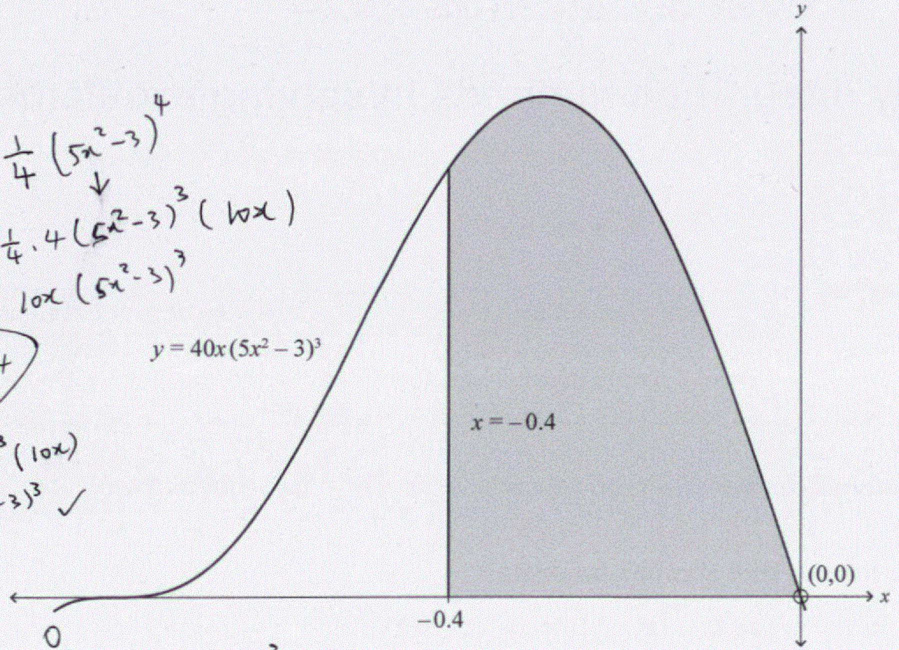
$10x(5x^2-3)^3$

$(5x^2-3)^4$

$\downarrow$

$4(5x^2-3)^3(10x)$

$40x(5x^2-3)^3$ ✓



$\int_{-0.4}^{0} 40x(5x^2-3)^3\,dx$

$\left[(5x^2-3)^4\right]_{-0.4}^{0}$

$(5\cdot 0-3)^4 - (5\cdot(-0.4)^2-3)^4$

$= 81 - 23.4256$

$= 57.5744$ units^2.

$19.8t^{4/3}$
$\downarrow$
$19.8 \cdot \frac{4}{3} t^{1/3}$

(c) An object's velocity can be modelled by the equation $v(t) = 26.4\sqrt[3]{t}$,
where v is the velocity of the object in m s^{-1},
and t is the time in seconds since the start of timing.

Initially, the object was 360 metres from a point P.

Calculate the distance that this object has travelled from the point P when it has reached a velocity of 264 m s^{-1}.

You must use calculus and show the results of any integration needed to solve the problem.

$\int 26.4t^{1/3}$

$= 19.8t^{4/3} + c$

$s(t) = 19.8t^{4/3} + c$

$s(0) = 360$

$19.8 \times 0^{4/3} + c = 360$

$c = 360$

$\therefore s(t) = 19.8t^{4/3} + 360$

$v(t) = 264$

$26.4t^{1/3} = 264$

$t^{1/3} = 10$

$t = \sqrt[3]{10}$

$= 2.15\text{ s}$

$s(2.15) = 19.8 \times 2.15^{4/3} + 360$

$= 414.94$ m.

(d) Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 24\cos(3x)\sin(x)$.

Given that $y = 6$ when $x = \frac{\pi}{3}$, find the value(s) of y when $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

