No part of the candidate's evidence in this exemplar material may be presented in an external assessment for the purpose of gaining an NZQA qualification or award.

SUPERVISOR'S USE ONLY



+

91585







Mana Tohu Mātauranga o Aotearoa New Zealand Qualifications Authority

Level 3 Mathematics and Statistics (Statistics) 2024

91585 Apply probability concepts in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (<//>
(<//>
<//>
</

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.



Merit

© New Zealand Qualifications Authority, 2024. All rights reserved.

QUESTION ONE

Himen

78

Educators use writing detectors like Turnitin to detect if students have used AI (Artificial (a) Intelligence) to write their assignments. When designing writing detectors, scientists focus on accuracy; if they say AI writing is present in a piece of work, they want to be pretty certain that the work is AI-generated, to ensure that students are not falsely accused of misconduct. This means that the detector may not always detect all AI writing; some may be missed.

A false positive in AI writing detection refers to incorrectly identifying fully human-written text as AI-generated. Suppose that a particular AI writing detector has a 1% false positive rate but that 74% of the time it fails to detect AI-generated writing.

Based on a confidential survey of students in one teacher's course, it is thought that 22% of pieces of student writing contain content that is generated by AI.

- If 120 pieces of writing are screened by this AI detector, approximately how many would (i) be detected as potentially being generated by AI? Decheel Hum TONO1 Hum FA O. Altully 0 0.0 Detel Ar L Þ: (defet A.) = (6.22 × 0.26)+ 6.74 (0.78 × 0.01) 0.0 00000
 - A piece of writing is detected as potentially being generated by AI. (ii)

Comment on whether a teacher should be concerned that the student could be unfairly accused of cheating.

Support your answer with statistical reasoning.

p(Human(Ai) = 0.01×0.78 = 0.12
P(A:1A:): 0.72x0.26 =0.88
the feater shard not be too comenced a siden
TS infaithy alused for A: as given A: was defecte
ntleward, pobability it is human wrigten is anly
12, While majority of the time (0.88), given the
water Ai, the waters actually si about
0.88 porpunol tetme.

= 0.4065 x120

- Exam supervisors are sometimes concerned that students who complete tests quickly might (b) have cheated. Based on data collected from one school, for a particular NCEA standard with an allocated time of 60 minutes, it is known that:
 - 1% of students cheat on the assessment for this standard
 - 20% of students complete the assessment in less than 25% of the allocated time
 - 80% of students who cheat on the assessment complete it in less than 25% of the allocated time.
 - Comment on whether the events, 'student cheats' and 'student completes the assessment in (i) less than 25% of the allocated time', are independent of each other. Cheut Choest 02 8×10-3 less

Use statistical reasoning to support your answer.

1-1-1-1

7.7.7.7.1.

11-11-1

2 X1()-3 0.99 1 p(Chent n bess tunzer time) = 0.01 p (Cheert) xp (less trun 25% time) the protocoldility one cierto curs and Notindependentas 5 anares 0.008 = 0.01 × 0.2 depends in the other 0.008 \$0.002 Pier

Here

The events a Student agents and the event they can be asservent less then 25% of the then is not independent. AND interpret your answer in terms of the relationship between the events 'student cheats' and 'student completes the assessment in less than 25% of the allocated time'.

This news recents they areas and anglete in less then 25% of the time is not independent. The out care (probability are event happen) of are contalects re probability ofte other event occurry. So the propositionity the events occurring different gren bifferent autore office forst cent (he condition 1)

(ii) Estimate the proportion of students across New Zealand who cheat and complete the assessment for this standard in less than 25% of the allocated time.

Planent newsster 25%) = 0.008

Mathematics and Statistics (Statistics) 91585, 2024

(iii) Give TWO reasons why care should be taken when using this data to estimate the proportion of students being assessed for NCEA who will cheat and complete the assessment in less than 25% of the allocated time.

Reason one: This data is taken from an school any Units B not representative of cull NCEA stutents. This School my have mae translessare students or less adequile Cheers presentions affectly their properties of Students who are a properties of the properties of Students who Reason two: The properties to lime to ingerent ay NCEA Students reproprise to lime to ingerent ay NCEA Students report to cheering, ar trut standed mybe different Jecuted and behavaler a years standard mae screets feel pressent to Cheert to gain UE, so put affects the proprism 25% of the the.

