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Draw a cross through the box (☒) if you have NOT written in this booklet

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Mathematics and Statistics (Statistics) 2024

91585 Apply probability concepts in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⁄⁄⁄). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 12

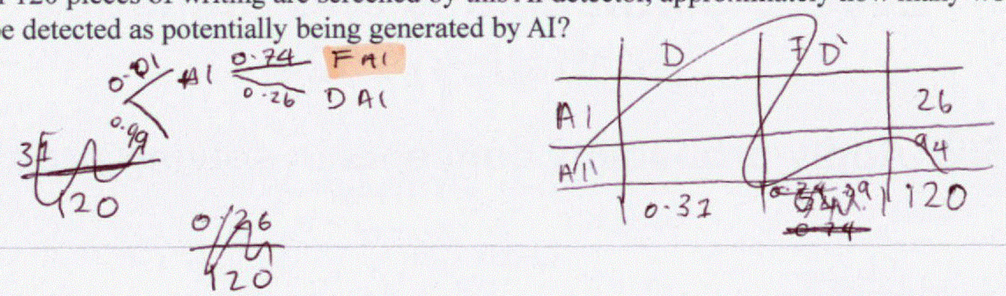
QUESTION ONE

(a) Educators use writing detectors like Turnitin to detect if students have used AI (Artificial Intelligence) to write their assignments. When designing writing detectors, scientists focus on accuracy; if they say AI writing is present in a piece of work, they want to be pretty certain that the work is AI-generated, to ensure that students are not falsely accused of misconduct. This means that the detector may not always detect all AI writing; some may be missed.

A false positive in AI writing detection refers to incorrectly identifying fully human-written text as AI-generated. Suppose that a particular AI writing detector has a 1% false positive rate but that 74% of the time it fails to detect AI-generated writing.

Based on a confidential survey of students in one teacher's course, it is thought that 22% of pieces of student writing contain content that is generated by AI.

(i) If 120 pieces of writing are screened by this AI detector, approximately how many would be detected as potentially being generated by AI?



$0.26 \times 120 = 31.2$

$= 31$

(ii) A piece of writing is detected as potentially being generated by AI.

Comment on whether a teacher should be concerned that the student could be unfairly accused of cheating.

Support your answer with statistical reasoning.

The AI detection only works 26% of the time, however it also has a 1% false positive rate, making it not reliable, So the teacher can ~~feel~~ be concerned about the Student being unfairly treated.

(b) Exam supervisors are sometimes concerned that students who complete tests quickly might have cheated. Based on data collected from one school, for a particular NCEA standard with an allocated time of 60 minutes, it is known that:

- 1% of students cheat on the assessment for this standard
- 20% of students complete the assessment in less than 25% of the allocated time
- 80% of students who cheat on the assessment complete it in less than 25% of the allocated time.

(i) Comment on whether the events, 'student cheats' and 'student completes the assessment in less than 25% of the allocated time', are independent of each other.

Use statistical reasoning to support your answer.

$P(SC) \times P(\leq 25)$

$P(\overset{0.01}{\cancel{0.1}}) \times P(0.20)$

$P(SC \cap A<25) = \cancel{P(SC) \times P(A<25)}$

$= \cancel{P(\ldots) \times P(0.1) \times P(0.8) = 0.08}$

$= \cancel{P(\ldots) \times P(\ldots) = 80}$

$P(0.01) \times (0.20) = \frac{1}{500} = 2 \times 10^{-3}$

$P(SC \cap <25) = 0.8$

$\therefore 0.8 \neq 2 \times 10^{-3}$ therefore events are not independent of each other.

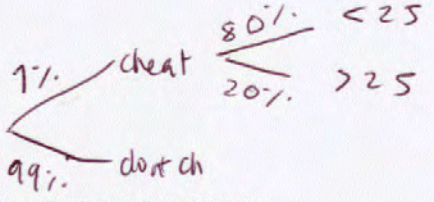
AND interpret your answer in terms of the relationship between the events 'student cheats' and 'student completes the assessment in less than 25% of the allocated time'.

mutually exclusive relationship between 'student cheats' + 'student complets assesment in less than 25% of allocated time'.

(ii) Estimate the proportion of students across New Zealand who cheat and complete the assessment for this standard in less than 25% of the allocated time.