$\int 24 \cos 3x \sin x$

$12 \int \sin 4x - \sin 2x$

$12\left(-\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x + c\right)$

$y = -3\cos 4x + 6\cos 2x + 12c$

when $x = \frac{\pi}{3}$, $y = 6$

$-3\cos\left(4\cdot\frac{\pi}{3}\right) + 6\cos\frac{2\pi}{3} + 12c = 6$

$-1.5 + 12c = 6$

$12c = 7.5$

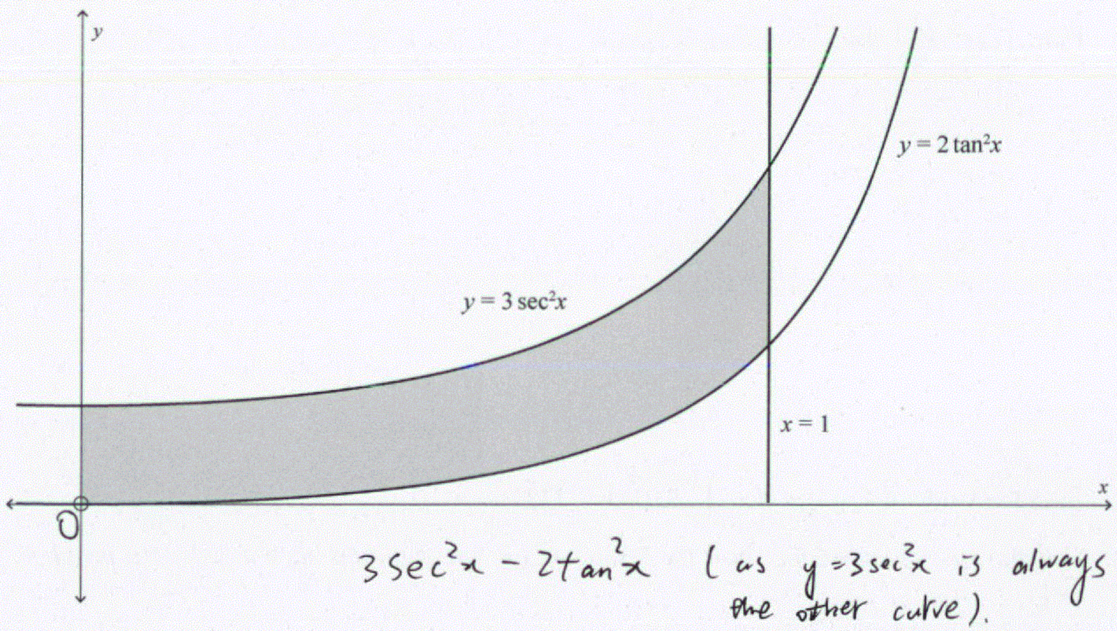
$c = 0.625$

$\therefore y = -3\cos 4x + 6\cos 2x + 7.5$

$y\left(\frac{\pi}{2}\right) = -3\cos(2\pi) + 6\cos(\pi) + 7.5$

$= -1.5$

(c) The graph below shows the curves $y = 3\sec^2 x$ and $y = 2\tan^2 x$.



$3\sec^2 x - 2\tan^2 x$ (as $y = 3\sec^2 x$ is always above the other curve).

Find the area of the shaded region enclosed by the two curves, $x = 1$, and the y-axis.

You must use calculus and show the results of any integration needed to solve the problem.

$\int_0^1 3\sec^2 x - 2\tan^2 x$

$\int_0^1 3\sec^2 x - (2\sec^2 - 1)$

$\int_0^1 \sec^2 x + 1$

$[\tan x + x]_0^1$

$(\tan(1) + 1) - (\tan(0) + 0)$

$= 2.5574 - 0$

$= 2.5574$ units^2

QUESTION TWO

(a) Find $\int(3x^4+4)^2\,dx$.

$\frac{1}{3}(3x^4+4)^3$

$\downarrow$

$3 \cdot \frac{1}{3}(3x^4+4)^2(12x^3)$

$12x^3(3x^4+4)^2$

$= \frac{1}{36x^3}(3x^4+4)^3 + c$

$2x^{3/2}$

$\downarrow$

$3x^{1/2}$

(b) Find the value of k, given that $\int_k^{16} 3\sqrt{x}\,dx = 112$.

You must use calculus and show the results of any integration needed to solve the problem.

$\int_k^{16} 3x^{1/2}\,dx = 112$

$\left[2x^{3/2}\right]_k^{16} = 112$

$(2 \cdot 16^{3/2}) - (2 \cdot k^{3/2}) = 112$

$128 - (2k^{3/2}) = 112$

$2k^{3/2} = 16$

$k^{3/2} = 8$

$k = \sqrt[3]{8^2}$

$k = 4$

(c) Consider the differential equation $\frac{dy}{dx} = 12y^2e^{3x}$.

Given that $y = 0.5$ when $x = 0$, find the value of y when $x = \frac{1}{3}$.