Mathematics and Statistics (Statistics) 91585, 2024

QUESTION TWO

(a) Data science is commonly thought of as the intersection of three main skill sets: coding, mathematical and statistical knowledge, and domain (or subject-specific) understanding, e.g. finance, biology, health.

From 35 applications for a data science role, where all three skill sets were desired, the following information is known:

- 7 applicants had none of the three skill sets
- 21 applicants had coding skills
- 16 applicants had mathematical and statistical knowledge
- 6 applicants had the necessary subject-specific understanding
- 9 applicants had exactly two of the three main skill sets
- 7 applicants were good candidates for the role except that they didn't have any subject-specific knowledge
- 1 applicant had only subject-specific understanding
- out of those with only one main skill set, $\frac{3}{8}$ had only mathematical and statistical knowledge.



(i) A shortlist of people with all three skill sets was created.

How many applicants were selected for the shortlist?

(ii) Calculate the proportion of applicants who had only one of the required skills.

16/35 = 0.457

(iii) It is claimed that, for those applicants with the required mathematical and statistical knowledge, applicants are twice as likely to have subject-specific understanding compared to coding skills.

Does this data support the claim?

Use calculations and statistical reasoning to support your answer.

P(P|M) = P(C|M)Leclamis . not supply twire as likey.

(b) 'In 2022, 82.4% of all 18-year-olds attained the equivalent of NCEA Level 2 or above. Of those who turned 18 in 2022, 78.3% of them attained at least NCEA Level 2 or above in school, and 4.1% of them attained at least NCEA Level 2 or above post-school in a tertiary or vocational setting.'





Source: www.educationcounts.govt.nz/statistics/18-year-olds-with-level-2-or-equivalent

(i) Based on the data, is the suggestion that the total proportion of 18-year-olds with NCEA Level 2 has increased between 2012 and 2022 correct?

Support your answer with statistical reasoning.

Base to redate it appears that the feter proprious of 18 year olds with reel or above here incessed (000 0.773 in 20/2 10 90.824 in 202). Hover, re rophinof18 year Olds u. In Just NCEA erel 2 is minumed as it's possible vents obtained terran a qualification erel 2 but not level 2 culhere, e proprim of 100 year olds with ne me menerse. But ne dent Zaulif e cresust here. SO derter . SIM

Mathematics and Statistics (Statistics) 91585, 2024

 (ii) Comment on the contribution of post-school qualifications to the total proportion of 18-year-olds with NCEA Level 2 or above between 2012 and 2022.

Support your answer with statistical reasoning.

QUESTION THREE

- (a) A player plays two different games, A and B, by rolling a pair of dice.
 - For game A, if the total of two dice rolls is between 5 and 10 (inclusive), then the player wins.

Calculate the probability that a player wins at least once when playing 3 times.

5,6,7,8,9,10 (1,4) (and) (1,5)(50) (2,3) (50) (4,2) (20) 8= (2,6), (3,5), (4,4) (1,6) (2,5)(3,4) 9=(4;5)(6,3) 10 - (5, 5)(6, 4)25 possibilities town $\frac{25}{36}$ che per sueto $\frac{11}{36}$ win $1(attentonce) = 1 - (\frac{11}{36})^3$ 3 7 3 (8 4 2 3 a 0.972 q 5

Mathematics and Statistics (Statistics) 91585, 2024

follows:

adds up to 7 is a winner.

•

• On the first roll, any dice total that equals 2, 3, 11, or 12	e is an immediate loser.
• If the first roll has not produced a winner or a loser, the known as the point.	total of the dice becomes
• For all successive rolls, the player will win a game if the However, if a 7 is rolled before the point is rolled, the p	e point is rolled again. layer loses.
Calculate the probability that the game is lost before the player third time. 34 - 36 - 36 - 36 - 36 - 36 - 36 - 36 -	er has to roll the dice for a $5 \times (\frac{6}{36}) = \frac{5}{36}$
$\frac{6}{36} + \frac{5}{36} = 0.3056$	
	Question Three continues on the next page.
Mathematics and Statistics (Statistics) 91585, 2024	03902

For game B, a version of the game called Hazard is played. The rules of this game are as

When a player rolls the dice for the first time, any combination of the two dice that

- (b) The player is concerned that one of their dice is biased. The outcomes from 1000 rolls of this die are summarised in the table below.

