National Certificate of Educational Achievement

2011 Assessment Report

Mathematics with Calculus Level 3
Statistics and Modelling Level 3

90635 Differentiate functions and use derivatives to solve problems

90636 Integrate functions and use integrals to solve problems

90638 Manipulate real and complex numbers, and solve problems

90639 Sketch graphs of conic sections and write equations related to conic sections

90642 Calculate confidence intervals for population parameters

90643 Solve straightforward problems involving probability

90644 Solve equations

90646 Use probability distribution models to solve straightforward problems

COMMENTARY

The 2011 exam followed the format of the two previous years. Each paper consisted of two multiple part questions with each question providing the opportunity for all grades of performance.

Successful candidates made the most of these opportunities. They also demonstrated good algebraic skills. The ability to differentiate (or integrate) alone is not sufficient to pass the differentiation (or integration) standards. Algebra is important in the manipulation of expressions into forms appropriate for differentiating or integrating. It is essential in solving a problem once the differentiation or integration step has been carried out.

Many candidates, in attempting to answer questions using graphics calculators, first did the required analytical steps, demonstrating sound understanding of what is required.

STANDARD REPORTS

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- found derivatives (including with the use of the chain rule)
- differentiated using the quotient rule (or rewrote the given function as a product and then used the product rule)
- found the gradient of a curve at a point on the curve
- found a rate of change.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not recognise tan2*x* as either a function of a function or a product for the purposes of differentiation
- rewrote a quotient as a product but did not use the product rule correctly
- did not answer questions fully e.g. differentiated but did not find the gradient at the specified point
- answered two or three of the first parts of Question One (“differentiate”) but did not answer either of the first two parts of Question Two (“solve problems / apply differentiation”).

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- differentiated a function implicitly
- differentiated a function from first principles
- found the conditions under which a function was maximised and answered the question fully (as they were asked)
- showed evidenc of having recognised the values of the independent variable for the graph of a function in relation to characteristics related to differentiation, and interpreted an aspect of the graph

- used mathematical symbolism appropriately.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- solved complex problems involving derivatives
- worked systematically and logically
- showed their reasoning clearly
- used mathematical symbolism and terminology correctly.

OTHER COMMENTS

Most candidates demonstrated sound differentiation skills but, as in previous years, a number found difficulty applying differentiation correctly to solve problems. The difficulty was usually in solving the algebraic problem resulting from the differentiation. Sound algebraic skills remain the main prerequisite for success in this subject.

In Question One (d), algebraic errors were common. Amongst the most common were the expansion of $-3(x + h)^2$ to contain the terms $+6xh$, and / or $+h^2$ or $+3h^2$, and errors in signs associated with the expansion of the term $-(5 - 3x^2)$.

Question One (e) was skipped by a significant number of candidates, including some who did well on the rest of the paper. Of those who attempted the question, a number of candidates used the distance between L and Motunui as the variable, making the working a bit more difficult. A few candidates used the distance between the oil well and L, which was not as problematic.

In Question Two (d), a number of candidates reversed the inequality signs. Also, a number of candidates gave discrete (integer) values of t in the cases involving an interval.

90636 Integrate functions and use integrals to solve problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- integrated trigonometric, exponential and surd expressions and $1/x$ correctly
- correctly manipulated and rearranged algebraic expressions
- showed evidence of having remembered the constant of integration and did not assume it was 0
- substituted correctly into Simpson's rule
- calculated distance from a velocity expression
- set their calculator in radians.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not integrate trigonometric expressions such as $\sin 2x$
- did not simplify rational expressions before integrating
- used the trapezium rule instead of Simpson's rule

- did not calculate the value of h in Simpson's rule
- did not manipulate an algebra expression into a form that could be integrated
- differentiated part or all of an expression instead of integrating
- did not manipulate negative signs correctly
- did not calculate numeric expressions correctly.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- integrated by substitution correctly, in particular changing from *dx* to *du*
- separated variables correctly
- recognised that 99% gone is the same as 1% remaining
- found a volume of revolution around the correct axis
- successfully integrated differential equations
- rearranged complicated algebraic expressions into the form required to solve the problem and correctly changed the subject of a formula.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- visualised complicated 3D problems correctly
- successfully carried through a long chain of mathematical reasoning
- demonstrated good algebraic skills and expanded and integrated surds correctly
- made up an equation needed from a given scenario and integrated it successfully
- found the volume of revolution about lines parallel to an axis.

OTHER COMMENTS

Lack of fundamental algebra skills was the most common impediment to achieving success.

Many candidates wrongly assumed that c = 0 in a question such as Question Two (b). The initial value will not always be 0, especially if exponential or trigonometric equations are involved.

In Question One (d), a number of candidates incorrectly used the limits from the x-axis although they were rotating around the y-axis.