You must use calculus and show the results of any integration needed to solve the problem.

$$\int \frac{1}{12y^2}\,dy = \int e^{3x}\,dx$$

$$\int (12y^2)^{-1}\,dy = \int \frac{1}{12}y^{-2} = -\frac{1}{12}y^{-1}$$

$$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + c$$

$$\therefore \quad -\frac{1}{12y} = \frac{1}{3}e^{3x} + c$$

$$-\frac{1}{12 \cdot 0.5} = \frac{1}{3}e^{3 \cdot 0} + c$$

$$-\frac{1}{6} = \frac{1}{3} + c$$

$$c = -0.5$$

$$\therefore \quad -\frac{1}{12y} = \frac{1}{3}e^{3x} - 0.5$$

when $x = \frac{1}{3}$

$$-\frac{1}{12y} = \frac{1}{3}e^{1} - 0.5$$

$$-\frac{1}{12}y^{-1} = 0.406094$$

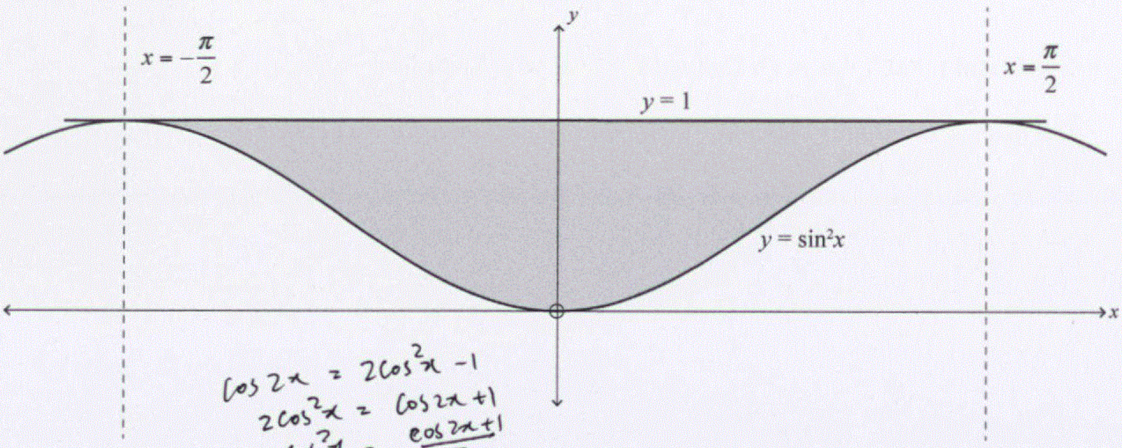
$$-1 \div 12y = 0.406094$$

$$12y = -2.4625$$

$$y = -0.205207$$

$$= -0.2052$$

(d) The graph below shows part of the graph of the function $y = \sin^2 x$.



$\cos 2x = 2\cos^2 x - 1$
$2\cos^2 x = \cos 2x + 1$
$\cos^2 x = \frac{\cos 2x + 1}{2}$

Find the shaded area enclosed between the lines $y = \sin^2 x$, $y = 1$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 - \sin^2 x \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x \, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos 2x + 1}{2} \, dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} \cos 2x + \frac{1}{2} \, dx$

$\left[\frac{1}{4} \sin 2x + \frac{1}{2} x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$\left(\frac{1}{4} \sin \pi + \frac{1}{2} \cdot \frac{\pi}{2} \right) - \left(\frac{1}{4} \sin(-\pi) + \frac{1}{2} \cdot \left(-\frac{\pi}{2}\right) \right)$

$= 0.7854 - -0.7854$

$= 1.5708$ units^2.

(e) The mass, M, of a spherical object, with radius p, can be approximated by

$$M = \int_0^p 4\pi r^2 \frac{a}{\left(1 + br^3\right)} \mathrm{d}r$$

where a, b, and p are all positive constants.

Using this formula, find an expression for the mass, M, of a spherical object, giving your answer in terms of a, b, p, and π.

QUESTION THREE

$3(e^{4x})^{-1}$ $\int 3e^{-4x} = -\frac{3}{4}e^{-4x}$ $= -\frac{3}{4}e^{-4x}$

(a) Find $\int \left(e^{2x} + \frac{3}{e^{4x}}\right) dx$.

$\frac{1}{2}e^{2x} - \frac{3}{4}e^{-4x} + c$

(b) Solve the differential equation $\frac{dy}{dx} = \frac{5}{4x-3}$, given that $y = 10$ when $x = 1$.

You must use calculus and show the results of any integration needed to solve the problem.