~~80/10~~ $0.8/1 = 0.8$

	cheat	cheat'	
>25	0.80		0.20
<25			
	0.01		



(iii) Give TWO reasons why care should be taken when using this data to estimate the proportion of students being assessed for NCEA who will cheat and complete the assessment in less than 25% of the allocated time.

Reason one: This data is only taken from one school, so the sample size is not enough ~~to~~ be reliable.

Reason two: This data is only taken from one, 60 minute exam, so different subjects / exams, & different ~~timing~~ times allowed for exams would make the results not reliable for all NCEA exams.

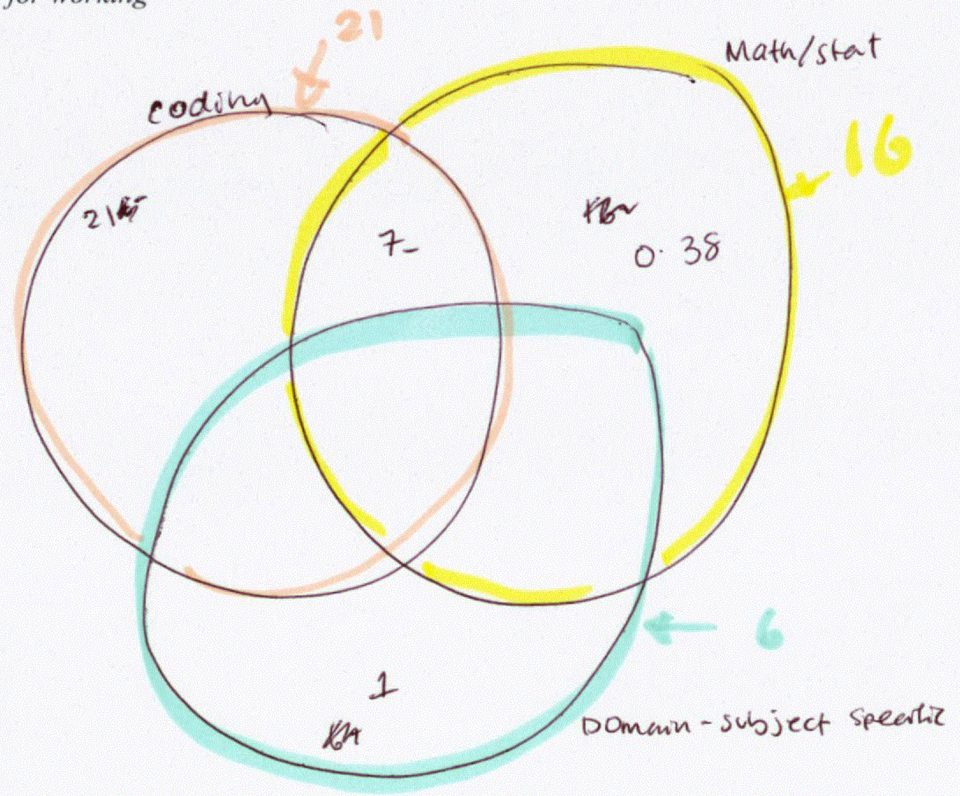
QUESTION TWO

(a) Data science is commonly thought of as the intersection of three main skill sets: coding, mathematical and statistical knowledge, and domain (or subject-specific) understanding, e.g. finance, biology, health.

From 35 applications for a data science role, where all three skill sets were desired, the following information is known:

- 7 applicants had none of the three skill sets ✓
- 21 applicants had coding skills ✓
- 16 applicants had mathematical and statistical knowledge ✓
- 6 applicants had the necessary subject-specific understanding ✓
- 9 applicants had exactly two of the three main skill sets
- 7 applicants were good candidates for the role except that they didn't have any subject-specific knowledge ✓
- 1 applicant had only subject-specific understanding ✓
- out of those with only one main skill set, $\frac{3}{8}$ had only mathematical and statistical knowledge.

space for working



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(i) A shortlist of people with all three skill sets was created.

How many applicants were selected for the shortlist?

(ii) Calculate the proportion of applicants who had only one of the required skills.

(iii) It is claimed that, for those applicants with the required mathematical and statistical knowledge, applicants are twice as likely to have subject-specific understanding compared to coding skills.

Does this data support the claim?

Use calculations and statistical reasoning to support your answer.

(b) 'In 2022, 82.4% of all 18-year-olds attained the equivalent of NCEA Level 2 or above. Of those who turned 18 in 2022, 78.3% of them attained at least NCEA Level 2 or above in school, and 4.1% of them attained at least NCEA Level 2 or above post-school in a tertiary or vocational setting.'