The great majority of candidates used correct analytical methods to integrate and solve the problems. Few candidates attempted to circumvent the questions by using graphics calculators then trying to 'fudge' their answers by going back and filling in the gaps.

90638 Manipulate real and complex numbers, and solve problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- manipulated and simplified surds
- simplified fractions by multiplying by the conjugate
- applied the Remainder Theorem

- used long division to find the quotient and remainder
- multiplied complex numbers in polar form
- added arguments that involved fractions of π
- multiplied moduli that involved parameters
- solved an equation by completing the square or using the quadratic formula
- showed understanding that an instruction asking for exact values meant that the answer had to involve surds simplified fully
- applied the simple logarithmic rules when solving both log and exponential equations
- changed numbers from rectangular to polar form
- substituted into $\frac{1}{z} + \frac{2}{\bar{z}}$ using $z = x + iy$ and simplify the resulting fraction.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not manipulate surds without making arithmetic errors
- did not understand the concept of a remainder when conducting polynomial division
- made careless numeric errors when applying algebraic long division to find the remainder
- did not demonstrate knowledge of having to multiply the moduli and add the arguments when multiplying complex numbers expressed in polar form
- showed a lack of sufficient understanding of fractions to simplify their answer when adding the arguments
- made careless errors when using the quadratic formula to solve a quadratic equation
- did not express their solutions for the quadratic equation as a simplified surd
- used their graphic calculators to find approximate rather than exact solutions for the quadratic equation
- did not show understanding of the logarithm rule: $\log a^n = n\log a$ sufficiently well to be able to work with $\ln 2e^5$
- did not use the natural logarithm appropriately when solving an exponential equation
- did not change a simple purely imaginary number expressed in pronumerals, *n*i, from rectangular to polar form
- did not show understanding of the symbolism for the conjugate.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- changed the subject of an equation involving surds
- simplified algebraic fractions and also set out their algebraic steps with clarity and precision
- showed understanding of how to apply de Moivre's Theorem when finding roots of an equation of the form: $z^n = a$ when the rectangular form of the complex number involved a pronumeral.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- communicated in a logical, clear, and precise algebraic manner
- applied the Factor Theorem and equated coefficients involving pronumerals
- applied algebraic long division in a situation involving parameters
- manipulated rational expressions by using the conjugate and equating terms
- solved simultaneous equations using substitution in a situation involving the intersection of two circles.

OTHER COMMENTS

The paper was constructed in a way that meant that the Question One Achievement level questions were intended to account for a candidate's ability to manipulate real and complex numbers and Question Two Achievement level questions were intended to account for a candidate's ability to solve equations. Candidates found Question One much more accessible than Question Two.

Many candidates hashed their answers by trying to give solutions in both rectangular and polar form when the instructions asked specifically for answers to be written in polar form.

90639 Sketch graphs of conic sections and write equations related to conic sections.

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- recognised the form of a graph from its equation and drew the graph
- showed understanding of the meaning of asymptote and showed this on their graph
- drew enough of a graph to clearly show all important features
- showed understanding of parametric form and how this could be used to sketch graphs or write equations
- completed the square to obtain the equation of a circle
- used the "a" and "b" values in the equations given to locate the critical points on the graphs.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- relied on a graphing calculator to produce partial graphs of conics
- did not check the form of equations carefully
- did not draw enough of the graph being attempted to show understanding of its shape
- showed a lack of basic understanding of algebraic manipulation techniques – for instance, by attempting to take the square root of whole equations.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- combined their knowledge of Conics with other areas of mathematics – for example, basic differentiation techniques and Pythagoras' Theorem to solve problems.
- showed understanding of when to substitute a specific point into a gradient function for the gradient of a tangent or normal
- showed understanding of implicit differentiation and completed it correctly with the necessary algebra to represent the derivative correctly.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- understood the chain and product rules for differential calculus
- were able to think holistically to form and equate gradients
- were confident with using indices, fractions and surds in their working
- demonstrated a logical chain of reasoning and had sufficient algebraic skills to complete a manipulation correctly.

OTHER COMMENTS

This standard, particularly at Excellence level, relies on algebraic and differentiation skills. Candidates who understood the process of implicit differentiation were at an advantage, although errors with rearrangement or constant differentiation were common. The alternative for candidates, to rearrange the equation into "y =" form, often was done incorrectly or the differentiation was incorrect.

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2011 Assessment Report

Statistics and Modelling Level 3

90642 **Calculate confidence intervals for population parameters**

90643 **Solve straightforward problems involving probability**

90644 **Solve equations**

90646 **Use probability distribution models to solve straightforward problems**

COMMENTARY

There has been a continuing trend towards the increased use of graphical calculators across all standards. This has resulted in candidates having the ability to appropriately manipulate the technology to an achieved standard. Achievement with Merit and Achievement with Excellence have proved more elusive as at this level the requirement is for understanding of the concepts involved in Level 8 of the New Zealand curriculum in Statistics. Understanding is not necessarily formulaic and, therefore, has been difficult for some candidates.