Outcome	1	2	3	4	5	6
Totals	138	189	197	143	179	154

(i) For these 1000 rolls, which is more likely? Rolling 3 or less OR rolling 4 or more?

Support your answer with statistical calculations. 30000 your answer with statistical calculations. 3 cr (e85: 524 4 cr mere: 476 = 0.524 i. Fertere (000 roll6, meliter, ford) 3 cr legs as 0.524 > 0.476.

The diagram below shows the results of 1000 trials of a simulation model. The simulation assumed that each outcome on the die was equally likely to occur.

The height of the blue vertical bars shows the relative frequencies of each observed digit outcome on the die, as shown in the table above. The grey band shows the variation expected for each outcome, based on simulating 1000 throws of a fair die.



(ii) Should the player be concerned that one of their dice is biased?

Use the results of the simulation model shown in the diagram on the previous page and the outcomes of the 1000 rolls given in the table, and refer to experimental, theoretical, and true probability as part of your answer.

The Similar vets the plyence versation in the results of rollingeen dice. The Similarian is made assurgeen dice has the keoreevical probability of beigrolled is 16. However, the similarly Shins only differential of the numer rolled (its Pequeng) Hissert Outside the expected versation (grey bails) which assues the experimental probability will full charrenge close to the Remotul probabous of 16. However, the expended probability for the dive being rolled is 3 is likely abare 16 as it free box (begrey of 3 beig rolled) is abare the expected probability box (begrey of 3 beig rolled) is abare the expected probability box bottom of he expected version to grey back box the recy bottom of he expected version of the statistical in the version of the statistical probability for the received bottom of he expected version of the statistical indications and we cononly estimate and menerties trugh simulatus and made

(iii) Explain what this result means for the chance of throwing a total of 2 using this particular die as one of the pair of dice in game B.

The begins of getty 2 very tris die is the Indhighest, after 3. This mens thue of getty cut for gove B is higher as the possible articles of getty aut is willy (2,1)(1,2) and points of 7, so (2,5)(5,2). This, are Objetty the out one trut results in cutose menses

	Extra space if required.	
QUESTION NUMBER	Write the question number(s) if applicable.	
	$(1)(1)(4)\neq 6$	
	(0)2)	
	~~~	
	$(10f)_{3}+b$	
	(9))	
	230	





# Merit

## Subject: Statistics

### Standard: 91585

### Total score: 17

Q	Grade score	Marker commentary
One	E7	1(a)(ii) – While the candidate correctly calculated that for those pieces of work that are indicated as being AI generated, 12% are produced by a human, they needed to make a correct comment that the teacher should be concerned to get the answer fully correct.
		1(b)(i) – the candidate correctly interpreted the given information and was able to use it to prove, in context, the events were not independent using the rule $P(A \cap B) = P(A) \times P(B)$ if A and B are independent. To gain the excellence grade they needed a clearer discussion of the relationship between the two events.
		1(b)(iii) – both reasons were identified and clearly explained in context.
Two	M5	The candidate was able to partially complete the Venn diagram in order to correctly answer 2(a)(i) and 2(a)(ii).
		2(b)(i) – The candidate has made a comparison of the proportions for 2012 and 2022 and concluded that the claim can be supported. To get the merit grade, they also needed to indicate that the change is not consistently positive year to year.
		2(b)(ii) – While the candidate compared the post-school proportions for 2012 and 2022 and stated that they have decreased, they needed to calculate the <b>relative</b> proportion and compare them to get the answer fully correct.
Three	M5	3(a)(i) – While the candidate had the correct approach to the problem, they missed counting a couple of the outcomes, getting 25/36 rather than 27/36.
		39a)(ii) – The probability of losing on the first roll was correctly calculated as 1/6 but the outcomes for losing in the second roll was not correctly calculated giving an incorrect total.
		3(b)(ii) – The candidate provided a discussion of how the simulation results allow one to compare what was observed with what was expected based on the theoretical probabilities. They needed to make a clear decision as to whether the candidate should be concerned that the die is biased.