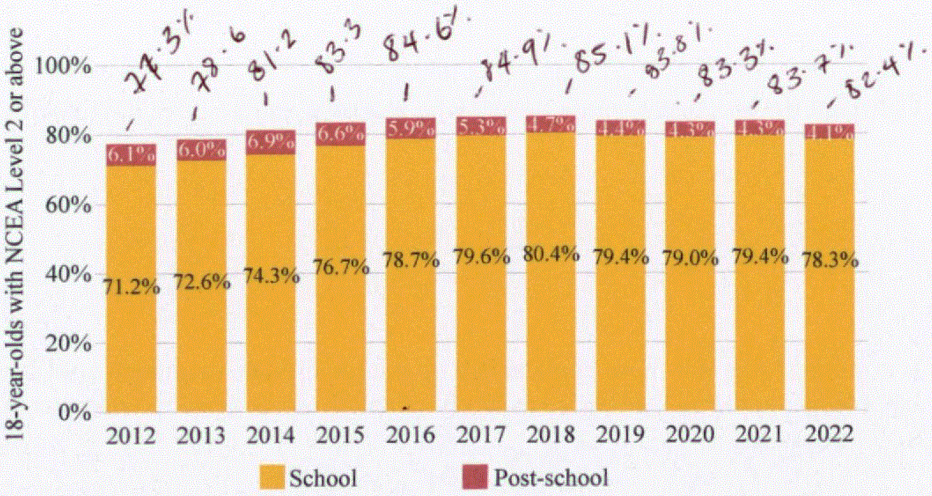


Figure 1: Most 18-year-olds with NCEA Level 2 or above attained their qualification in school

Source: www.educationcounts.govt.nz/statistics/18-year-olds-with-level-2-or-equivalent

(i) Based on the data, is the suggestion that the total proportion of 18-year-olds with NCEA Level 2 has increased between 2012 and 2022 correct?

Support your answer with statistical reasoning.

in 2012 the total proportion of 18 yr olds with NCEA L2 was ~~77.3%~~ 0.773 (77.3%). And in 2022 the total proportion of 18 yr olds with NCEA L2 was 0.824 (82.4%) this is an increase of 5.1% (0.051). However the optimum number of 18 yr olds with NCEA L2 was in 2018, when they reached 0.851 (85.1%), before dropping back down as the years increased. So yes the suggestion is correct, because the total proportion of 18 yr olds with NCEA L2 has increased by 5.1% between 2012 + 2022.

(ii) Comment on the contribution of post-school qualifications to the total proportion of 18-year-olds with NCEA Level 2 or above between 2012 and 2022.

Support your answer with statistical reasoning.

71.2%

in 2012 the ~~number~~ percentage of students who turned 18, ~~[illegible]~~ of them attained at least L2≥; however 6.1%. attained through post-School. The highest %. of students who gained L 2≥ from post-school is in 2014 with 6.9%., this increased the total number of 18 yr olds in 2014 with L2 as its total = 81.2%. In 2022 78.3%. got L2≥, with the additional 4.1%. after school, increasing their number to 82.4%.

QUESTION THREE

(a) A player plays two different games, A and B, by rolling a pair of dice.

(i) For game A, if the total of two dice rolls is between 5 and 10 (inclusive), then the player wins.

Calculate the probability that a player wins at least once when playing 3 times.

~~2/6 = 0.3331~~

PTI

2/6

$= 0.333^3$

$= 0.037$

(ii) For game B, a version of the game called Hazard is played. The rules of this game are as follows:

- When a player rolls the dice for the first time, any combination of the two dice that adds up to 7 is a winner. 6 + 7
- On the first roll, any dice total that equals 2, 3, 11, or 12 is an immediate loser.
- If the first roll has not produced a winner or a loser, the total of the dice becomes known as the point.
- For all successive rolls, the player will win a game if the point is rolled again. However, if a 7 is rolled before the point is rolled, the player loses.

Calculate the probability that the game is lost before the player has to roll the dice for a third time.

Question Three continues on the next page.

(b) The player is concerned that one of their dice is biased. The outcomes from 1000 rolls of this die are summarised in the table below.

Outcome	1	2	3	4	5	6
Totals	138	189	197	143	179	154

(i) For these 1000 rolls, which is more likely? Rolling 3 or less OR rolling 4 or more?