Candidates at this level are exhibiting errors in basic number skills and knowledge (rounding, BEDMAS, calculator usage, fractions).

STANDARD REPORTS

90642 Calculate confidence intervals for population parameters

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- showed understanding of and correctly identified at least two of the three types of confidence intervals
- calculated the answer to the given question correctly, using the appropriate confidence interval formula and notation, to at least two significant figures
- wrote the confidence interval using appropriately accepted conventions
- used or calculated correct z-scores.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not use their calculators (graphical or scientific) effectively to yield an appropriate solution
- did not read questions carefully
- gave more than one answer to a question
- showed a lack of understanding of the concept of ‘proportion’ being a number between 0 and 1
- did not select or calculate confidence intervals using correct z-scores or ignored or left out the z-score in calculations
- prematurely rounded intermediate working.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- explained in context what the difference between two means confidence intervals indicates
- recognised that a minimum sample size should be a whole number

- showed understanding of the concept and purpose of confidence intervals
- gave reasons for insufficient evidence for a statistical conclusion based on a calculated confidence interval and interpreted confidence intervals correctly
- rearranged given equations to find an unknown(s).

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- applied the principles underpinning the central limit theorem
- communicated (including diagrammatically) what they were doing in intermediate steps – showing logic and reasoning and statements connected to calculations
- demonstrated understanding of calculating the parameters for the distribution of sample totals and the effect that the change in the confidence level had on the size and the boundaries of the interval
- showed a good understanding of the usefulness and relevance of confidence intervals
- exhibited the ability to process information and to logically complete an extended chain of reasoning
- applied confidence interval and normal distribution knowledge to analysing a practical issue.

OTHER COMMENTS

Successful candidates were more responsive to validating claims based on confidence intervals in the context of the situation. Poor written communication skills by some candidates when explaining or interpreting of the confidence interval did not allow them to show that they understood how to interpret a confidence interval e.g. “values do not pass through zero”, “2.95 is outside the interval”.

A number of candidates spoiled good responses by trying to write too much and then writing a comment that showed a lack of understanding or contradicted earlier statements made.

A lack of working is an inhibiting factor for attainment of the Merit and Excellence grades.

90643 Solve straightforward problems involving probability

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- calculated the expected value from a table
- attempted multiple parts of the questions
- modelled data using Venn Diagrams
- determined the independence of two events
- recognised situations involving conditional probability.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- misread questions and calculated the wrong parameter
- confused situations that involved conditional probability
- found difficulty in modelling data using the most appropriate technique
- showed evidence of having misunderstood basic probability concepts e.g. probabilities not being greater than 1.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- selected the appropriate probability technique to model a situation
- applied combinatorial mathematics correctly
- set out a logical argument e.g. proving two events were independent
- used the formula for conditional probability appropriately

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- constructed a probability distribution table to represent a situation
- showed a clear understanding of expected value
- used problem solving techniques accurately and recognised that not everything has a tidy solution

OTHER COMMENTS

Many candidates did not understand the concept of E(X) as a mean and not an exact value. The reality of the answer should be a thought that underpins all the candidate work.

90644 Solve equations

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- solved simultaneous equations (either on a graphical calculator or algebraically)
- accurately substituted numbers into a formula (for NR or Bisection) on their calculator
- answered questions in context
- used Newton-Raphson method correctly for two iterations
- graphed simple inequalities
- solved a simple linear programming problem.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not differentiate correctly
- did not relate a solution to a system of equations in context
- did not use the Newton-Raphson formula as asked, or correctly did only one step of the N-R method
- did not read the correct coordinates from their linear programming graph
- did not show their working.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- formed and solved simultaneous equations in context from a word problem
- solved an equation correctly, using an appropriate method, to a given rounding
- had good graphic calculator skills.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- recognised and described the geometrical solution to the given set of three equations
- outlined an advantage and disadvantage of using the bisection method
- optimised a difficult set of constraints to solve a linear programming problem
- displayed good mathematical communication skills

OTHER COMMENTS

Candidates should always check solutions and use correct rounding at the appropriate time. Candidates who didn't have graphical calculators were disadvantaged in this topic.

90646 Use probability distribution models to solve straightforward problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- recognised different distributions and calculated simple probabilities
- attempted only the first parts of the questions

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- seemed to know only how to apply the normal distribution
- did not show understanding of the meaning of “at least”.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- combined probabilities to answer a problem
- worked with all three distributions
- knew the meaning of a quartile and could calculate the inverse normal
- solved inverse Poisson problems.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- attempted all of the paper
- provided clear working and gave their answers in context.