$\int \frac{5}{4x-3} = \int 5(4x-3)^{-1}$

$= \frac{5}{4}\ln(4x-3) + c$

$10 = \frac{5}{4}\ln(4-3) + c$

$10 = 0 + c$

$c = 10$

$\therefore y = \frac{5}{4}\ln(4x-3) + 10$

(c) Find the value of m, given that $\int_{-1}^{m}\left(\frac{4x+5}{2x+3}\right)dx = 2m$.

$$\int_{-1}^{m} \cancel{4x+5} \; \frac{2(2x+3)-1}{2x+3}$$

$$\int_{-1}^{m} 2 - \frac{1}{2x+3}$$

$$\left[2x - \frac{1}{2}\ln(2x+3)\right]_{-1}^{m}$$

$$\left(2m - \frac{1}{2}\ln(2m+3)\right) - \left(-2 - \frac{1}{2}\ln(1)\right)$$

$$= 2m - \frac{1}{2}\ln(2m+3) - -2$$

$$2m - \frac{1}{2}\ln(2m+3) + 2 = 2m$$

$$2 = \frac{1}{2}\ln(2m+3)$$

$$4 = \ln(2m+3)$$

$$e^4 = 2m+3$$

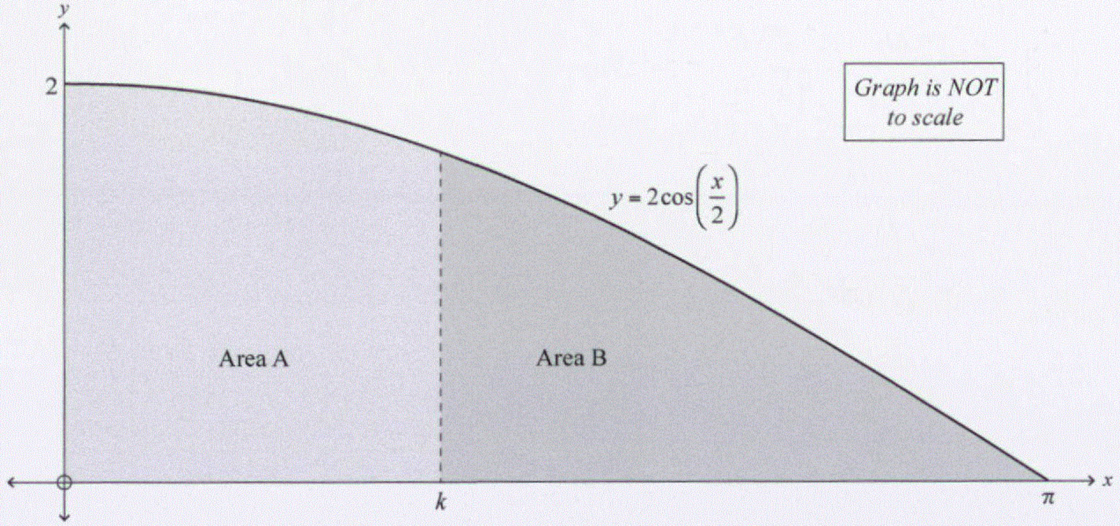
$$54.6 = 2m+3$$

$$2m = 51.6$$

$$m = 25.8$$

Question Three continues on the next page.

(d) The graph below shows the function $y = 2\cos\left(\frac{x}{2}\right)$.



Find the value of k so that the shaded Area A will be equal to the shaded Area B.

You must use calculus and show the results of any integration needed to solve the problem.

(e) A teacher makes a cup of coffee at the start of interval. The teacher leaves the cup of coffee in the staff room where the temperature is 18 °C.

After 30 minutes, the temperature of the cup of coffee is 50 °C, but the teacher believes that the coffee is still too hot to drink.

When the teacher returns again, after a further one hour, the cup of coffee has cooled down to a temperature of 30 °C.

The rate at which the temperature of the cup of coffee changes at any instant is proportional to the difference between the temperature of the cup of coffee, N, and the temperature of the room.

Write a differential equation that models this situation, and then solve it to calculate the temperature of the cup of coffee when it was made.

You must use calculus and show the results of any integration needed to solve the problem.

Merit

Subject: Calculus

Standard: 91579

Total score: 16

Q	Grade score	Marker commentary
One	M5	Candidate correctly used the product to sum trigonometric identity, and then correctly integrated this expression and used the expression to solve the problem.
Two	M6	Candidate correctly split the variables and used trigonometric identities to solve problems. Candidate found the area under a curve.
Three	M5	Candidate correctly rearranged a rational expression with linear expressions on numerator and denominator into a form to integrate. They then correctly integrated this expression, and used it to solve a problem.