Support your answer with statistical calculations.

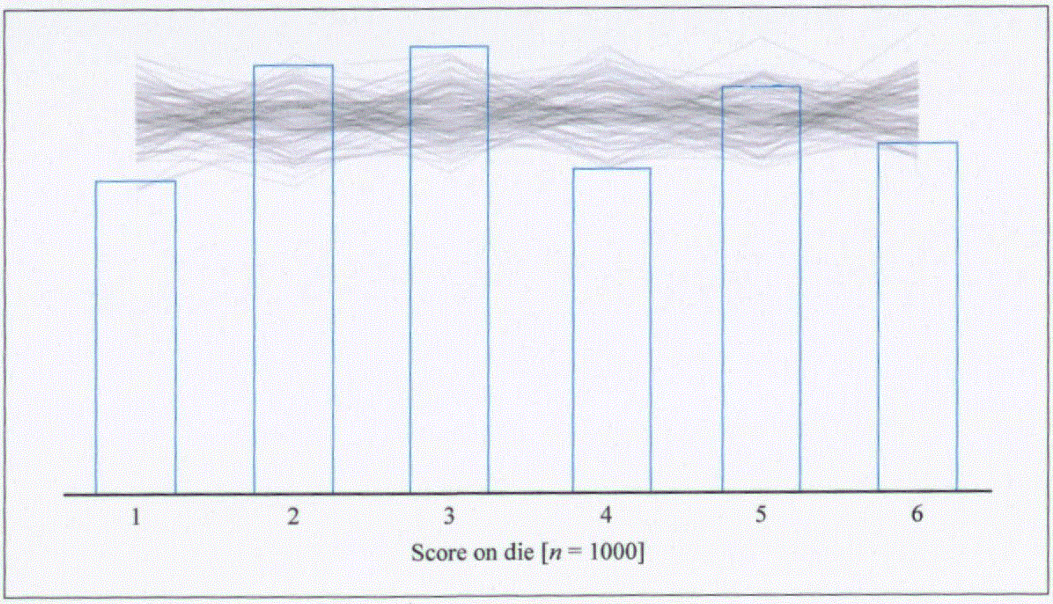
$\cancel{P(R3\geq}$ $P(R3\leq) = 524/1000 = 0.524$

$P(R4\geq) = 476/1000 = 0.476$

~~Ro~~ the probability of rolling a 3 or less is more likely than rolling a 4 or more.

The diagram below shows the results of 1000 trials of a simulation model. The simulation assumed that each outcome on the die was equally likely to occur.

The height of the blue vertical bars shows the relative frequencies of each observed digit outcome on the die, as shown in the table above. The grey band shows the variation expected for each outcome, based on simulating 1000 throws of a fair die.



(ii) Should the player be concerned that one of their dice is biased?

Use the results of the simulation model shown in the diagram on the previous page and the outcomes of the 1000 rolls given in the table, and refer to experimental, theoretical, and true probability as part of your answer.

This simulation only consists of 1000 as their sample size, this sample size is too small, & likely to ~~have~~ show not as much variation. And this isn't a fair enough test to say whether a dice is 'biased'. The dice numbers 3, 2 & 5 have similar numbers of total times they were rolled. And same for 1, 4 & 6. If they continued this simulation say 10,000 times, this would begin to show that dice, cannot be 'biased' & numbers of times certain numbers were rolled would even up.

(iii) Explain what this result means for the chance of throwing a total of 2 using this particular die as one of the pair of dice in game B.

this meaning throwing a 2 has the same change as throwing any other number

Achievement

Subject: Statistics

Standard: 91585

Total score: 12

Q	Grade score	Marker commentary
One	M5	1(b)(i) – The candidate made a partial attempt at proving the events are not independent, correctly calculating 0.01 × 0.2 = 0.002. However, they incorrectly compared this to the 0.8, which is the conditional probability rather than the intersection. 1(b)(ii) – The candidate correctly identified and explained one reason (data taken from one standard).
Two	M5	The candidate was not able to process the information given into a Venn diagram and consequently was not able to correctly answer the three parts of question 2(a). 2(b)(i) – The candidate has made a comparison of the proportions for 2012 and 2022 and concluded that the claim can be supported. They have also indicated that the change is not consistently positive year to year.
Three	N2	3(b)(i) – The candidate calculated P(3 or less) and compared it to P(4 or more) to make a correct conclusion.